IEEE Signal Processing Society
Distinguished Lecture

Sparse Sensor Array Processing for High-Resolution Sensing

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Advanced
Signal
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Laboratory

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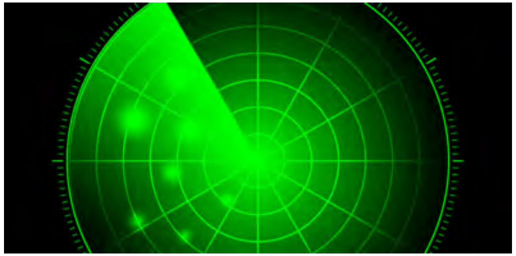
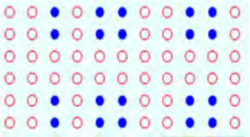


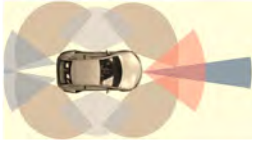
Sparse Sensor Array Processing for High-Resolution Sensing

Introduction

Array Signal Model and DOA Estimation



Sparse Array Design and Processing



New Directions and Applications

Concluding Remarks

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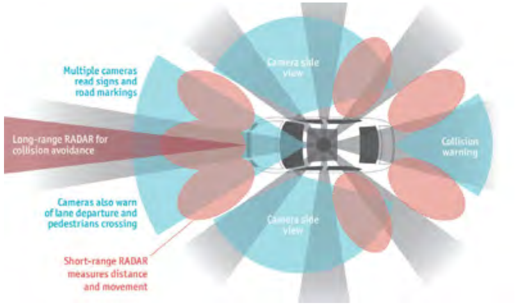
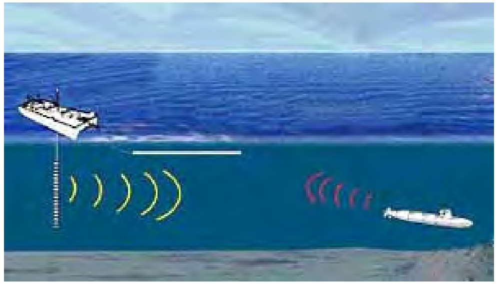
Applications of array processing

- Array processing uses multiple sensors (antennas, microphones, transducers) and plays a fundamental role in wireless communications, radar and sonar sensing, autonomous driving, speech separation, and medical imaging
- **Beamforming**
    - Signal enhancement
    - Interference cancellation
    - Multi-user detection
    - Multiple-input multiple-output (MIMO) systems
    - Increased channel capacity
- **Sensing: Localization/imaging**
    - Ground-to-air radar
    - Automotive radar
    - Sonar
    - Ultrasonic imaging

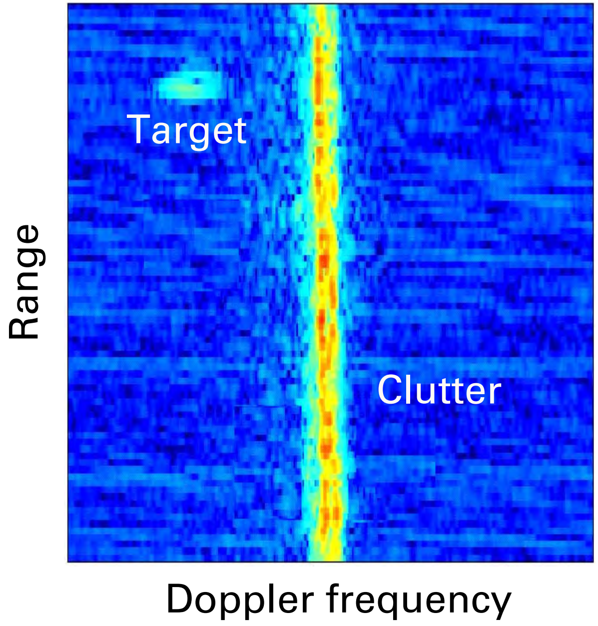




Four-dimensional sensing

Radar sensing often requires high-resolution results in four dimensions (**4-D imaging**):

- **Range:** range resolution and accuracy are determined by signal bandwidth
- **Doppler frequency:** corresponding to radial velocity with its resolution determined by pulse repetition frequency
- **Azimuth angle**
- **Elevation angle**



This talk focuses on the angle estimation problem:

- **Angular resolution** is determined by the array aperture
- Number of detectable signals is determined by the number of **degrees of freedom (DOFs)** which is related to the number of sensors
- Mainly consider azimuth angle estimation using linear arrays
- Extension to 2-D array for 2-D direction-of-arrival (DOA) estimation

Signal model for a ULA

Consider an N-element uniform linear array (ULA) with inter-element spacing d.

Time delay for far-field signals: $\tau = (d \sin\theta)/c$

c: speed of propagation

Phase delay: $\phi = 2\pi f_c \tau = 2\pi f_c d \sin\theta / c = 2\pi(d/\lambda)\sin\theta$

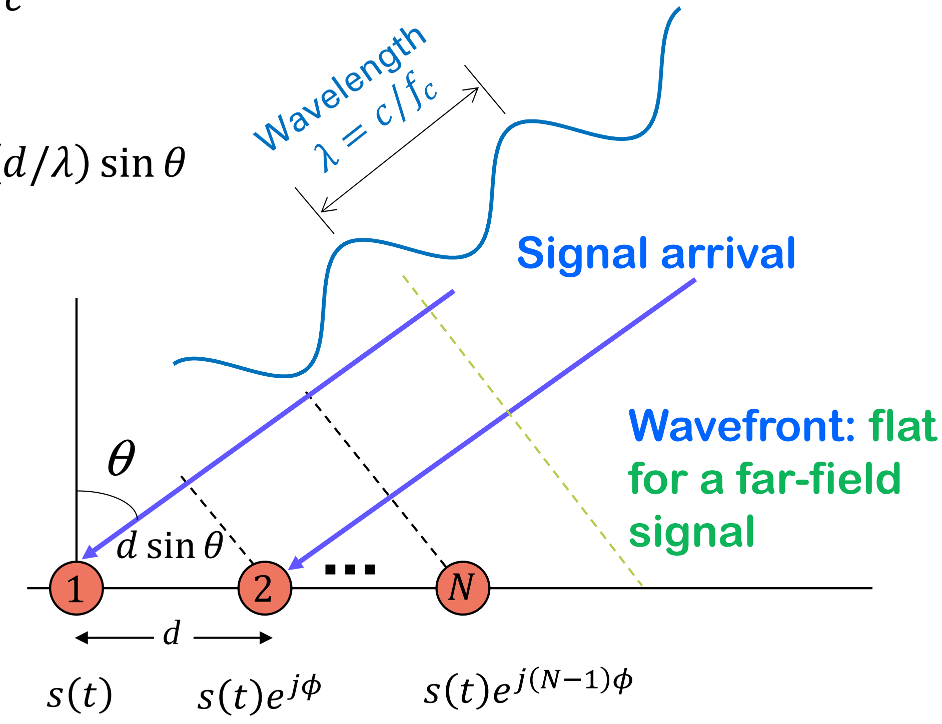
f_c: carrier frequency

$\lambda = c/f_c$: wavelength

Received signal vector under the narrowband signal assumption $s(t-\tau) \approx s(t)$:

$$\boldsymbol{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_N(t) \end{bmatrix} = s(t) \begin{bmatrix} 1 \\ e^{j\phi} \\ \vdots \\ e^{j(N-1)\phi} \end{bmatrix} = s(t)\boldsymbol{a}(\theta)$$

$\boldsymbol{a}(\theta)$: steering vector



$\phi = 2\pi(d/\lambda)\sin\theta$

Signal model for a ULA

Array signal model in the presence of K stationary signals

$$\boldsymbol{x}(t) = \sum_{k=1}^{K} s_k(t)\boldsymbol{a}(\theta_k) + \boldsymbol{n}(t) = \boldsymbol{A}\boldsymbol{s}(t) + \boldsymbol{n}(t)$$

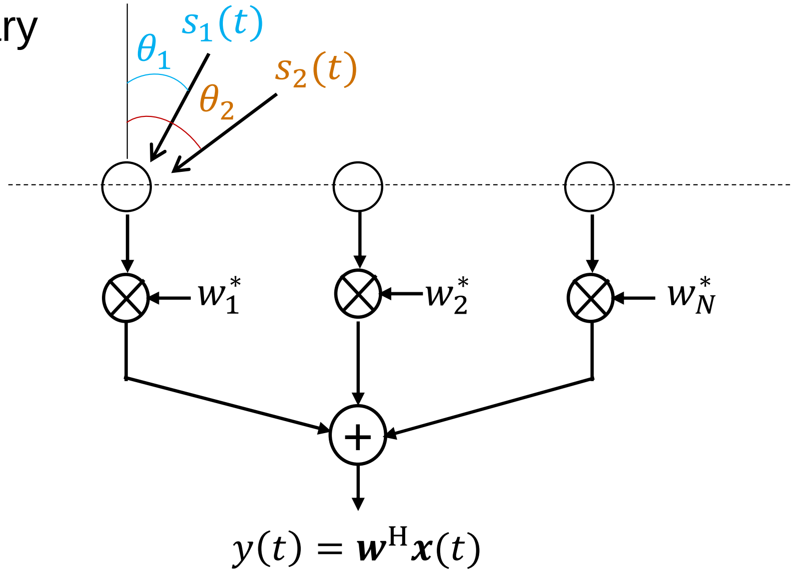
where

$\boldsymbol{a}(\theta_k) = \left[1, \mathrm{e}^{j\phi_k}, \mathrm{e}^{j2\phi_k}, \cdots, \mathrm{e}^{j(N-1)\phi_k}\right]^{\mathrm{T}}$: steering vector

$\phi_k = 2\pi(d/\lambda)\sin\theta_k$: phase delay between adjacent sensors

$\mathbf{A} = [\boldsymbol{a}(\theta_1), \boldsymbol{a}(\theta_K), \cdots, \boldsymbol{a}(\theta_K)]$: array manifold matrix

$\boldsymbol{s}(t) = [s_1(t), \cdots, s_K(t)]^{\mathrm{T}}$: signal vector



Beamforming for signal enhancement / interference cancellation: apply a weight vector $\boldsymbol{w}$ to $\boldsymbol{x}(t)$ such that $|\boldsymbol{w}^{\mathrm{H}}\boldsymbol{a}(\theta_d)|$ for some desired signal takes a high value and $|\boldsymbol{w}^{\mathrm{H}}\boldsymbol{a}(\theta_i)|$ for some interference signals takes a small value

DOA estimation: determine the directions of signal arrivals, $\theta_1, \cdots, \theta_K$, from the received signal vector $\boldsymbol{x}(t)$ over (typically) multiple samples $t = 1, \cdots, T$.

DOA estimation

Traditional DOA estimation approach through beamforming (~Fourier transform):

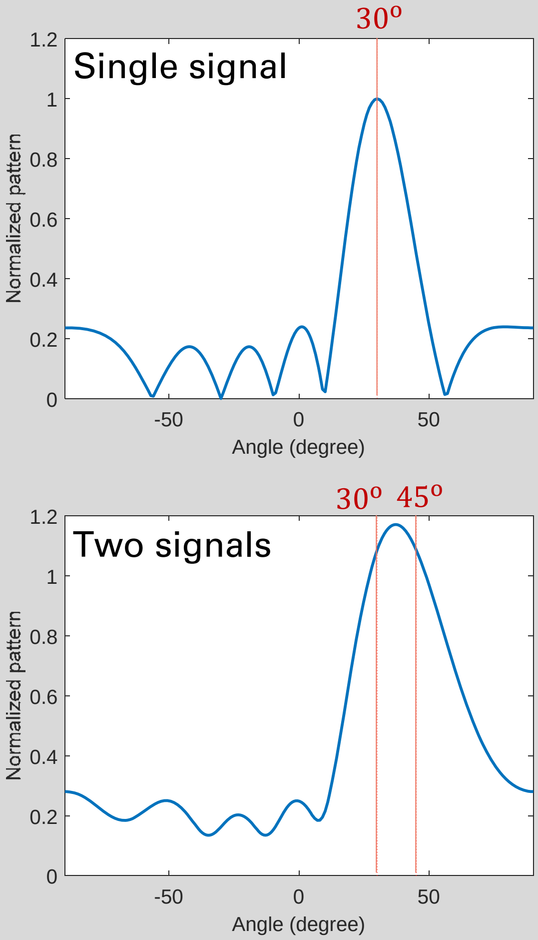
- For $\mathbf{x}(t) = s(t)\mathbf{a}(\theta_0)$, the magnitude of $y(t,\theta) = \boldsymbol{a}^{\mathrm{H}}(\theta)\boldsymbol{x}(t) = s(t)\boldsymbol{a}^{\mathrm{H}}(\theta)\boldsymbol{a}(\theta_0)$ is peaked at θ_0.
- This approach (~Fourier transform) has a low resolution.

Subspace-based DOA estimation techniques based on the subspace analysis of the covariance matrix are commonly used to achieve a high resolution.

$$\boldsymbol{R}_{\boldsymbol{xx}} = \mathrm{E}[\boldsymbol{x}(t)\boldsymbol{x}^{\mathrm{H}}(t)]$$

$$= \begin{bmatrix} \mathrm{E}[x_1(t)x_1^*(t)] & \mathrm{E}[x_1(t)x_2^*(t)] & \mathrm{E}[x_1(t)x_3^*(t)] & \mathrm{E}[x_1(t)x_4^*(t)] \\ \mathrm{E}[x_2(t)x_1^*(t)] & \mathrm{E}[x_2(t)x_2^*(t)] & \mathrm{E}[x_2(t)x_3^*(t)] & \mathrm{E}[x_2(t)x_4^*(t)] \\ \mathrm{E}[x_3(t)x_1^*(t)] & \mathrm{E}[x_3(t)x_2^*(t)] & \mathrm{E}[x_3(t)x_3^*(t)] & \mathrm{E}[x_3(t)x_4^*(t)] \\ \mathrm{E}[x_4(t)x_1^*(t)] & \mathrm{E}[x_4(t)x_2^*(t)] & \mathrm{E}[x_4(t)x_3^*(t)] & \mathrm{E}[x_4(t)x_4^*(t)] \end{bmatrix}$$

Beamforming-based DOA estimation has a low resolution (Example of 6-element ULA)



Subspace-based DOA estimation

Eigen-decomposition of the covariance matrix

$$\boldsymbol{R}_{\boldsymbol{xx}} = \underbrace{\sum_{i=1}^{K} \lambda_i \boldsymbol{v}_{\boldsymbol{i}} \boldsymbol{v}_{\boldsymbol{i}}^{\mathrm{H}}}_{\text{Signal subspace}} + \underbrace{\sum_{i=K+1}^{N} \sigma_n^2 \boldsymbol{v}_{\boldsymbol{i}} \boldsymbol{v}_{\boldsymbol{i}}^{\mathrm{H}}}_{\text{Noise subspace}} = \boldsymbol{U}_{\boldsymbol{s}} \boldsymbol{\Sigma}_{\boldsymbol{s}} \boldsymbol{U}_{\boldsymbol{s}}^{\mathrm{H}} + \boldsymbol{U}_{\boldsymbol{n}} \boldsymbol{\Sigma}_{\boldsymbol{n}} \boldsymbol{U}_{\boldsymbol{n}}^{\mathrm{H}}$$

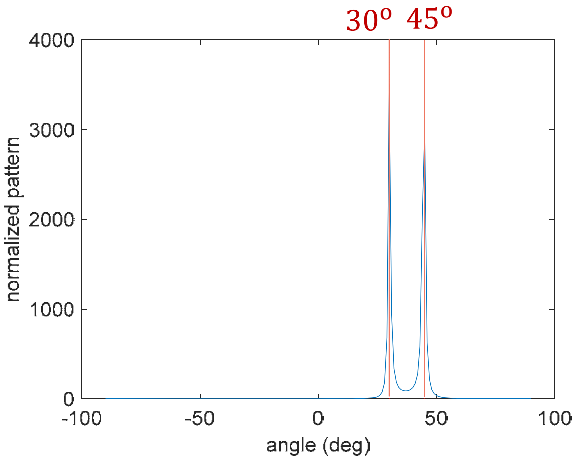
Observations:

- The signal subspace and the noise subspace are orthogonal: $\boldsymbol{U}_{\boldsymbol{s}}^{\mathrm{H}} \boldsymbol{U}_{\boldsymbol{n}} = \boldsymbol{0}$.
- Valid signal steering vectors are orthogonal to the noise subspace: $\boldsymbol{A}^{\mathrm{H}} \boldsymbol{U}_{\boldsymbol{n}} = \boldsymbol{0}$.

Pseudo spatial spectrum of **MUSIC** (MUltiple SIgnal Classification):

$$P(\theta) = \frac{1}{\boldsymbol{a}^{\mathrm{H}}(\theta) \boldsymbol{U}_n \boldsymbol{U}_n^{\mathrm{H}} \boldsymbol{a}(\theta)} = \left\| \boldsymbol{a}^{\mathrm{H}}(\theta) \boldsymbol{U}_n \right\|^{-2}$$

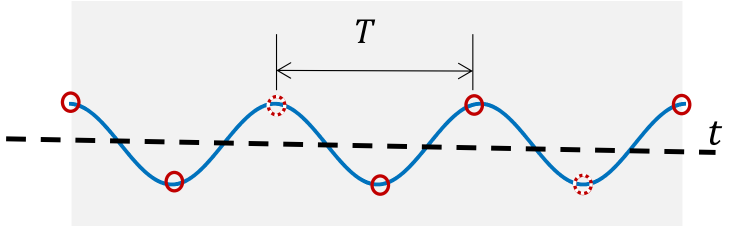
- MUSIC is popular because only 1-D search is needed.
- An N-element ULA can detect $N-1$ signals.
- Knowledge of the number of signals K is required.



R. O. Schmidt, "Multiple emitter location and signal parameter estimation," *IEEE Trans. Antennas and Propagation*, 1986.

Uniform and sparse sampling

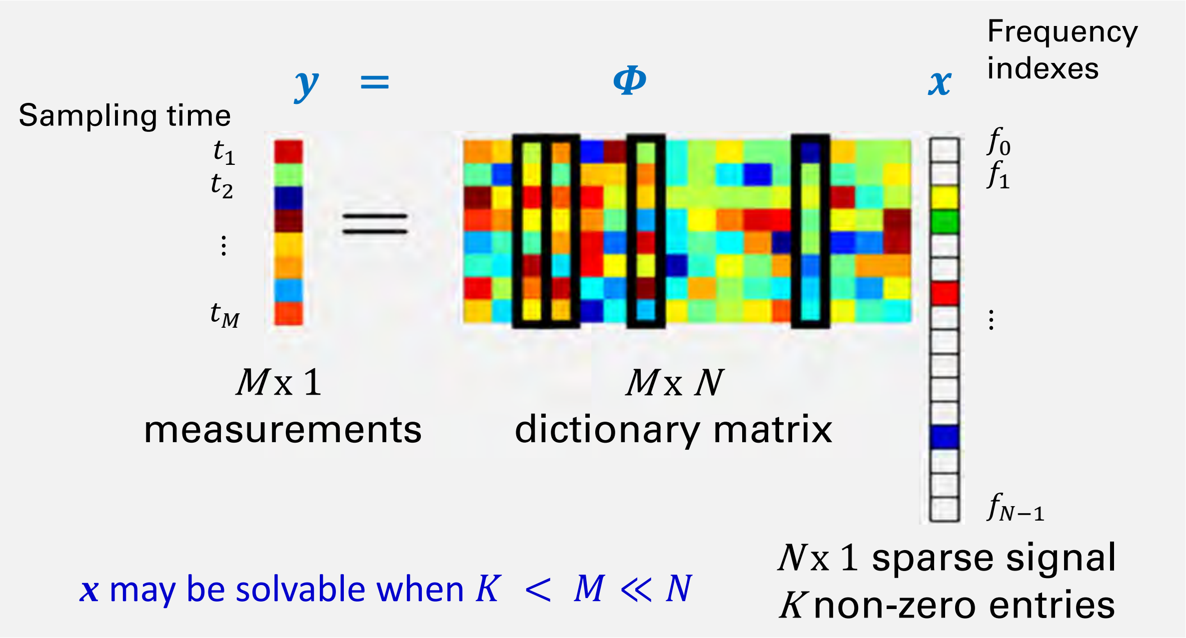
Nyquist theorem: For periodic signals, sampling interval should satisfy $T_s \leq T_{\min}/2$.



Sparse sampling:

If we know that there are multiple periodic components, their parameters may be estimated from sparse samples using **compressive sensing**.

- For a limited number of sinusoids, several sparse samples are sufficient to estimate parameters, i.e., frequencies, initial phases, and magnitudes.
- Requires dictionary matrix with columns of low correlations



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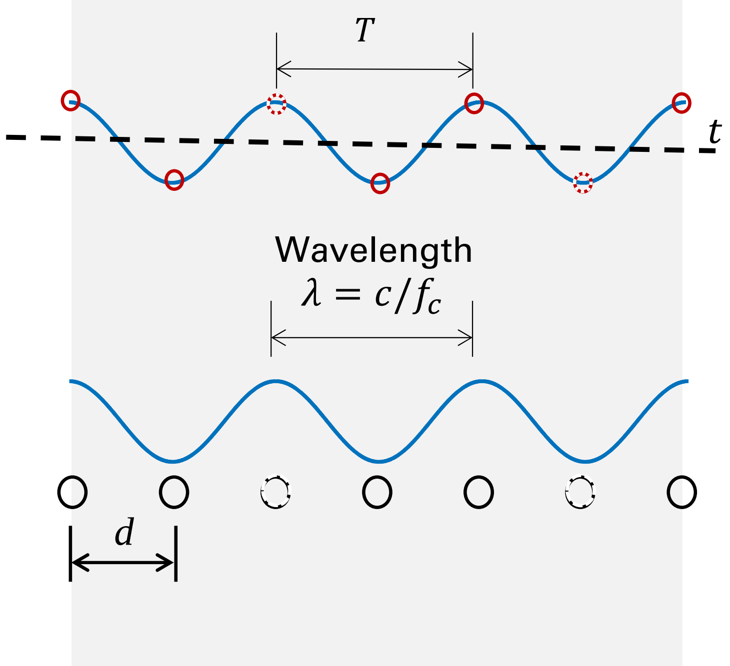
Uniform and sparse sampling

Similarly, array sensors are typically placed with inter-element spacing of $d = \lambda/2$ (ULA).

Sparse arrays:

- In **direct DOA estimation**, we may sparsely place the array sensors with $N > K$ satisfied.
  - Achieve a larger aperture but do not increase the number of DOFs
  - High sidelobe effects
- **Underdetermined DOA estimation:** A more popular approach is to utilize second-order statistics to perform DOA estimation with $N < K$
  - It increases both the array aperture and the number of DOFs
  - May achieve consecutive lags to effectively suppress sidelobe issues



$$\boldsymbol{x}(t) = \sum_{k=1}^{K} s_k(t)\boldsymbol{a}(\theta_k) + \boldsymbol{n}(t)$$

$$\boldsymbol{a}(\theta_k) = \left[1, \mathrm{e}^{j\phi_k}, \mathrm{e}^{j2\phi_k}, \cdots, \mathrm{e}^{j\,(N-1)\phi_k}\right]^{\mathrm{T}}$$

$$\phi_k = 2\pi(d/\lambda)\sin\theta_k$$

Difference coarray

Subspace-based DOA estimation exploits the data covariance matrix $\boldsymbol{R_{xx}}$.

For a ULA with uncorrected signals:

- $\boldsymbol{R_{xx}}$ is Toeplitz (diagonal-constant) and Hermitian
- $\boldsymbol{R_{xx}}$ is highly redundant: Only N elements are unique in the $N \times N$ covariance matrix
- We may not need N sensors to estimate the $N \times N$ covariance matrix

Consider the same array example with one sensor removed:

- All the entries of the covariance matrix can be restored: e.g., $\mathrm{E}[x_2 x_3^*] \Rightarrow \mathrm{E}[x_1 x_2^*]$
- The 4-element ULA and the 3-element sparse array are **different coarray equivalent** because they generate the same number of correlation lags.
- For physical array $\mathbb{G}$, The difference lags are given as: $\mathbb{C}_G = \{\boldsymbol{z} | \boldsymbol{z} = \boldsymbol{u} - \boldsymbol{v}, \boldsymbol{u}, \boldsymbol{v} \in \mathbb{G}\}$.

$$\boldsymbol{R_{xx}} = \begin{bmatrix} \mathrm{E}[x_1 x_1^*] & \mathrm{E}[x_1 x_2^*] & \mathrm{E}[x_1 x_3^*] & \mathrm{E}[x_1 x_4^*] \\ \mathrm{E}[x_2 x_1^*] & \mathrm{E}[x_2 x_2^*] & \mathrm{E}[x_2 x_3^*] & \mathrm{E}[x_2 x_4^*] \\ \mathrm{E}[x_3 x_1^*] & \mathrm{E}[x_3 x_2^*] & \mathrm{E}[x_3 x_3^*] & \mathrm{E}[x_3 x_4^*] \\ \mathrm{E}[x_4 x_1^*] & \mathrm{E}[x_4 x_2^*] & \mathrm{E}[x_4 x_3^*] & \mathrm{E}[x_4 x_4^*] \end{bmatrix}$$

Correlation lags (difference coarray)

-3 -2 -1 0 1 2 3

Minimum redundancy array

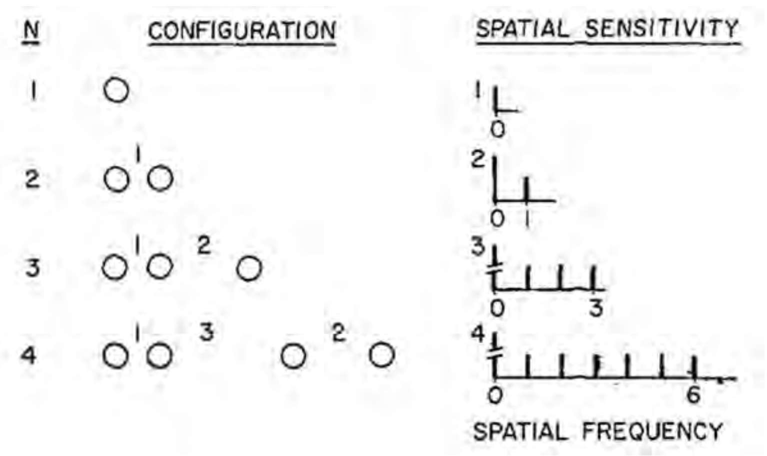
Approaches for sparse array design with consecutive lags lead to the **minimum redundancy array (MRA)**: For a given number of physical sensors, maximizes the number of consecutive virtual sensors in the resulting difference coarray.

- **Restricted arrays:** All lags are consecutive
- **General arrays:** Not all lags are consecutive

The redundancy is defined as $R = \frac{\frac{1}{2}N(N-1)}{N_{\max}}$, where $N_{\max}$ is the maximum number of obtained consecutive lags.

- R is found to be $1.217 \lesssim R \lesssim 1.674$.

However, MRA cannot be systematically designed.



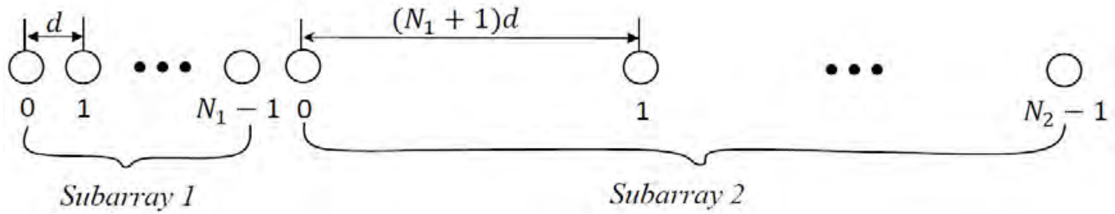
N	$N_{\max}$	R	Configuration
Restricted Arrays:			
5	9	1.11	·1·3·3·2·
6	13	1.16	·1·5·3·2·2
7	17	1.24	·1·3·6·2·3·2·
8	23	1.22	·1·3·6·6·2·3·2·
9	29	1.24	·1·3·6·6·6·2·3·2·
10	36	1.25	·1·2·3·7·7·7·4·4·1·
General Arrays:			
5	9	1.11	·4·1·2·6·
6	13	1.16	·6·1·2·2·8·
7	18	1.17	·14·1·3·6·2·5·
8	24	1.17	·8·10·1·3·2·7·8·
10	37	1.22	·16·1·11·8·6·4·3·2·22·

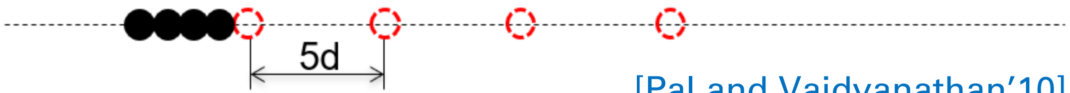
A. Moffet, "Minimum-redundancy linear arrays," *IEEE Trans. Antennas and Propagation*, 1968.

M. Ishiguro, "Minimum redundancy linear arrays for a large number of antennas," *Radio Science*, 1980.

Systematical sparse array design: Nested array

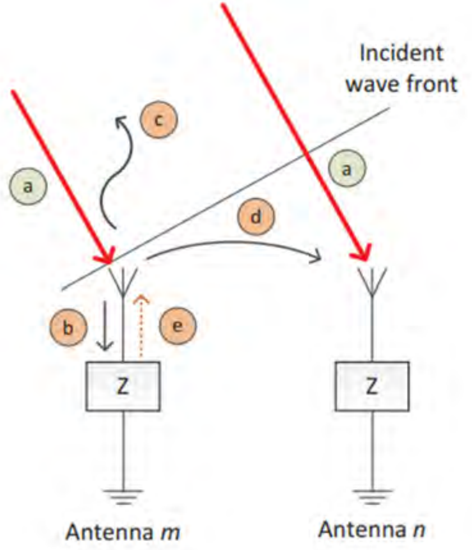
Systematical design: **Nested array** is a simple sparse array configuration which consists of two uniform linear subarrays, one of which has a unit spacing.





[Pal and Vaidyanathan'10]

- Depending on the applications, the high number of consecutive physical sensors may cause a high mutual coupling effect, degrading the performance.
- Mutual coupling brings highest impact when the spacing between the sensor is small (e.g., half-wavelength spacing).
- The coprime array is proposed as an alternative.

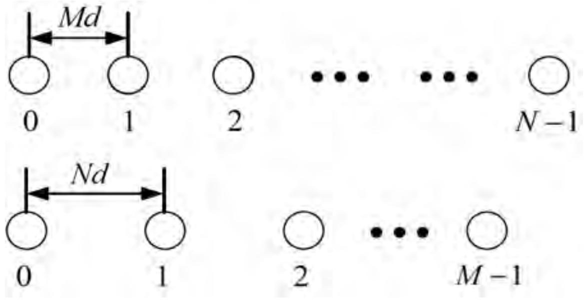


P. Pal and P. P. Vaidyanathan, "Nested arrays: A novel approach to array processing with enhanced degrees of freedom," *IEEE Trans. Signal Processing*, 2010.

Systematical sparse array design: Coprime array

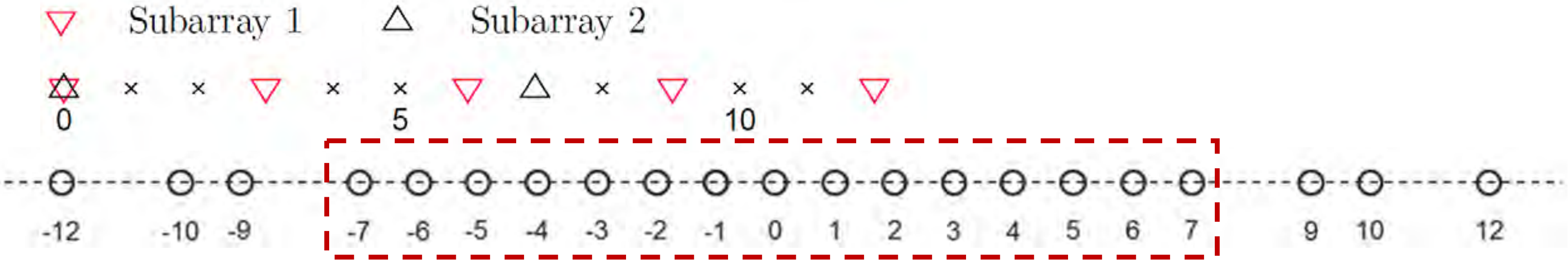
Coprime array: utilizes a pair of uniform linear subarrays with M and N being coprime integers (greatest common divisor $\gcd(M,N) = 1$)

Example: $M = 3$ and $N = 5$ (6 elements)



Physical array

Coarray



- Unlike nested arrays, coprime arrays have holes in the resulting lags.
- Direct MUSIC only uses consecutive lags $[-7{:}1{:}7]$ and detect up to 7 signals.
- In this context, optimum design of parse arrays is to
  - Have a high number of consecutive lags
  - With low mutual coupling (few elements are separated by lag-1 and lag-2)

P. P. Vaidyanathan and P. Pal, “Sparse sensing with co-prime sampler and arrays,” *IEEE Trans. Signal Processing*, 2011

Direct MUSIC-based DOA estimation

Vectorizing $\boldsymbol{R}_{\boldsymbol{xx}}$ yields

$$\boldsymbol{z} = \mathrm{vec}(\boldsymbol{R}_{\boldsymbol{xx}}) = \widetilde{\boldsymbol{A}}\,\boldsymbol{b} + \sigma_n^2\,\tilde{\boldsymbol{\imath}} = \boldsymbol{A}^o\boldsymbol{b}^o$$

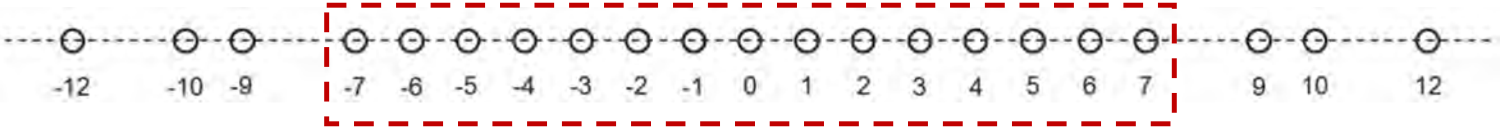
$\widetilde{\boldsymbol{A}} = [\boldsymbol{a}(\theta_1)\otimes\boldsymbol{a}^*(\theta_1), \ldots, \boldsymbol{a}(\theta_Q)\otimes\boldsymbol{a}^*(\theta_Q)]$: Manifold matrix for the difference coarray

$\boldsymbol{b} = [\sigma_1^2, \ldots, \sigma_Q^2]^{\mathrm{T}}$: Source power vector

$\tilde{\boldsymbol{\imath}} = \mathrm{vec}(\boldsymbol{I}_N)$

$\boldsymbol{A}^o = [\widetilde{\boldsymbol{A}}, \widetilde{\boldsymbol{I}}]$

$\boldsymbol{b}^o = [\boldsymbol{b}^{\mathrm{T}}, \sigma_{\mathrm{n}}^2]^{\mathrm{T}}$



$\boldsymbol{z}$ acts as received data of a virtual array (difference coarray)

- Manifold matrix corresponds to more virtual sensors than physical antennas
- Only has a single snapshot corresponding to vector $\boldsymbol{b}$
- To perform direct MUSIC, we can use **consecutive lags** to form a full-rank Hermitian and Toeplitz matrix which acts as a covariance matrix

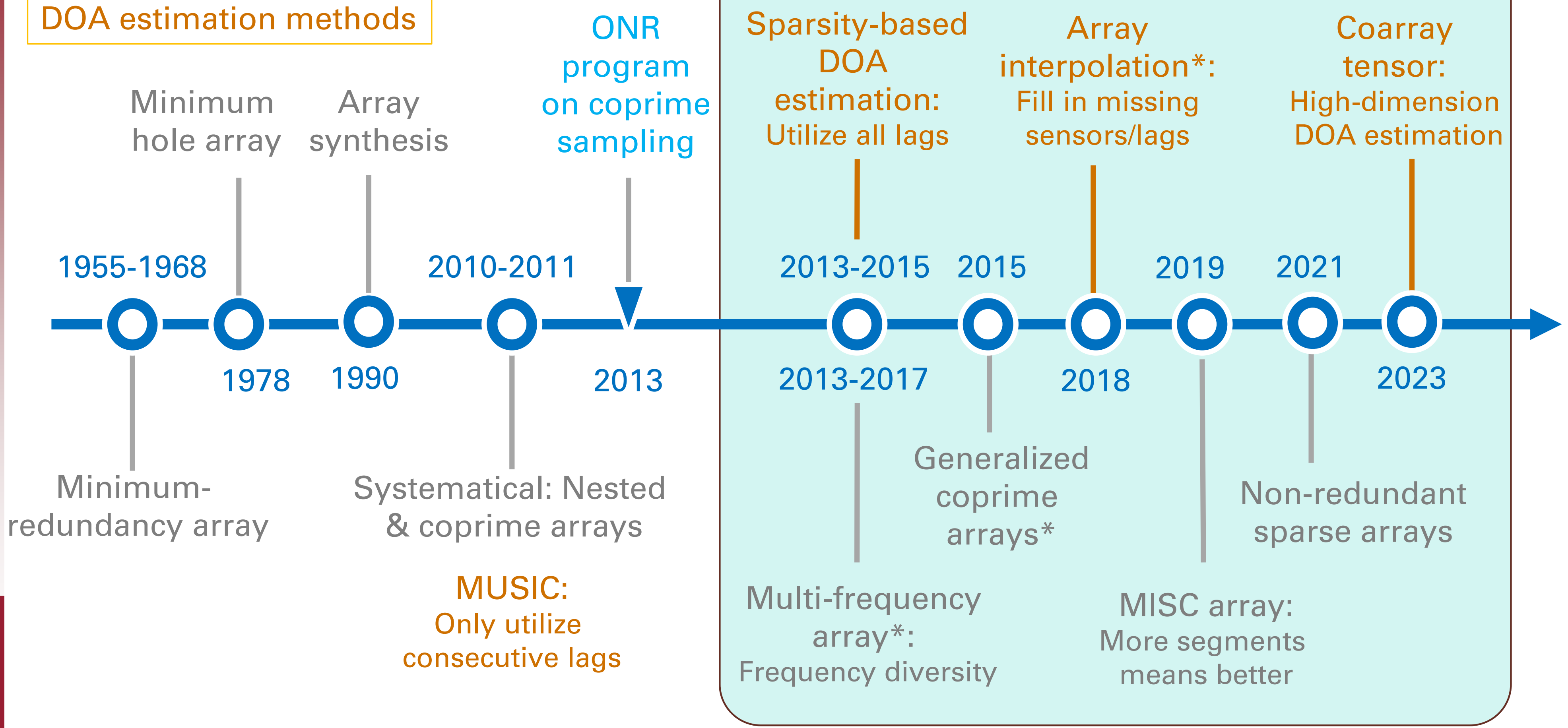
$$\widetilde{\boldsymbol{R}}_{xx} = \begin{bmatrix} z_0 & z_1 & \cdots & z_7 \\ z_{-1} & z_0 & \cdots & z_6 \\ \vdots & \vdots & \ddots & \vdots \\ z_{-7} & z_{-6} & \cdots & z_0 \end{bmatrix}$$

P. Pal and P. P. Vaidyanathan, “Coprime sampling and the MUSIC algorithm,” *IEEE Digit. Signal Process. Workshop/ IEEE Signal Process. Educ. Workshop*, 2011.

C.-L. Liu and P. P. Vaidyanathan, "Remarks on the Spatial Smoothing Step in Coarray MUSIC," *IEEE Signal Processing Letters*, 2015.

Sparse array design and processing

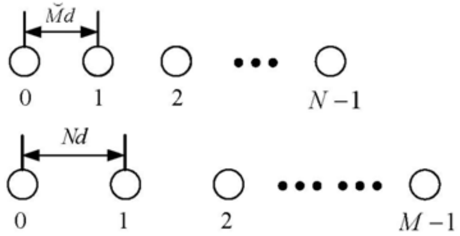
Sparse array designs
DOA estimation methods



Sparse array: Generalized coprime and nested arrays

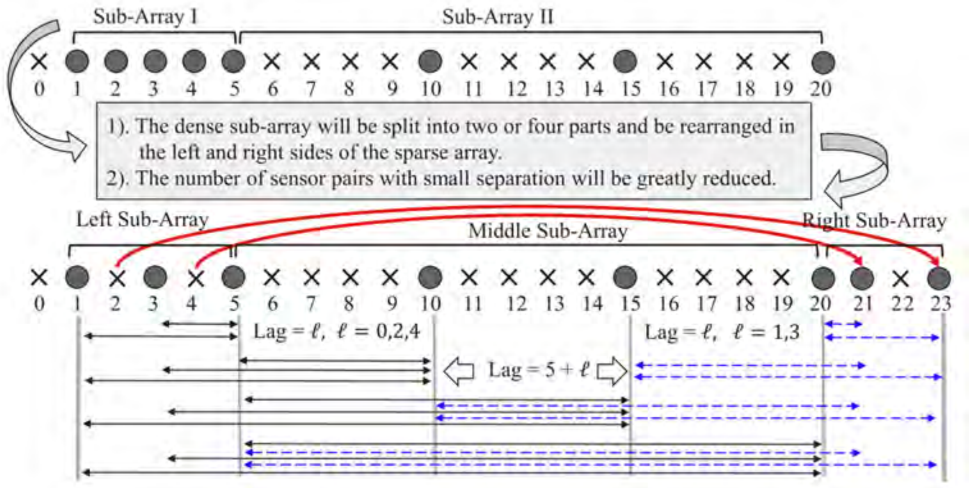
Problems:

- Coprime array: have holes in the lags
- Nested array: high mutual coupling



Modified coprime and nested arrays:

- **CACIS** (Coprime array with compressed inter-element spacing)**:** compresses the interelement spacing of one subarray $\breve{M} = M/p$ with $2 \le p \le M$ to increase the number of consecutive lags
- **Augmented nested array:** Split the densely located elements in inner subarray to reduce the mutual coupling. Several variations.



S. Qin, Y. D. Zhang, and M. G. Amin, "Generalized coprime array configurations for direction-of-arrival estimation," *IEEE Trans. Signal Processing*, 2015.

J. Liu, Y. Zhang, Y. Lu, S. Ren and S. Cao, "Augmented nested arrays with enhanced DOF and reduced mutual coupling," *IEEE Trans. Signal Processing*, 2017.

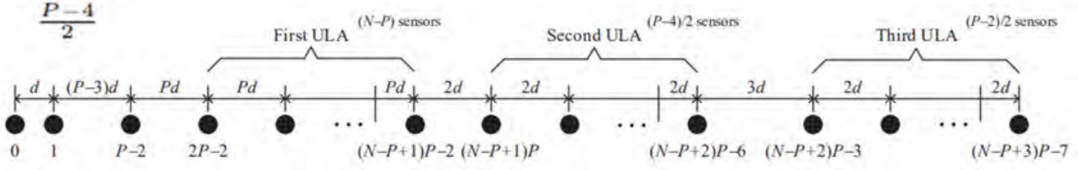
Sparse array: MISC family

MISC (maximum interelement spacing constraint): Achieve a high number of consecutive lags with low mutual coupling

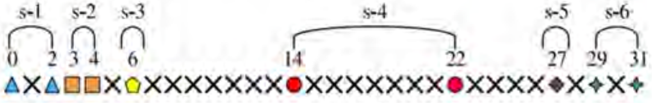
$$\mathbb{A}_{\mathrm{MISC}} = \{1, P-3, \underbrace{P,\dots,P}_{N-P}, \underbrace{2,\dots,2}_{\frac{P-4}{2}}, 3, \underbrace{2,\dots,2}_{\frac{P-4}{2}}\}$$ with $P = 2\lfloor N/4 \rfloor + 2 \;\; (N \geq 5)$

Modified versions:

- Improved MISC (I-MISC)
- Enhanced MISC (EMISC)
- Symmetry improved MISC (S-IMISC)
- Extended MISC (xMISC)



MISC (4 segments)



I-MISC (6 segments)

Z. Zheng, W-Q. Wang, Y. Kong, and Y. D. Zhang, "MISC Array: A new sparse array design achieving increased degrees of freedom and reduced mutual coupling effect," *IEEE Trans. Signal Processing*, 2019.

W. Shi, Y. Li, and R. C. de Lamare, "Novel sparse array design based on the maximum inter-element spacing criterion," *IEEE Signal Processing Letters*, 2022.

X. Sheng, D. Lu, Y. Li, and R. C. de Lamare, "Enhanced MISC-based sparse array with high uDOFs and low mutual coupling," *IEEE Trans. Circuits and Systems II: Express Briefs*, in press.

X. Li, H. Yang, J. Han, and N. Dong, "A novel low-complexity method for near-field sources based on an S-IMISC array model," *Electronics*, 2023.

S. Wandale and K. Ichige, "xMISC: Improved sparse linear array via maximum inter-element spacing concept," *IEEE Signal Processing Letters*, 2023.

Sparse array: Performance evaluation

DOF ratio:

$$\gamma(N) = \frac{N^2}{\mathcal{S}_u}$$

$\mathcal{S}_u$: one-side uniform DOF (uDOF)

Coupling leakage:

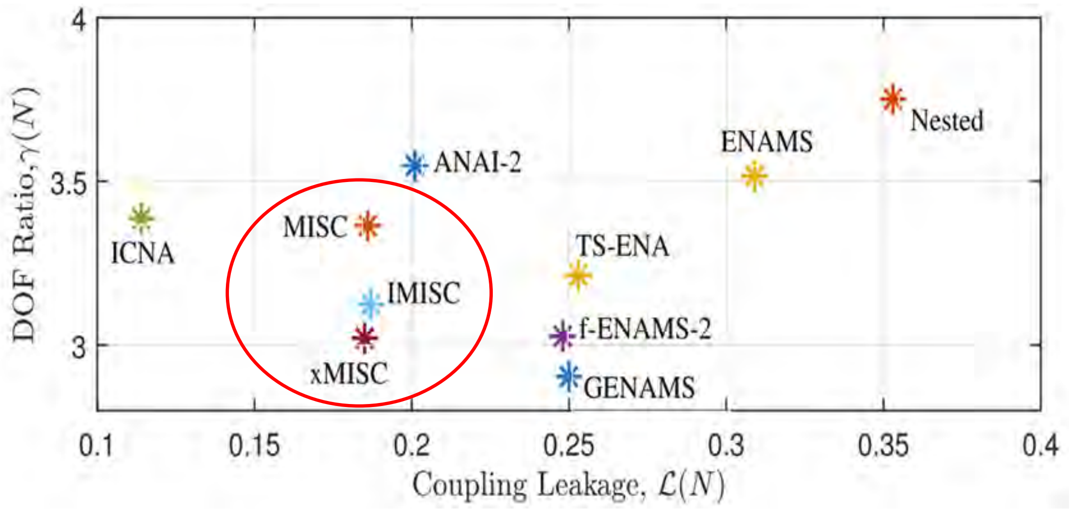
$$\mathcal{L}(N) = \frac{\|\boldsymbol{H} - \mathrm{diag}(\boldsymbol{H})\|_F}{\|\boldsymbol{H}\|_F}$$

$\boldsymbol{H}$: mutual coupling matrix whose elements depends on the distance between elements

Simulations assumed

$$\langle \boldsymbol{H} \rangle_{j,l} = \begin{cases} c_{|j-l|}, & \text{if } |j-k| \leq V \\ 0, & \text{otherwise} \end{cases}$$

with $c_0 = 1$, $c_1 = 0.2e^{j\pi/3}$, $\left|\frac{c_b}{c_l}\right| = \frac{l}{b}$



(N=30)

S. Wandale and K. Ichige, "xMISC: Improved sparse linear array via maximum inter-element spacing concept," *IEEE Signal Processing Letters*, 2023.

Sparsity-based DOA estimation

Sparsity-based DOA estimation:

$$\boldsymbol{z} = \mathrm{vec}(\boldsymbol{R}_{xx}) = \widetilde{\boldsymbol{A}}\,\boldsymbol{b} + \sigma_n^2 \tilde{\boldsymbol{\imath}} = \boldsymbol{A}^o \boldsymbol{b}^o$$

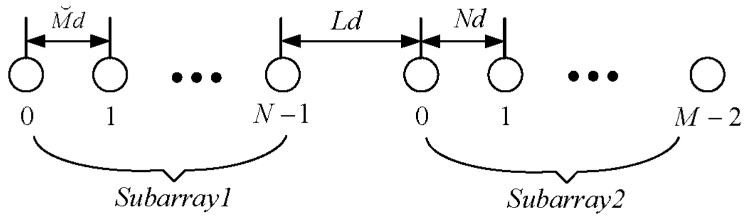
- The linear coarray model well fits into the compressive sensing (CS) problem by defining dense dictionary matrix $\boldsymbol{A}^g$ over a grid, e.g., $[-90{:}\,1{:}\,90]$:

$$\min_{\boldsymbol{z}} \|\boldsymbol{b}^g\|_0 \ \text{subject to} \ \|\boldsymbol{z} - \boldsymbol{A}^g \boldsymbol{b}^g\| \le \epsilon$$

- The positions of the nonzero solutions of $\boldsymbol{b}^g$ represent the signal DOA
- This approach does not require a specific array structure and all difference lags can be utilized in sparsity-based DOA estimation: Unique lags

CADiS (Coprime array with displaced subarrays):

- Displaces two subarrays to increases unique lags and reduces mutual coupling
- In general, the resulting lags are disconnected in the center region



$\breve{M} = M/p$ with $2 \le p \le M$

S. Qin, Y. D. Zhang, and M. G. Amin, "Generalized coprime array configurations for direction-of-arrival estimation," *IEEE Trans. Signal Processing*, 2015.

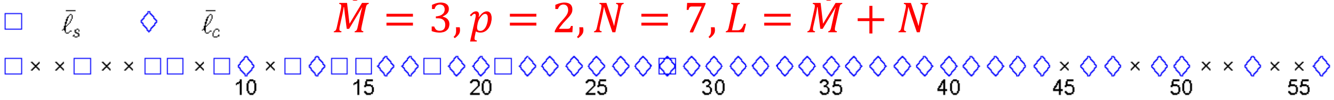
Sparse array: CADiS

In CADiS configurations, the self-lags are less likely to coincide with the cross-lags:

(a) $L > (M-2)N$ achieves the maximum number of unique lags

(b) $L = \breve{M} + N$ yields the largest number of consecutive lags

$\breve{M} = 3, p = 2, N = 7, L = \breve{M} + N$



$\breve{M} = 3, p = 2, N = 7, L = \breve{M} + N, \eta_c = 33, \eta_u = 89$

$\breve{M} = 2, p = 3, N = 7, L = \breve{M} + N, \eta_c = 38, \eta_u = 87$

$\breve{M} = 1, p = 6, N = 7, L = \breve{M} + N, \eta_c = 85, \eta_u = 85$

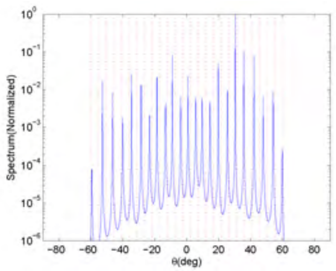
- A smaller value of $\breve{M}$ reduces the unique lags and reduces the number of holes
- The lags become consecutive when $\breve{M} = 1$ (nested array)

S. Qin, Y. D. Zhang, and M. G. Amin, "Generalized coprime array configurations for direction-of-arrival estimation," *IEEE Trans. Signal Processing*, 2015.

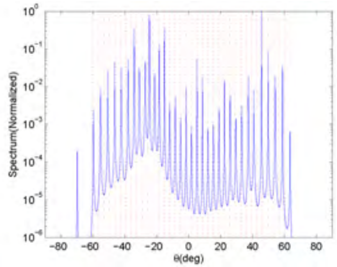
Sparse arrays: Comparison

Consider $M = 6$ and $N = 7$ with $M + N - 1 = 12$ physical sensors

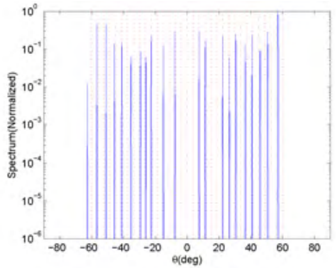
- CADiS generally outperforms CACIS
- The CS based method achieves better DOA estimation performance



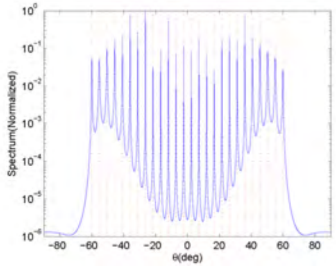
(a) CACIS with $\breve{M} = 3$ ($\eta_c = 47$)



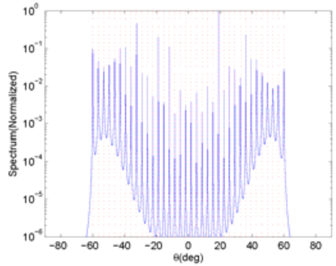
(a) CACIS with $\breve{M}$ =1 ($\eta_c = 71$)



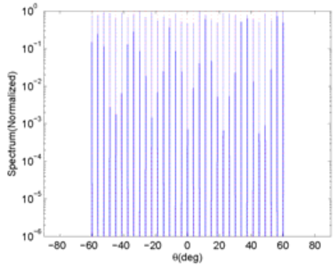
(a) CACIS with $\breve{M} = 2$ ($\eta_u = 65$)



(b) CACIS with $\breve{M} = 2$ ($\eta_c = 59$)



(b) CADiS with $\breve{M} = 1$ ($\eta_c = 85$)



(b) CADiS with $\breve{M} = 2$ ($\eta_u = 87$)

MUSIC ($Q = 26$)

MUSIC ($Q = 36$)

CS ($Q = 33$)

Off-grid problem

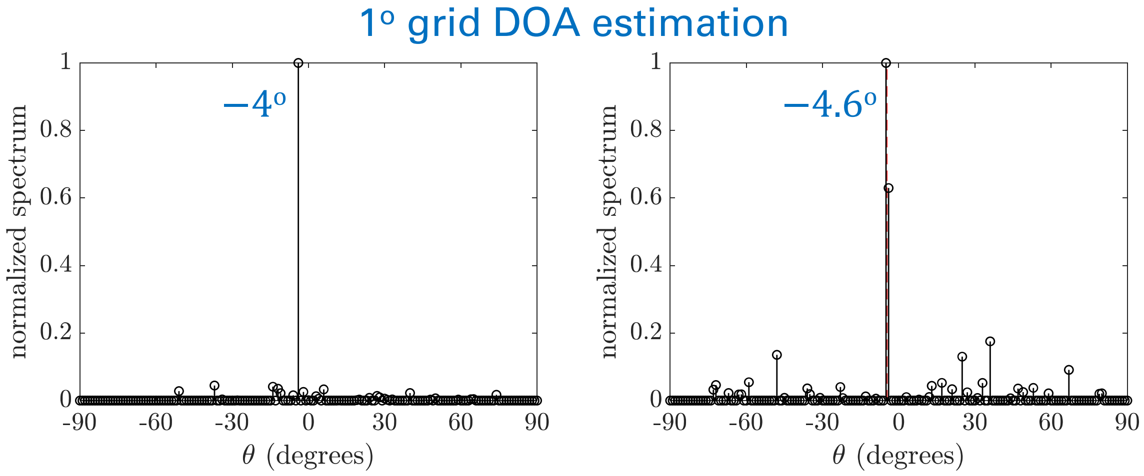
A major problem with the CS-based DOA estimation approach is that the DOAs must be on the defined grid, e.g., $[-90^{\circ} : 1^{\circ} : 90^{\circ}]$.

Signals arriving from other DOAs will suffer the **off-grid problem**, e.g., signal from 43.6°.

- Less sparse solution
- Difficult to converge

Solutions in the context of CS:

- Finer grid resolution
- Grid refining
- Off-grid estimation
- Atomic decomposition



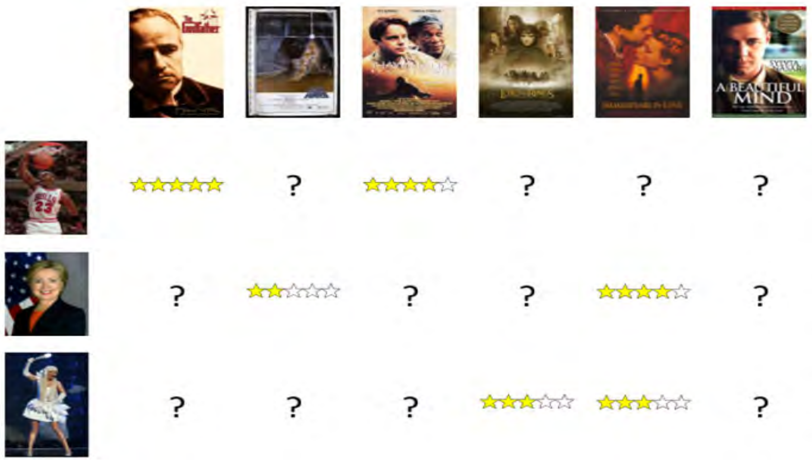
An attractive method is to complete the covariance matrix (matrix completion) so that conventional subspace-based methods (e.g., MUSIC) can be applied.

Matrix completion

Netflix problem: Predict unknown scores.

The data is **low-rank**, but the dictionary is unknown (unlike CS).

Let Ω be the region where the elements of matrix $\boldsymbol{M}$ are observed (i.e., $\{M_{ij}|(i,j) \in \Omega\}$), matrix completion finds a low-rank full matrix $\boldsymbol{X}$ which matches $\boldsymbol{M}$:



$$\min_{\boldsymbol{X}} \quad \operatorname{rank}(\boldsymbol{X}) \quad \text{subject to} \quad X_{ij} = M_{ij} \quad \forall\, (i,j) \in \Omega$$

Because the problem involves matrix rank, it is non-convex and NP-hard.

Therefore, the matrix rank is often relaxed, e.g., to its **nuclear norm**:

$$\min_{\boldsymbol{X}} \quad ||\boldsymbol{X}||_* \quad \text{subject to} \quad X_{ij} = M_{ij} \quad \forall\, (i,j) \in \Omega$$

$||\boldsymbol{X}||_* = \sum_{i=1}^{r} \sigma_i$: nuclear norm of matrix $\boldsymbol{X}$ (σ_i are singular values)

Structured matrix completion of covariance matrix

A matrix cannot be completed when an entire row or column is missing in the observed matrix.

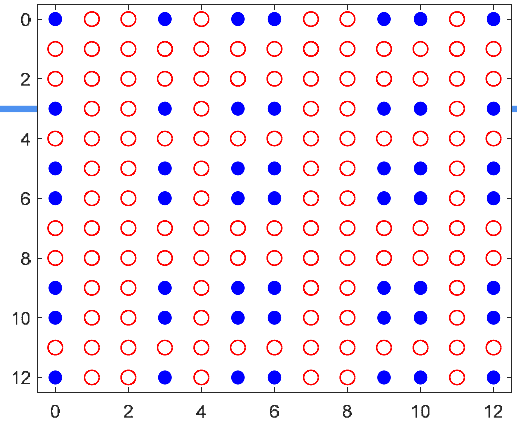
- Cannot complete covariance matrix of physical array
- However, for ULA, we can recover the covariance matrix utilizing its Toeplitz and Hermitian structure
- In this case, the completed covariance matrix can be defined by only one column vector $\boldsymbol{z}$ as $\mathcal{T}(\boldsymbol{z})$, and the nuclear norm minimization becomes

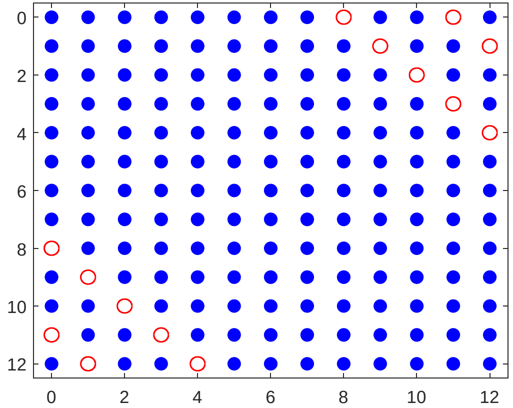
$$\min_{\mathbf{z}} \quad \|[\mathcal{T}(\boldsymbol{z}) - \boldsymbol{M}] \circ \boldsymbol{B}_{\Omega}\|_F^2 + \tau \|\mathcal{T}(\boldsymbol{z})\|_*$$

$$\text{subject to} \quad \mathcal{T}(\boldsymbol{z}) \succcurlyeq \mathbf{0}$$

$\boldsymbol{B}_{\Omega}$: mask matrix with $[\boldsymbol{B}_{\Omega}]_{ij} = \begin{cases} 1, & \text{if } (i,j) \in \Omega \\ 0, & \text{otherwise} \end{cases}$

τ: regularization parameter





- Other methods: atomic norm minimization and dual-variable rank minimization.

C. Zhou, Y. Gu, Z. Shi, and Y. D. Zhang, "Off-grid direction-of-arrival estimation using coprime array interpolation," *IEEE Signal Processing Letters*, 2018.
C. Zhou, Y. Gu, X. Fan, Z. Shi, G. Mao, and Y. D. Zhang, "Direction-of-arrival estimation for coprime array via virtual array interpolation," *IEEE Trans. Signal Processing*, 2018.
S. Liu, Z. Mao, Y. D. Zhang, and Y. Huang, "Rank minimization-based Toeplitz reconstruction for DoA estimation using coprime array," *IEEE Communications Letters*, July 2021.

A S P Lab

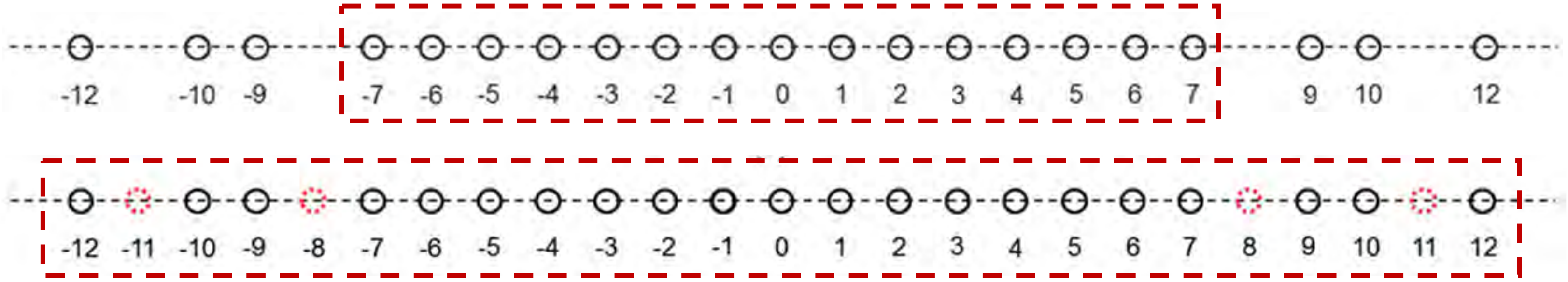
Matrix completion-aware sparse array design

Matrix completion

- Fills in information in missing lags
- Changes missing holes in the lag from obstacles in consecutive-lag construction into a resource for aperture extension
- Enabling off-grid DOA estimation with larger array apertures

Direct MUSIC

MUSIC with matrix completion



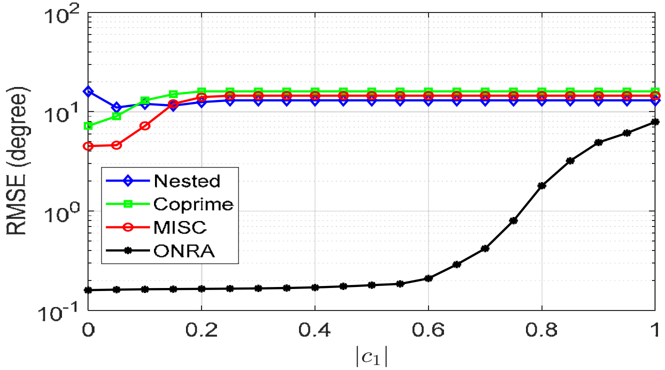
With such capability, how shall we consider the "optimality" of a sparse array? We introduce **optimized non-redundant array (ONRA)**:

- Redundancy-free: Each lag only appears once
- Introduce holes in the lag for reducing mutual coupling and enlarging array aperture

A. Ahmed and Y. D. Zhang, "Generalized non-redundant sparse array designs," *IEEE Trans. Signal Processing*, 2021.

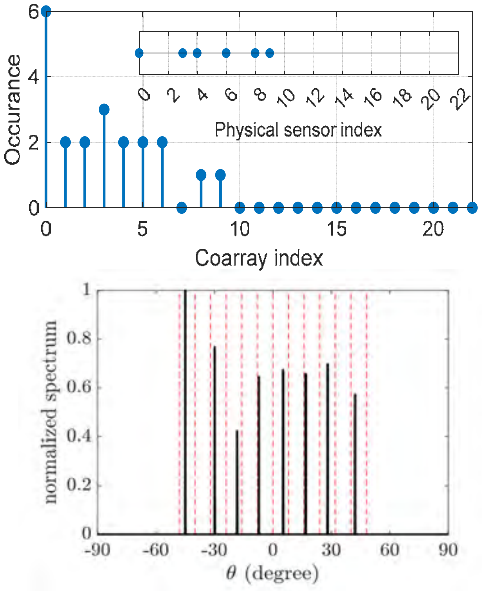
Non-redundant sparse array: Comparison

- Comparison for 6-sensor arrays (DOA estimation for 13 sources; LASSO)
- ONRA has very low mutual coupling effect as the minimum interelement spacing is 2 units

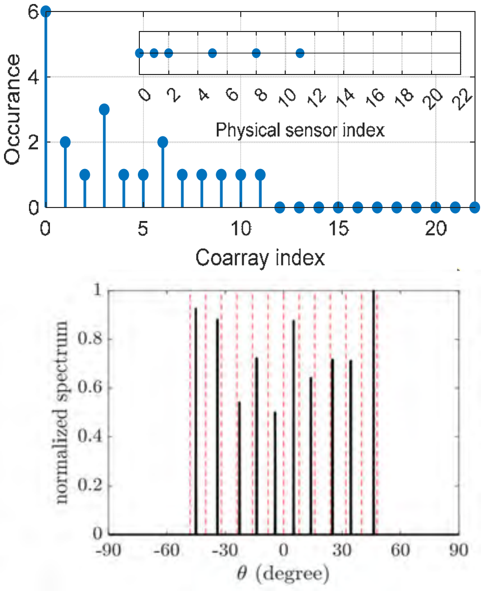


RMSE for two closely spaced source case

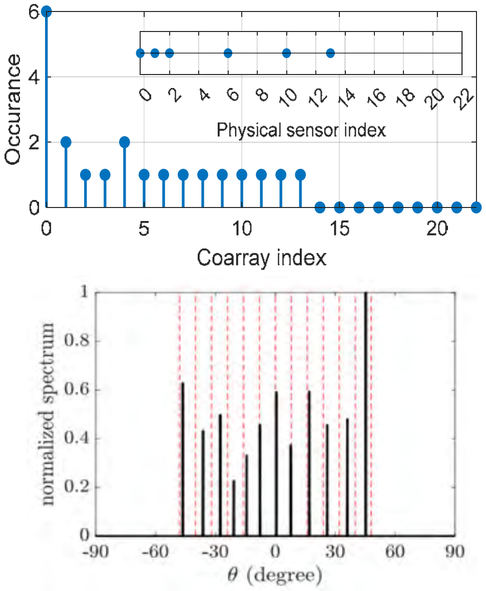
Coprime array (9 lags; max lag 9)



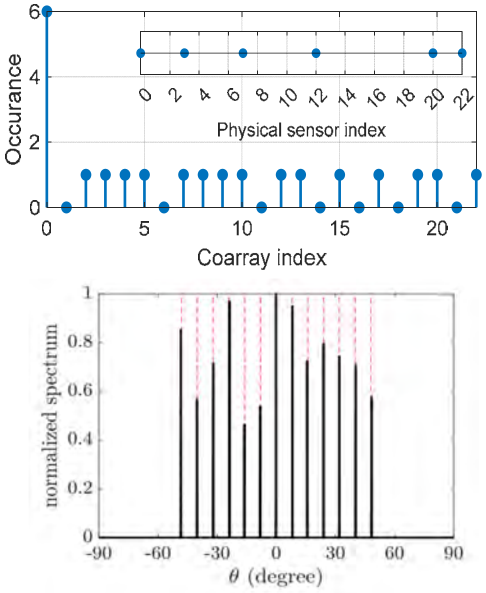
Nested array (12 lags; max lag 11)



MISC array (14 lags; max lag 13)



ONRA (16 lags; max lag 22)

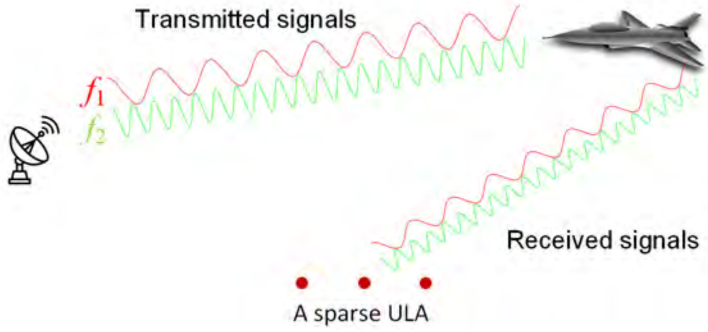


A. Ahmed and Y. D. Zhang, "Generalized non-redundant sparse array designs," *IEEE Trans. Signal Processing*, 2021.

ASP Lab

Multi-frequency sparse array

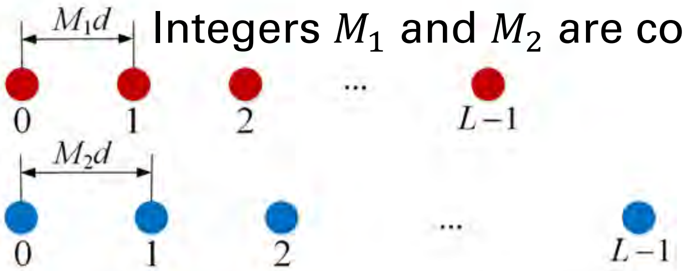
- Difference coarray is obtained from array data covariance matrix, which requires (time-domain) snapshots.
- Can we utilize resources in the other domain, e.g., frequency?
- Multi-frequency sparse array exploits two or more frequencies to obtain virtual arrays.



$$D = M_i \frac{\lambda_i}{2}, i = 1, \cdots, I$$

Sparse ULA

Integers M_1 and M_2 are coprime

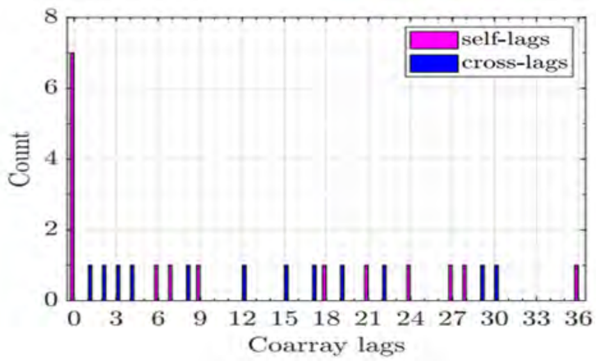


Equivalent structure with two coprime frequencies

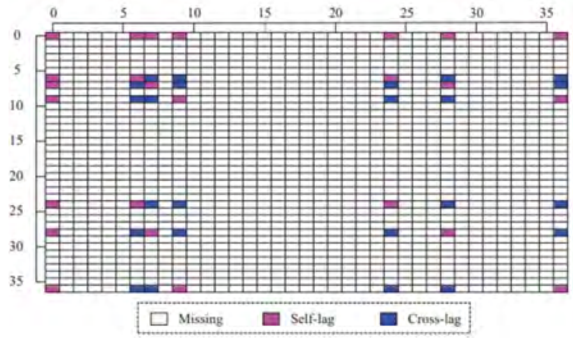
S. Qin, Y. D. Zhang, M. G. Amin, and B. Himed, "DOA estimation exploiting a uniform linear array with multiple co-prime frequencies," *Signal Processing*, 2017.

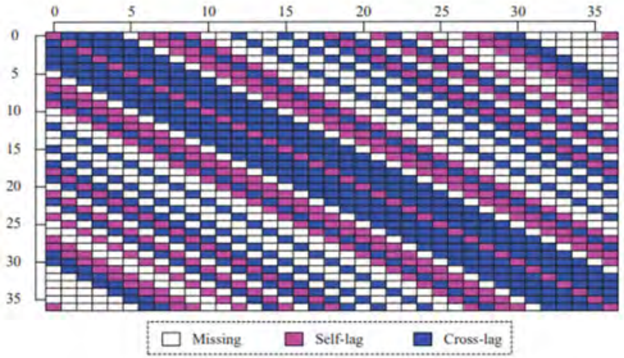
Multi-frequency sparse array

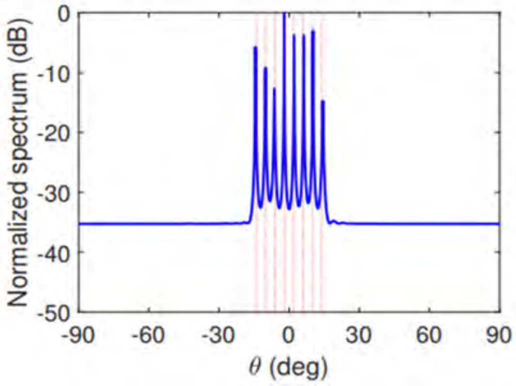
Example: 3 antennas, 3 frequencies



- Same physical array appears as different virtual arrays in different frequencies, rendering 7 virtual sensors
- Covariance matrix of 37×27 is reconstructed
- 8 targets uniformly distributed in $[-14^{\circ}, 14^{\circ}]$ are detected





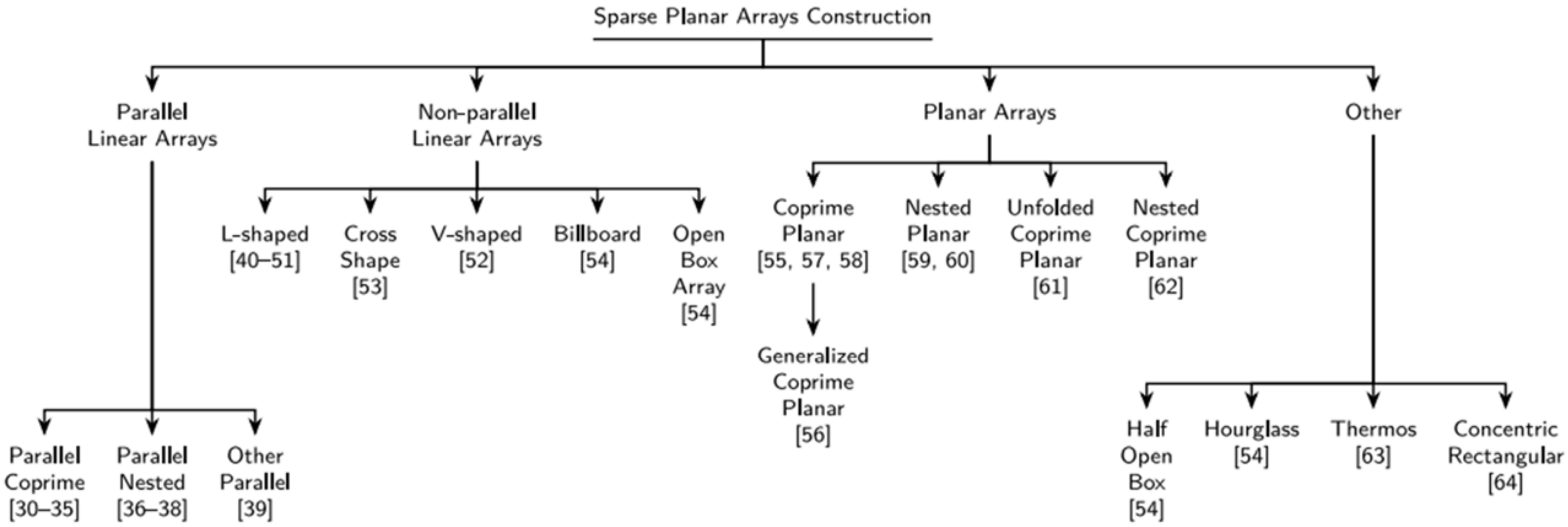


S. Zhang, A. Ahmed, Y. D. Zhang, and S. Sun, "Enhanced DOA estimation exploiting multi-frequency sparse array," *IEEE Trans. Signal Processing*, 2021.

2-D sparse arrays

Extension to 2-D (planar) sparse arrays: Sparse array design and processing concepts apply for 2-D array but need additional considerations.

- Sparse arrays may have additional rooms for sensor reduction
- Covariance statistics becomes a tensor with high redundancies
- Multi-dimensional processing may require decoupling for reduced complexity

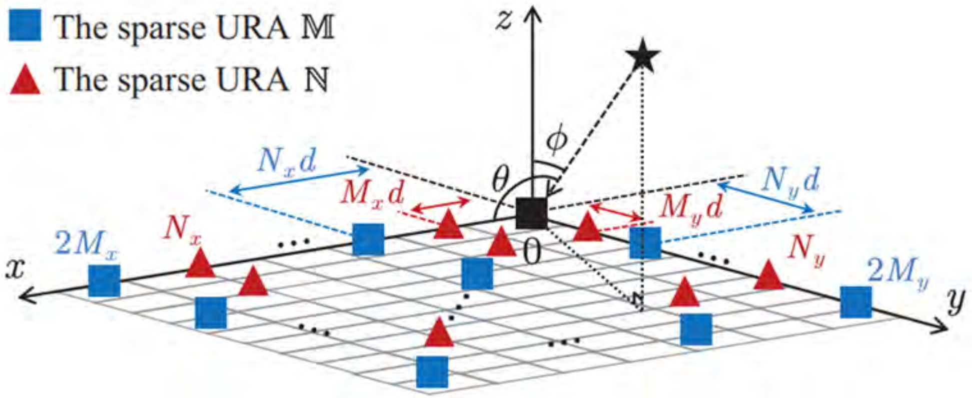


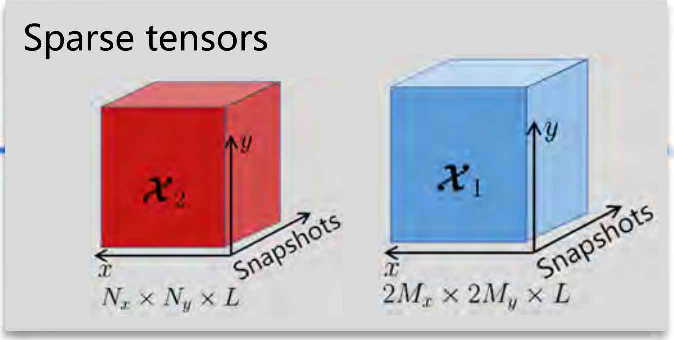
I. Aboumahmoud, A. Muqaibel, M. Alhassoun and S. Alawsh, "A Review of sparse sensor arrays for two-dimensional direction-of-arrival estimation," *IEEE Access*, 2021.

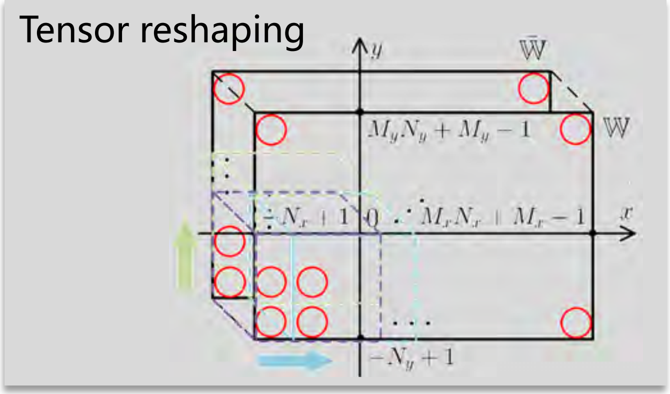
Coarray tensor DOA estimation

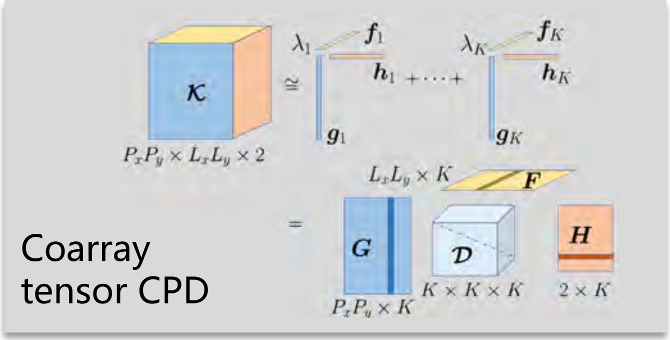
Consider a coprime planar array with coprimality deployment along both x-axis and y-axis.

- The signal received at the two sparse uniform rectangular arrays form two 3-D tensors, which are combined and interpolated into a complete 3-D tensor.
- The coarray tensor can be flexibly manipulated for dimension expansion and enhanced signal detectability.
- Tensor canonical polyadic decomposition (CPD) is used to perform DOA estimation.



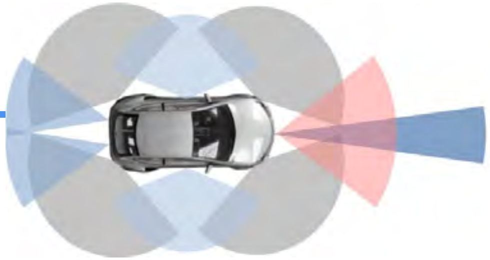






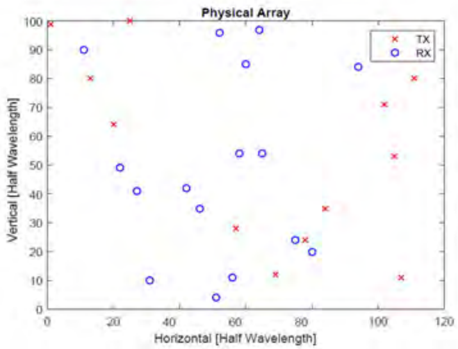
H. Zheng, C. Zhou, Z. Shi, Y. Gu, Y. D. Zhang, "Coarray tensor direction-of-arrival estimation," *IEEE Trans. Signal Processing*, 2023.

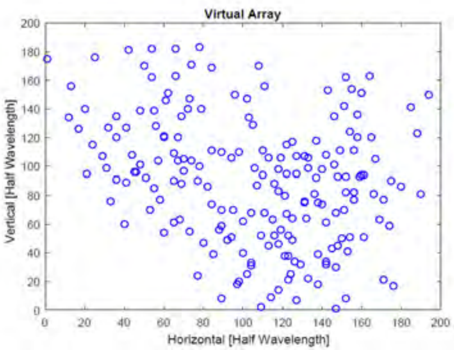
Automotive radar application

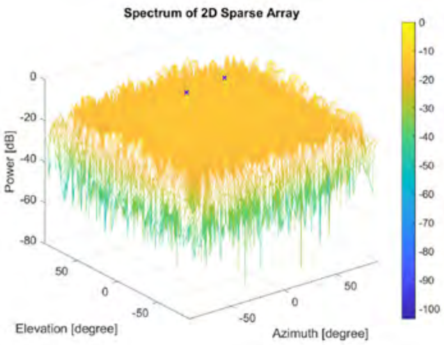


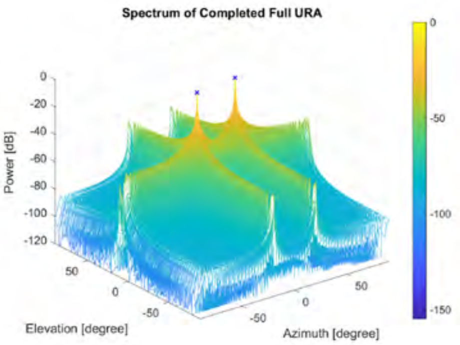
Automotive radar:

- We may only use few (even one) data samples
- A large aperture in both azimuth and elevation is important to identify objects and enable drive-over and drive-under
- Consider a sparse 2-D multiple-input multiple-output (MIMO) radar using 12 transmit antennas and 16 receive antennas
- MIMO radar enjoys sum coarray with 196 virtual antennas
- Data completion is important to reduce the sidelobes









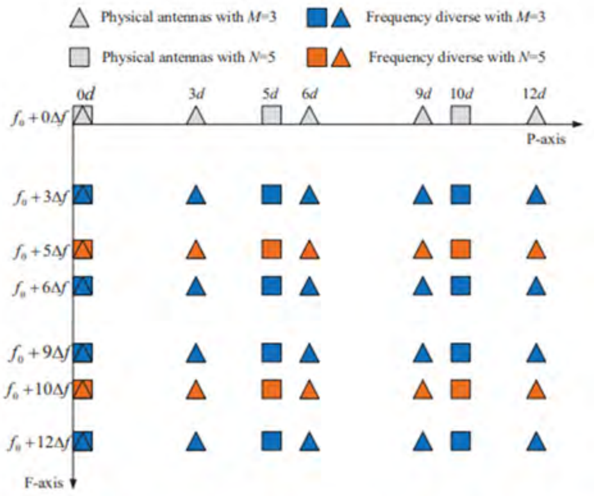
S. Sun and Y. D. Zhang, “4D automotive radar sensing for autonomous vehicles: A sparsity-oriented approach,” *IEEE Journal of Selected Topics in Signal Processing*, 2021.

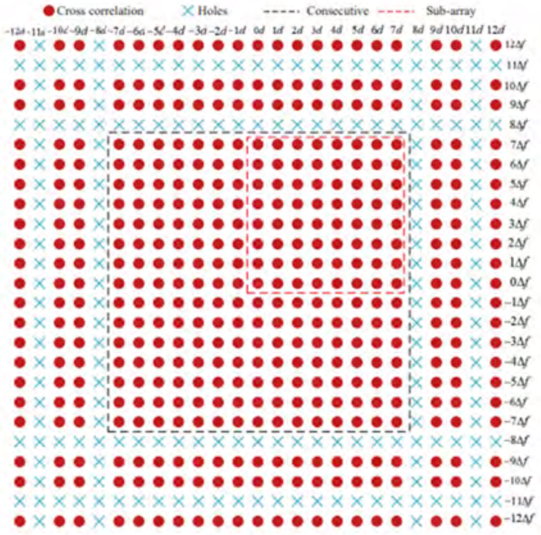
S. Sun and Y. D. Zhang, "Redefining radar perception for autonomous driving: The role of sparse array and waveform design in 4D automotive radar," IEEE Signal Processing Society Webinar Series, Sept. 2023.

Joint DOA-range estimation

Consider frequency diverse array (FDA) which localize targets in both angle and range dimensions by using a small frequency increment across the array elements.

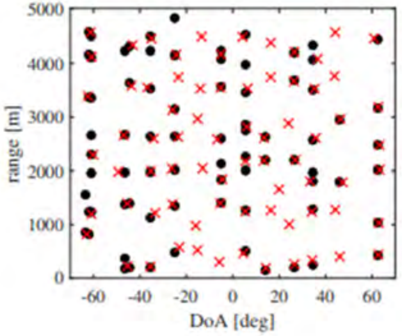
A decoupled atomic norm minimization (DANM) approach is developed to achieve effective interpolation of the doubly-Toeplitz covariance matrix.



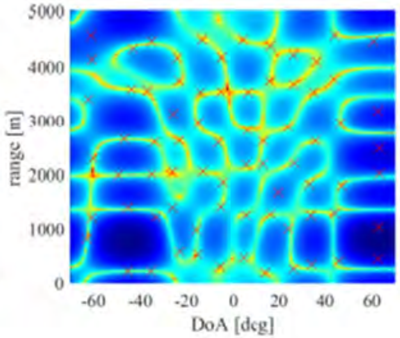


Detect 74 targets using a coprime FDA with 7 sensor and 7 frequencies

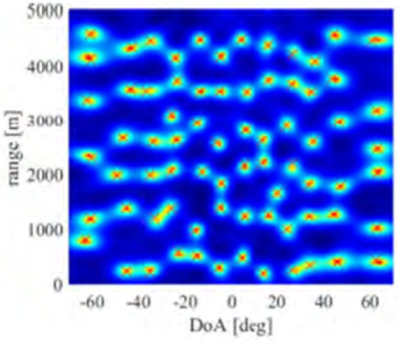
Bayesian CS



Spatial smoothing

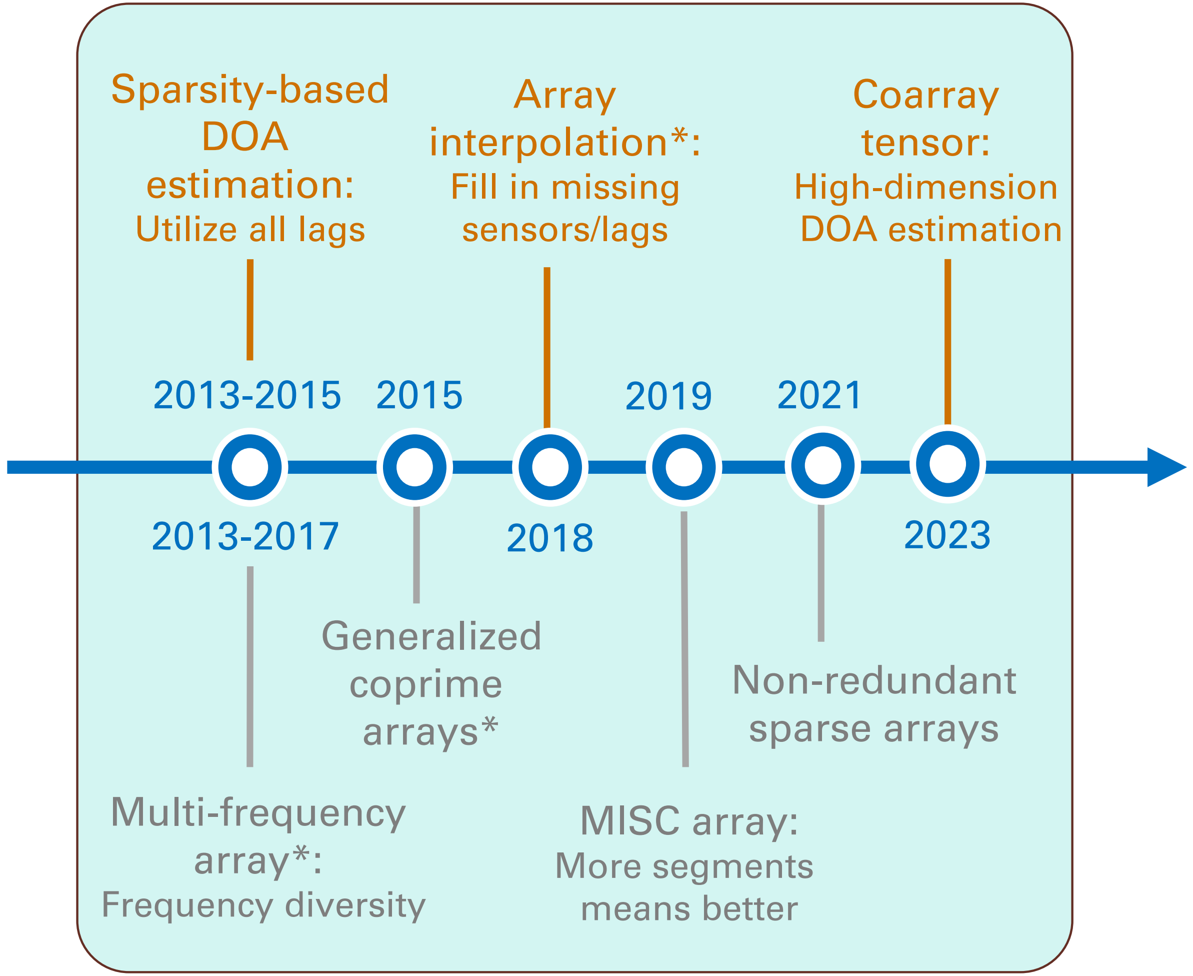


DANM



Z. Mao, S. Liu, Y. D. Zhang, L. Han, and Y. Huang, "Joint DoA-range estimation using space-frequency virtual difference coarray," *IEEE Trans. Signal Processing*, 2022.

Sparse array design and processing



Two/multi-dimensional arrays

- Array design
- Extreme sparse arrays
- Tensor-based processing

Bandwidth exploitation

- DOF for wideband signals
- Low-complexity
- Rational sparse arrays

Signal coherency

- Coherent/correlated signals
- Mixed uncorrelated/coherent signals

Analysis of performance, bounds, and robustness

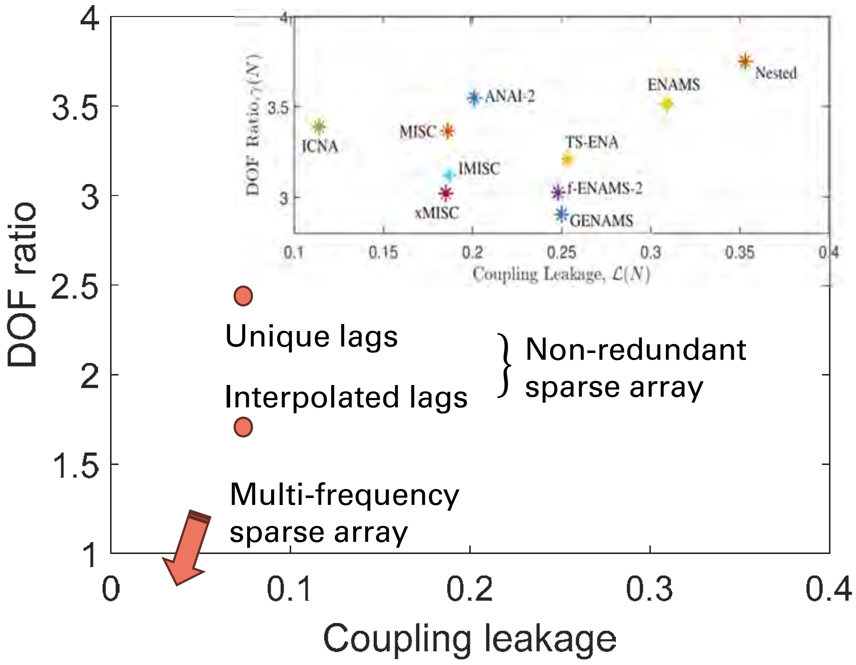
Concluding remarks

Modern sensing applications require higher resolution (larger aperture), resolve more signals (more DOFs), low mutual coupling effect (avoid close placement).

The fundamental goals of sparse array design and processing are to answer these needs while keeping a low system complexity.

Sparse array designs are enabled by the signal processing techniques, such as compressive sensing, matrix completion, and tensor analysis.

While the last decade witnessed significant process in this area, many challenging issues remain to be explored.



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- https://www.youtube.com/watch?v=3fn8OPBNczI

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