Signaling Strategies and Array Processing for Sensing and Communications

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Outline

- Introduction
- **Joint radar-communications:**
  Signaling strategies
- **Radar-communication coexistence:**
  Robust beamforming and DOA estimation
- **Passive radar:**
  Sparsity-based imaging, localization, and STAP
- Concluding remarks

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Spectrum sharing: Introduction

- Radio frequency (RF) spectrum is a finite resource to be shared by various applications.
- The explosive growth in wireless communications and other applications resulted in soared demand for wireless broadband, making spectrum increasingly congested.
- Spectrum sharing among disparate wireless systems is a solution to this problem: Particular interest between radar and communications.
- Many different ways are considered for spectrum sharing:
  - **Joint radar-communications**: Radar-centric or communication-centric
  - **Coexistence**: Radar and communication subsystems transmit respective waveforms
  - **Passive radar**: Radar is secondary with no control to the sources of opportunity

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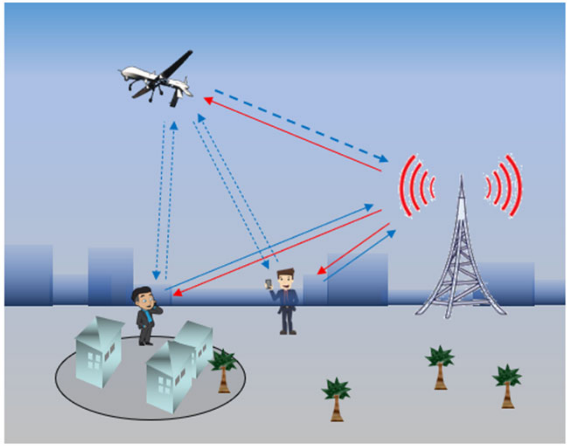
I. Joint radar-communications

Transmit system is shared by radar and communication functions

- Advantages:
  - Effective use of the signal spectrum and hardware
  - No or minimal mutual interference
- **Communication-centric:** Achieving radar sensing in a primary communication system
  - Automotive radar considers sensing using IEEE 802.11 family signals designed for wireless network
- **Radar-centric:** Embedding communication information in a primary radar system



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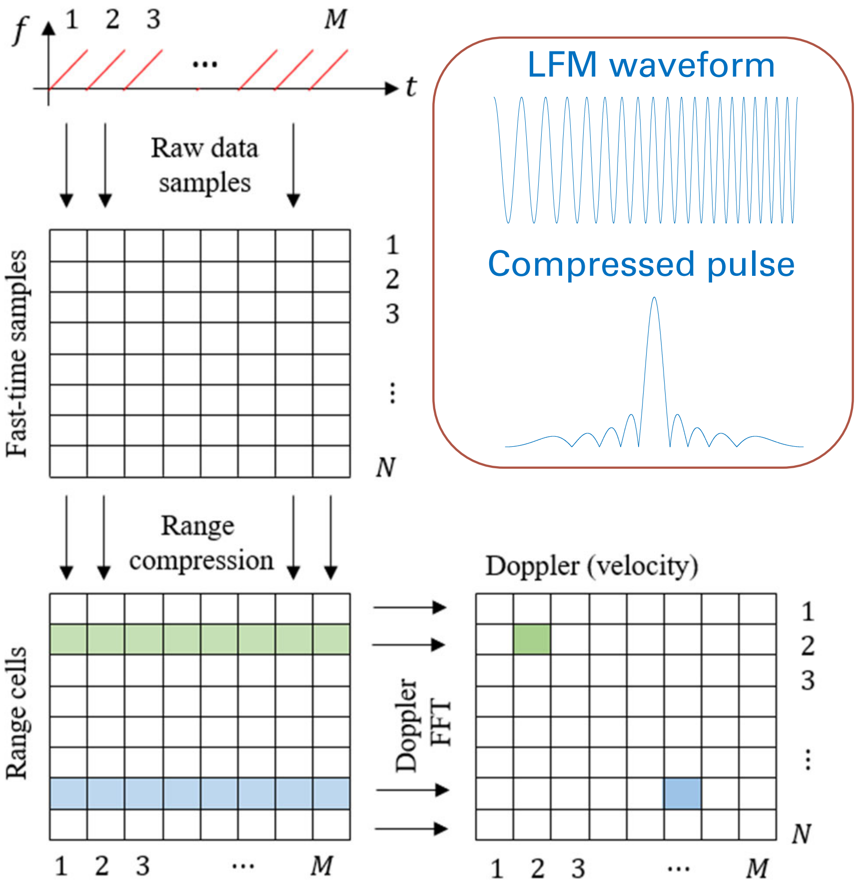
Radar operation

Typical radar operations:

- Periodically transmit the same probing waveforms
- Matched filtering of received signals provides range information
- Fourier transform of slow-time data yields target Doppler (velocity) information



Information embedding in radar signals

- Information embedding in radar signals
  - Different domains can be used: fast-time, slow-time, spatial
  - Depends on the level of radar flexibility
- **Fast-time:**
  - Traditional linear frequency modulated (LFM)
  - Coded sparse frequency / frequency-hopping
  - Orthogonal frequency-division multiplexing (OFDM)
  - Phase-modulation continuous-wave (PMCW)
- Factors to be considered
  - Transmit waveform and receiver filter maintain identical $s_i(t) * h_i(t)$?
  - Narrowband vs wideband intermediate frequency (IF) signals
  - Tolerance to peak-to-average power ratio (PAPR)
  - Ambiguity characteristics

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Information embedding in radar signals

Slow-time:

- On/off, phase …

Spatial:

- Sidelobe levels (SLLs), phase, index modulation…

- We mainly focus on spatial domain signaling that does not require major changes to the radar signal waveforms and the receiver structure
  - Radar mainbeams remain the same
  - May change SLLs and/or signal phase

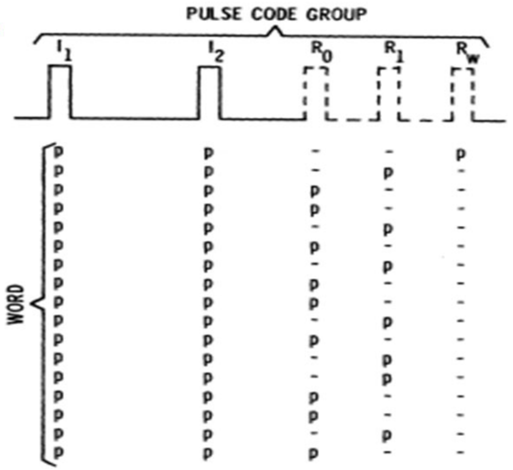
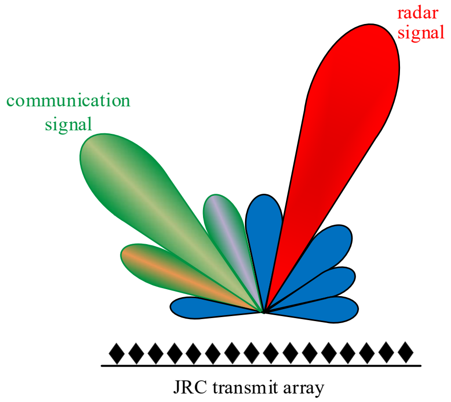


Image: Mealey



R. M. Mealey, "A method for calculating error probabilities in a radar communication system," IEEE Trans. Space Electronics and Telemetry, 1963.

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Sidelobe-based information embedding

Consider a communication user at a sidelobe direction θ_c and the steering vector is $\mathbf{a}(\theta_c)$.

Design two beamforming vectors $\mathbf{w}_1$ and $\mathbf{w}_2$ with

$$\mathbf{w}_1^{\mathrm{H}}\mathbf{a}(\theta_c) = \Delta_H, \qquad \mathbf{w}_2^{\mathrm{H}}\mathbf{a}(\theta_c) = \Delta_L$$

Convex optimization-based design (similar for $\mathbf{w}_2$):

$$\min_{\mathbf{w}_1} \max_{\theta} \left|G_{\mathrm{d}}(\theta)e^{j\phi(\theta)} - \mathbf{w}_1^H \boldsymbol{a}(\theta)\right|, \qquad \theta \in \boldsymbol{\Theta}$$

$$\text{subject to } \left|\mathbf{w}_1^H \boldsymbol{a}(\theta)\right| \le \epsilon, \qquad \theta \in \overline{\boldsymbol{\Theta}}$$

$$\mathbf{w}_1^H \boldsymbol{a}(\theta_c) = \Delta_H$$

$G_{\mathrm{d}}(\theta)$: Designed transmit radar beampattern

$\phi(\theta)$: Phase profile of choice

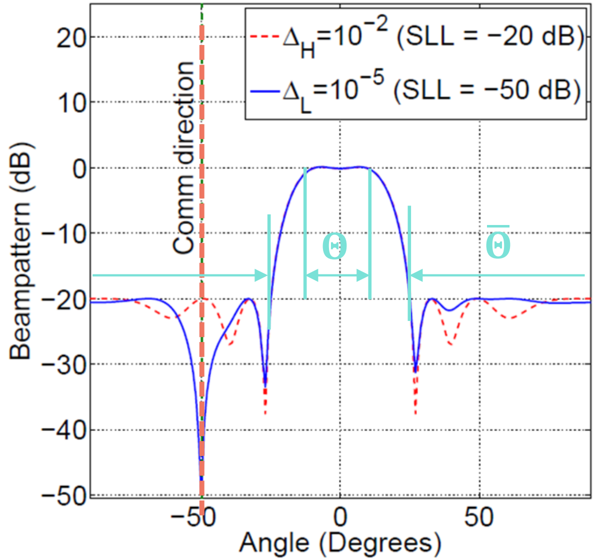
ϵ: Maximum SLL

$\boldsymbol{\Theta}$ and $\overline{\boldsymbol{\Theta}}$: Mainbeam and sidelobe regions

Illustrative example:

$\Delta_H = -20$ dB, $\Delta_L = -50$ dB

$\epsilon = -20$ dB, $\theta_c = -50^{o}$



A. Hassanien, M. G. Amin, Y. D. Zhang, and F. Ahmad, "Dual-function radar-communications: Information embedding using sidelobe control and waveform diversity," IEEE Trans. Signal Processing, 2016.

Multi-waveform signaling

Transmitted signal vector using waveform $\psi(t)$:

$$\mathbf{x}(t) = [(1-b)\mathbf{w}_1^* + b\mathbf{w}_2^*]\psi(t)$$

b: Information bit

Multiple orthogonal waveforms $\psi_1(t)$, …, $\psi_K(t)$:

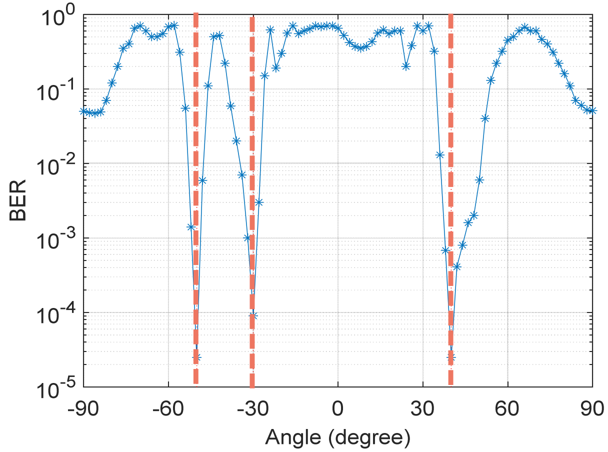
$$\mathbf{x}(t) = \sum_{k=1}^{K} [(1-b_k)\mathbf{w}_1^* + b_k\mathbf{w}_2^*]\psi_k(t)$$

b_1, …, b_K: Information bits

Features:

- Can support multiple communication users
- Low bit error rate (BER) only in specified communication directions
- Secure against eavesdropping in other directions

3 communication receivers at -50^o, -30^o, and 40^o



Phase-based signaling

Limitation of amplitude-based signaling:

- Communication enabled only in sidelobe region
- Sensitive to channel fading

Phase-based information embedding

- Enable information delivery to both mainbeam and sidelobe regions
- Less sensitive to channel fading

For single-pulse signaling, a reference waveform $\psi_0(t)$ is used to provide reference phase

$$\mathbf{s}(t) = \mathbf{w}^* \psi(t) + \mathbf{w}_0^* \psi_0(t)$$

$\psi(t)$: Waveform orthogonal to $\psi_0(t)$

$\mathbf{w}_0$: Beamforming weights provides the reference phase

$\mathbf{w}$: Beamforming weight vector carrying phase information

A. Hassanien, M. G. Amin, Y. D. Zhang, and F. Ahmad, “Phase-modulation based dual-function radar-communications," IET Radar, Sonar and Navigation, 2016.

Multiuser QAM scheme

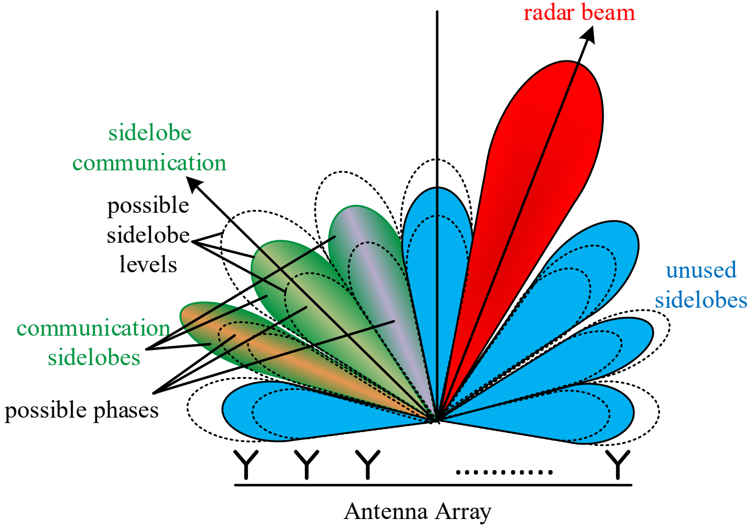
- Information embedding in radar waveforms can be extended to deliver separate information to multiple users with QAM scheme.

$$\min_{\mathbf{w}_n} \max_{\theta} \left| G_{\mathrm{d}}(\theta) e^{j\phi(\theta)} - \mathbf{w}_n^H \mathbf{a}(\theta) \right|, \qquad \theta \in \boldsymbol{\Theta}$$

$$\text{subject to } \left| \mathbf{w}_n^{\mathrm{H}} \mathbf{a}(\theta) \right| \leq \epsilon, \qquad \theta \in \bar{\boldsymbol{\Theta}}$$

$$\mathbf{w}_n^{\mathrm{H}} \mathbf{a}(\theta_c) = \Delta_{n,c} e^{j\phi_n(\theta_c)}, \qquad \forall \theta_c \in \boldsymbol{\Theta}_c$$

- To support C communication users with QAM of L amplitude levels and Q distinct phases, it requires $N = (LQ)^C$ beamforming weight vectors.
- Total number of embedded information is $C \log_2(LQ)$ bit per pulse.



A. Ahmed, Y. D. Zhang, and Y. Gu, "Dual-function radar-communications using QAM-based sidelobe modulation," Digital Signal Processing, 2018.

Optimum sensor selection

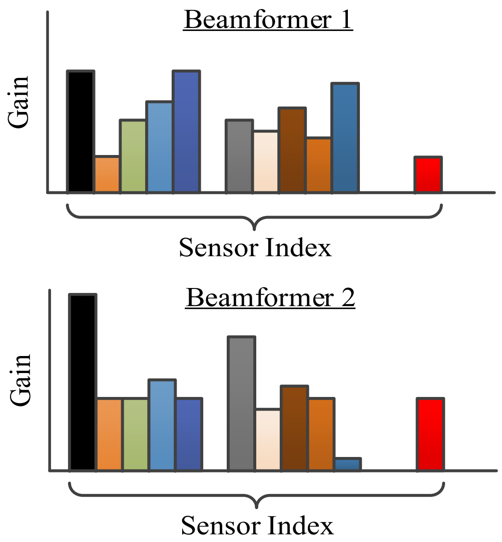
- For a large transmit array, we can perform antenna selection to reduce the number of RF chains.
- The following optimization uses a small number of antennas and exploits the minimum transmit power:

$$\min_{\mathbf{w}_n} \quad \sum_{n=1}^{N} \|\mathbf{w}_n\|_2^2 + \eta \|\mathbf{v} \odot \mathbf{w}_n\|_{1,2}$$

$$\text{subject to} \quad \left|G_{\mathrm{d}}(\theta)e^{j\phi(\theta)} - \mathbf{w}_n^H \mathbf{a}(\theta)\right| < \gamma_{\text{total}}, \theta \in \mathbf{\Theta}, n = 1, \cdots, N$$

$$\left|\mathbf{w}_n^{\mathrm{H}} \mathbf{a}(\theta)\right| \le \epsilon, \theta \in \overline{\mathbf{\Theta}}$$

$$\mathbf{w}_n^{\mathrm{H}} \mathbf{a}(\theta_c) = \Delta_n e^{j\phi_n(\theta_c)}, \forall \theta_c \in \mathbf{\Theta}_c$$



η: Parameter trading off between two objectives

$\mathbf{v}$: Weighting coefficients for re-weighted minimization

A. Ahmed, S. Zhang, and Y. D. Zhang, "Antenna selection strategy for transmit beamforming-based joint radar-communication system," Digital Signal Processing, 2020.

E. J. Candes, M. B. Wakin, S. P. Boyd, "Enhancing sparsity by reweighted l1 minimization," Journal of Fourier Analysis and Applications, 008

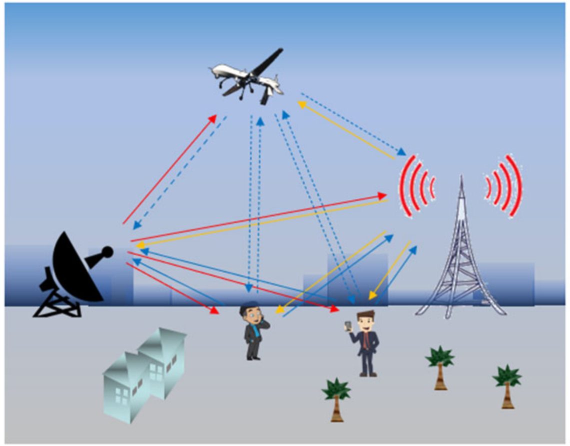
Remarks: Joint radar-communications

- We considered information embedding in radar-centric systems.
- In general, radar-centric information embedding can use fast-time, slow-time, and spatial domain resources in the radar system.
- We mainly focused on spatial domain information embedding that requires minimal modifications of existing radar operations and radar systems.
- Other forms of information embedding are possible as radar systems become more flexible in accepting different operation modes and system configurations.

II. Radar-communication coexistence

Coexistence of radar and communication systems permits flexible operations.

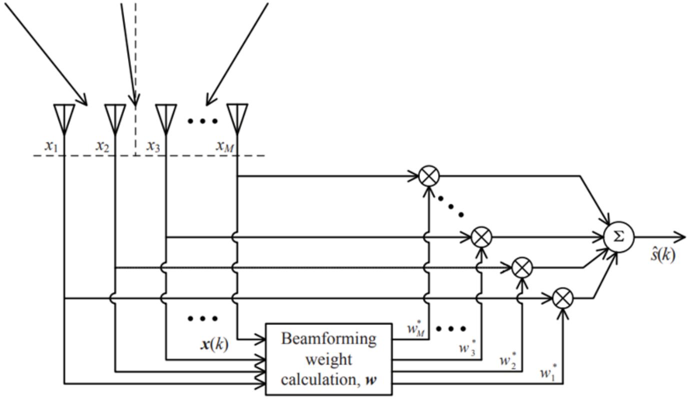
- Mutual interference exists between radar and communications
- Important to perform robust beamforming to mitigate mutual interference



Received array signal vector:

$$\mathbf{x}(t) = \mathbf{x}_s(t) + \mathbf{x}_j(t) + \mathbf{n}(t)$$

$\mathbf{x}_s(t) = \mathbf{a}(\theta_s)s(t)$: Desired signal vector
$\mathbf{a}(\theta_s)$: Steering vector of desired signal
$\mathbf{x}_j(t)$: Interference signal vector



Desired beamformer:

$$y(t) = \mathbf{w}^{\mathrm{H}}\mathbf{x}(t) = \hat{s}(t)$$

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Adaptive beamforming: MVDR

Consider minimum variance distortionless response (MVDR) beamformer:

$$\min_{\mathbf{w}} \mathbf{w}^{\mathrm{H}}\mathbf{R}_{\mathrm{j+n}}\mathbf{w} \quad \text{subject to} \quad \mathbf{w}^{\mathrm{H}}\mathbf{a}(\theta_s) = 1$$

$$\mathbf{R}_{\mathrm{j+n}} = \mathrm{E}\left[\left(\mathbf{x}_j(t) + \mathbf{n}(t)\right)\left(\mathbf{x}_j(t) + \mathbf{n}(t)\right)^{\mathrm{H}}\right]$$: Interference-plus-noise covariance matrix

- MVDR minimizes interference-plus-noise power while keeping the desired signal unaffected.
- The solution is given as

$$\mathbf{w}_{\mathrm{MVDR}} = \frac{\mathbf{R}_{\mathrm{j+n}}^{-1}\mathbf{a}(\theta_s)}{\mathbf{a}^{\mathrm{H}}(\theta_s)\mathbf{R}_{\mathrm{j+n}}^{-1}\mathbf{a}(\theta_s)}$$

- In practice, the interference-plus-noise covariance matrix $\mathbf{R}_{\mathrm{j+n}}$ is difficult to obtain.

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Adaptive beamforming: MPDR

Replacing $\mathbf{R}_{\mathrm{j+n}}$ with the data covariance matrix, $\mathbf{R}_{\mathbf{xx}} = \mathrm{E}[\mathbf{x}(t)\mathbf{x}^{\mathrm{H}}(t)]$, yields minimum power distortionless response (MPDR) beamformer:

$$\min_{\mathbf{w}} \mathbf{w}^{\mathrm{H}}\mathbf{R}_{\mathbf{xx}}\mathbf{w} \quad \text{subject to} \quad \mathbf{w}^{\mathrm{H}}\mathbf{a}(\theta_s) = 1$$

with solution given as

$$\mathbf{w}_{\mathrm{MPDR}} = \frac{\mathbf{R}_{\mathbf{xx}}^{-1}\mathbf{a}(\theta_s)}{\mathbf{a}^{\mathrm{H}}(\theta_s)\mathbf{R}_{\mathbf{xx}}^{-1}\mathbf{a}(\theta_s)}$$

- MPDR is commonly used because it is easier to implement and is equivalent to MVDR in the statistical sense with accurate estimate of $\mathbf{R}_{\mathbf{xx}}$ and $\mathbf{a}(\theta_s)$
- However, MPDR is not robust and its performance degrades when
  - The estimated covariance matrix $\widehat{\mathbf{R}}_{\mathbf{xx}}$ is inaccurate due to, e.g., insufficient snapshots
  - The presumed signal steering vector is inaccurate due to, e.g., imperfect estimation, multipath fading, or calibration errors

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Robust beamforming: Existing methods

Robust beamforming techniques by looking at

- Covariance matrix
- Steering vector

Examples:

- Diagonal loading in covariance matrix

$$\mathbf{R}_{\mathrm{DL}} = \widehat{\mathbf{R}}_{\mathbf{xx}} + \xi \mathbf{I}$$

ξ: diagonal loading factor (chosen ad hoc way)

- Worst-case beamforming accounting for errors in steering vector

$$\min_{\mathbf{w}} \mathbf{w}^{\mathrm{H}} \widehat{\mathbf{R}}_{\mathbf{xx}} \mathbf{w} \quad \text{subject to} \min_{\|\mathbf{e}\| \leq \epsilon} \mathbf{w}^{\mathrm{H}} (\bar{\mathbf{a}}_s + \mathbf{e}) \geq 1$$

$\bar{\mathbf{a}}_s$: Presumed steering vector of the desired signal

Robust beamforming: Proposed method

Robust adaptive beamforming based on reconstruction of the interference-plus-noise covariance matrix.

The interference-plus-noise covariance matrix can be estimated through direction-of-arrival (DOA) and power estimation for all signals except the desired one (or outside the desired signal region).

$$\mathbf{R}_{\mathrm{j+n}} = \sum_{k=1}^{K} \sigma_k^2 \mathbf{a}(\theta_k)\mathbf{a}^{\mathrm{H}}(\theta_k) + \sigma_{\mathrm{n}}^2 \mathbf{I}$$

K: number of interferers

σ_k^2: Interference power

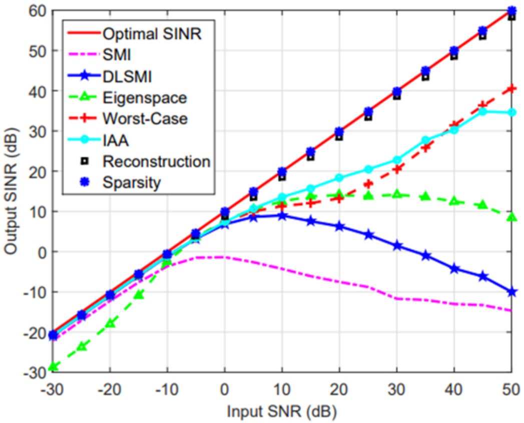
σ_{n}^2: noise power

Y. Gu, N. A. Goodman, and Y. D. Zhang, "Adaptive beamforming via sparsity-based reconstruction of covariance matrix," in A. De Maio, Y. C. Eldar, and A. Haimovich (eds.), Compressed Sensing in Radar Signal Processing, Cambridge University Press, 2019. Available at http://yiminzhang.com

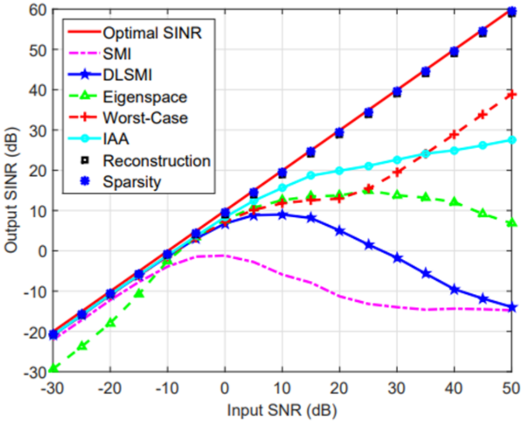
Robust beamforming: Proposed method

Example: 10-antenna uniform linear array (ULA), 1 desired signal with presumed direction 5^{o} and 2 interferers from -50^{o} and -20^{o}. 30 snapshots.

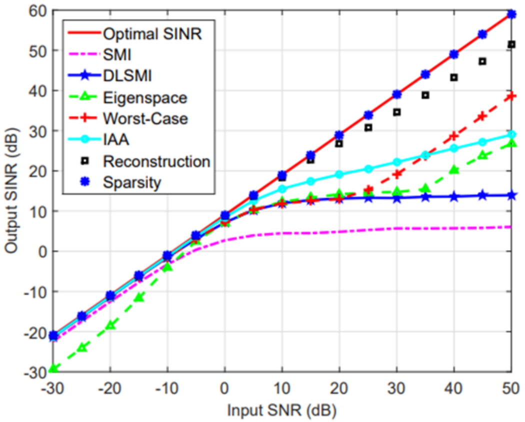
Random DOA mismatch (uniform in $[1^{\mathrm{o}}, 9^{\mathrm{o}}]$)



Coherent local scattering (4 paths $N(5^{\mathrm{o}}, (4^{\mathrm{o}})^2)$)



Incoherent local scattering (4 paths $N(5^{\mathrm{o}}, (4^{\mathrm{o}})^2)$)



- Proposed method consistently offers robust output signal-to-interference-plus-noise ratio (SINR) performance and outperforms existing methods

Y. Gu, N. A. Goodman, and Y. D. Zhang, "Adaptive beamforming via sparsity-based reconstruction of covariance matrix," in A. De Maio, Y. C. Eldar, and A. Haimovich (eds.), Compressed Sensing in Radar Signal Processing, Cambridge University Press, 2019. Available at http://yiminzhang.com

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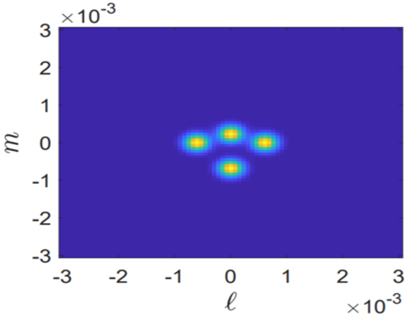
Robust beamforming

When steering vector has mismatch, we can estimate the true steering vector:

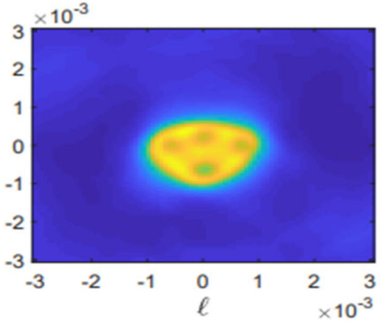
$$
\begin{aligned}
\min_{\mathbf{e}_\perp} \quad & (\bar{\mathbf{a}}_s + \mathbf{e}_\perp)^{\mathrm{H}} \mathbf{R}_{\mathrm{j+n}} (\bar{\mathbf{a}}_s + \mathbf{e}_\perp) \\
\text{Subject to} \quad & \bar{\mathbf{a}}_s^{\mathrm{H}} \mathbf{e}_\perp = 0 \\
& (\bar{\mathbf{a}}_s + \mathbf{e}_\perp)^{\mathrm{H}} \mathbf{R}_{\mathrm{j+n}} (\bar{\mathbf{a}}_s + \mathbf{e}_\perp) \leq \bar{\mathbf{a}}_s^{\mathrm{H}} \mathbf{R}_{\mathrm{j+n}} \bar{\mathbf{a}}_s
\end{aligned}
$$

- This is a convex quadratically constrained quadratic program (QCQP) problem.

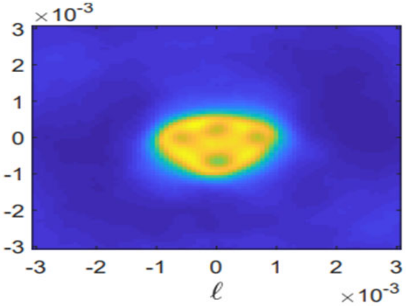
Example: Radio astronomy images using adaptive angular response (AAR)



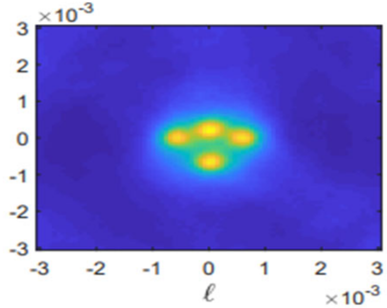
True image



Mismatch image



Worst-case image



QCQP-based image

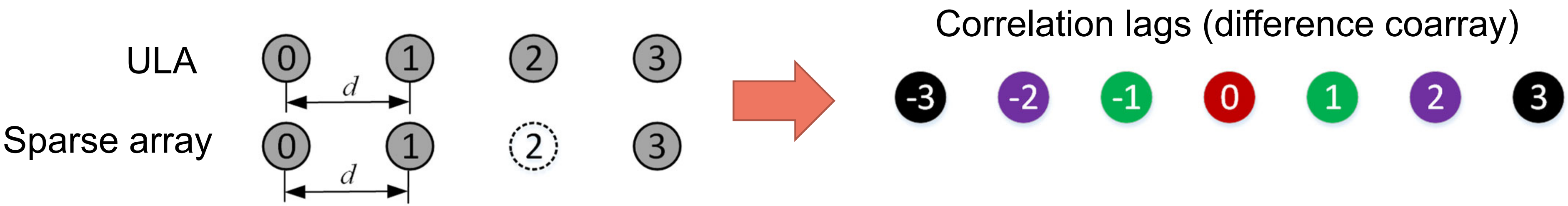
S. Zhang, Y. Gu, and Y. D. Zhang, "Robust astronomical imaging in the presence of radio frequency interference," Journal of Astronomical Instrumentation, 2019.

Sparse array and sparsity-based DOA estimation

For sparse arrays, the difference lags between sensor positions yield a large virtual array (difference coarray).

Consider a ULA, possibly with missing elements.

Due to the Toeplitz and Hermitian properties of the covariance matrix, the following two arrays yield the same covariance matrix.



$$\mathbf{R}_{\mathbf{xx}} = \mathrm{E}[\mathbf{x}\mathbf{x}^{\mathrm{H}}] = \begin{bmatrix} E[x_0x_0^*] & E[x_0x_1^*] & E[x_0x_2^*] & E[x_0x_3^*] \\ E[x_1x_0^*] & E[x_1x_1^*] & E[x_1x_2^*] & E[x_1x_3^*] \\ E[x_2x_0^*] & E[x_2x_1^*] & E[x_2x_2^*] & E[x_2x_3^*] \\ E[x_3x_0^*] & E[x_3x_1^*] & E[x_3x_2^*] & E[x_3x_3^*] \end{bmatrix}$$

Sparse array and sparsity-based DOA estimation

Assume Q uncorrelated signals, $s_q(t)$, impinging from angles $\Theta = [\theta_1, \dots, \theta_Q]^{\mathrm{T}}$.

Received signal vector at an N-element array

$$\mathbf{x}(t) = \sum_{q=1}^{Q} \mathbf{a}(\theta_q) s_q(t) + \mathbf{n}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t)$$

$\mathbf{A}$=$[\mathbf{a}(\theta_1), \dots, \mathbf{a}(\theta_Q)]$: Manifold matrix with $\mathbf{a}\left(\theta_Q\right) = [1, \dots, e^{j2\pi d_N \sin(\theta_q)/\lambda}]^T$

$\mathbf{s}(t) = [s_1(t), \dots, s_Q(t)]^{\mathrm{T}}$: Signal vector

$\mathbf{n}(t) \sim CN(0, \sigma_{\mathrm{n}}^2 \mathbf{I})$: Noise vector

Covariance matrix of $\mathbf{x}(t)$

$$\mathbf{R}_{\mathbf{xx}} = \mathrm{E}[\mathbf{x}(t)\mathbf{x}^{\mathrm{H}}(t)] = \mathbf{A}\, \mathbf{R}_{\mathbf{ss}} \mathbf{A}^{\mathrm{H}}$$

$$\mathbf{R}_{\mathbf{ss}} = \mathrm{E}[\mathbf{s}(t)\mathbf{s}^{\mathrm{H}}(t)] = \mathrm{diag}([\sigma_1^2, \dots, \sigma_Q^2])$$

Sparse array and sparsity-based DOA estimation

Vectorizing $\mathbf{R}_{\mathbf{xx}}$ yields

$$\mathbf{z} = \mathrm{vec}(\mathbf{R}_{\mathbf{xx}}) = \widetilde{\mathbf{A}}\,\mathbf{b} + \sigma_n^2\,\tilde{\mathbf{I}} = \mathbf{A}^{\mathrm{o}}\mathbf{b}^{\mathrm{o}}$$

$\widetilde{\mathbf{A}} = \left[\boldsymbol{a}(\theta_1)\otimes\boldsymbol{a}^*(\theta_1), \dots, \boldsymbol{a}(\theta_Q)\otimes\boldsymbol{a}^*(\theta_Q)\right]$: Manifold matrix for the difference coarray

$\mathbf{b} = \left[\sigma_1^2, \dots, \sigma_Q^2\right]^{\mathrm{T}}$: Source power vector

$\tilde{\mathbf{I}} = \mathrm{vec}(\mathbf{I}_N)$

$\mathbf{A}^{\mathrm{o}} = [\widetilde{\mathbf{A}}, \tilde{\mathbf{I}}]$

$\mathbf{b}^{\mathrm{o}} = [\mathbf{b}^{\mathrm{T}}, \sigma_{\mathrm{n}}^2]^{\mathrm{T}}$

$\mathbf{z}$ amounts to the received data of a virtual array (difference coarray)

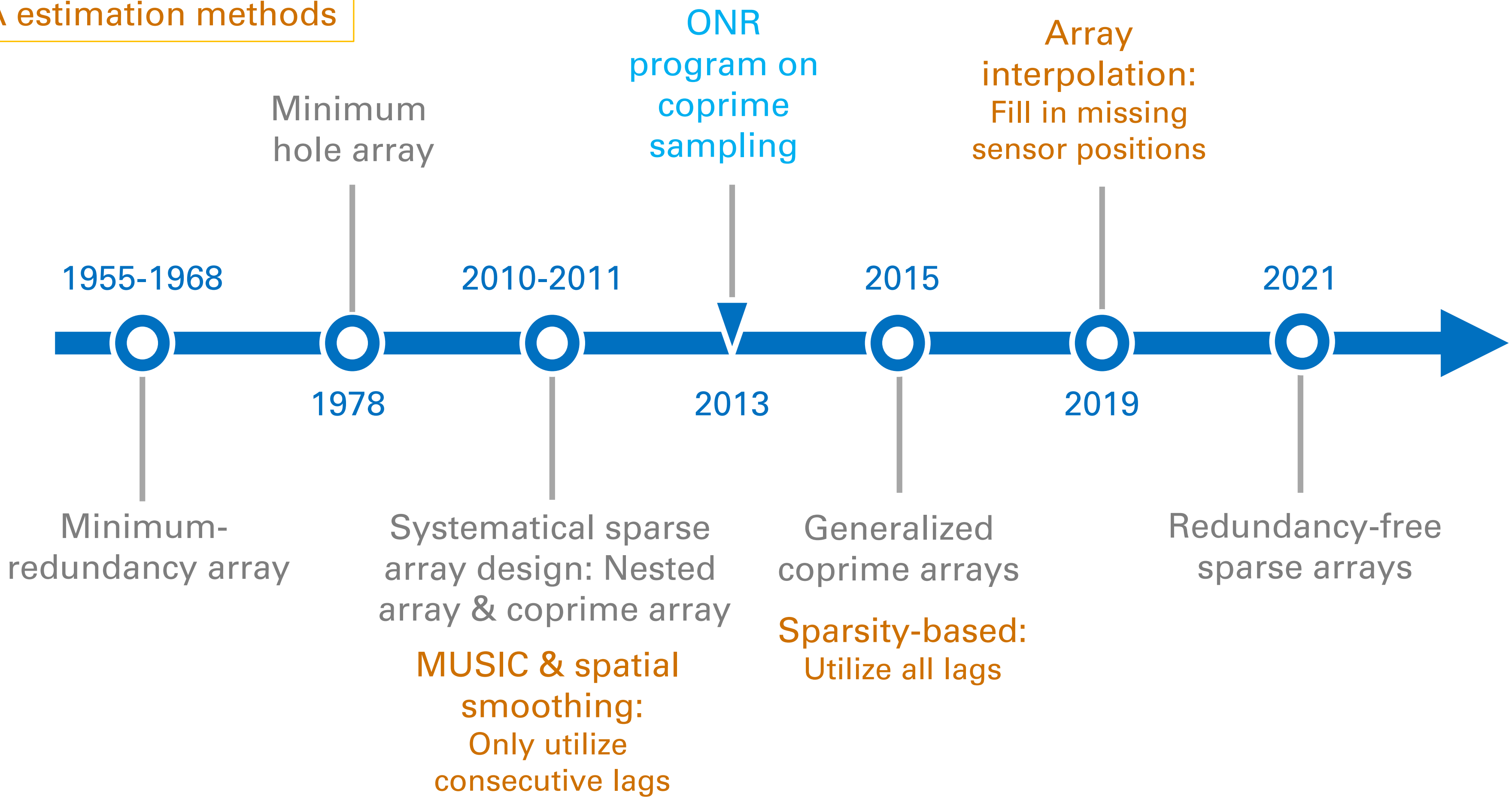
- Manifold matrix corresponds to more virtual sensors than physical antennas
- Only has a single snapshot corresponding to vector $\mathbf{b}$

Sparse array design and DOA estimation problems:

- How to design the sparse arrays?
- How to perform DOA estimation from the coarray data $\mathbf{z}$?

Sparse array and sparsity-based DOA estimation

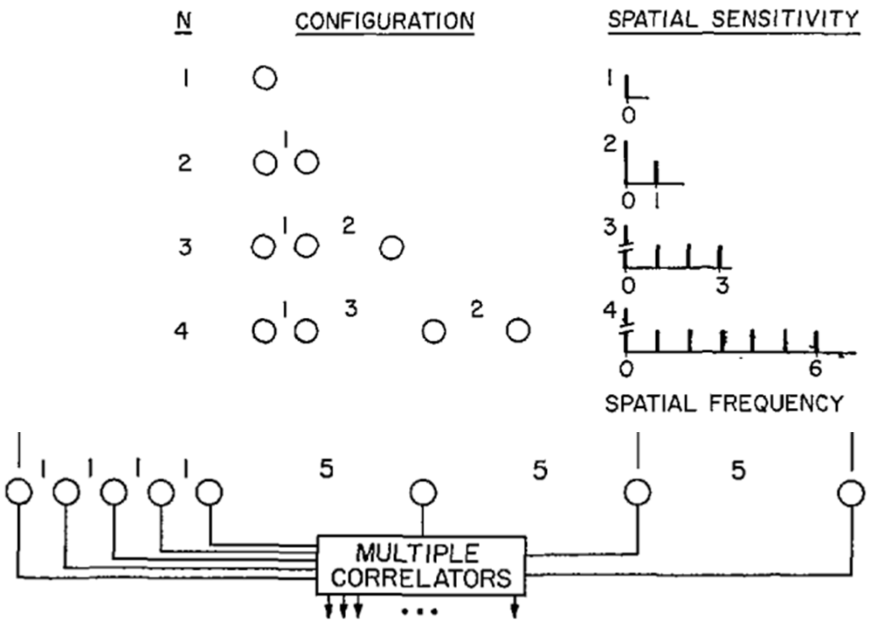
Sparse array designs
DOA estimation methods



Sparse array: Classical sparse arrays

Minimum-redundancy array: For a given number of antennas, maximizes the number of consecutive virtual sensors in the resulting difference coarray

- Finding the array configuration is not easy except for some "nested" structures
- Redundancy $R = \frac{1}{2}N(N-1)/N_{\max}$ varies and is $1.217 \leq R \leq 1.332$ for large N
- Restricted array: All lags are consecutive
- General array: Lags contain holes

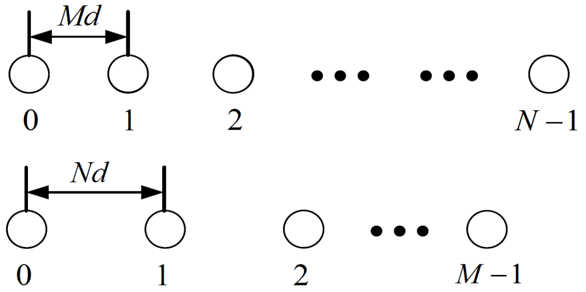


J. Arsac, "Nouveau reseau pour l'observation radioastronomique de la brilliance sur le soleil a 9350 Mc/s," Comptes Rendus de l'Académie des Sciences, 1955.

A. Moffet, "Minimum-redundancy linear arrays," IEEE Trans. Antennas and Propagation, 1968.

Sparse array: Coprime array

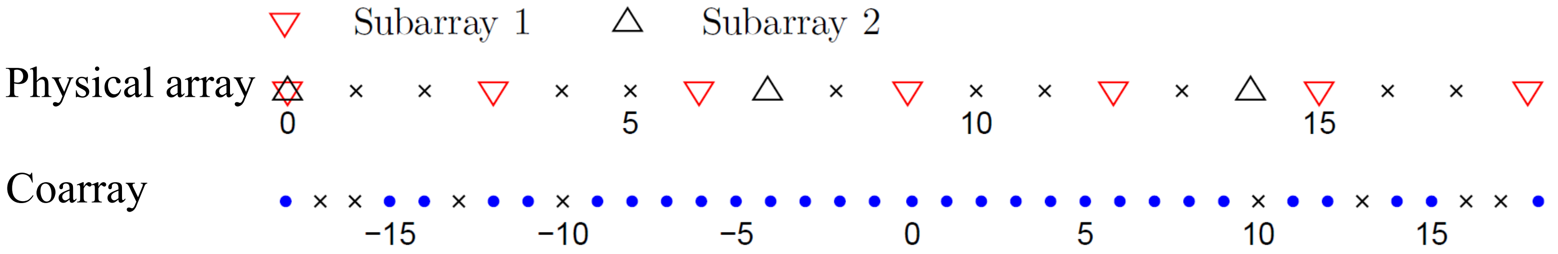
Coprime array: utilizes a pair of uniform linear subarrays with M and N being coprime integers



MUSIC-based DOA estimation:

- Spatial smoothing is applied to $\mathbf{z}\mathbf{z}^H$ to obtain a full-rank matrix (only consecutive lags are used)

Example: $M = 3$ and $N = 7$



P. P. Vaidyanathan and P. Pal, "Sparse sensing with co-prime sampler and arrays," IEEE Trans. Signal Processing, 2011

P. Pal and P. P. Vaidyanathan, "Coprime sampling and the MUSIC algorithm," IEEE Digit. Signal Process. Workshop/ IEEE Signal Process. Educ. Workshop, 2011

Sparse array: Generalized coprime array

Sparsity-based DOA estimation:

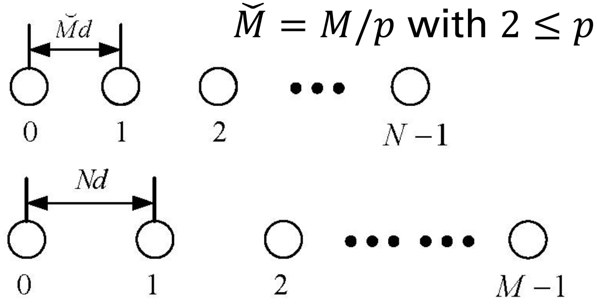
$$\min_{\mathbf{z}} \|\mathbf{b}^{o}\|_0 \ \text{subject to} \ \|\mathbf{z} - \mathbf{A}^{o}\mathbf{b}^{o}\| \le \epsilon$$

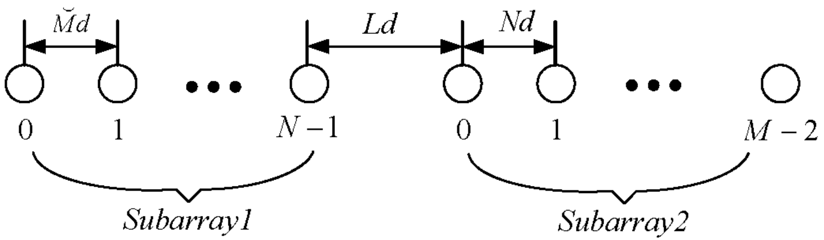
- All difference lags can be utilized in sparsity-based DOA estimation

Generalized coprime arrays: Flexible sparse array design

- CACIS compresses the interelement spacing of one subarray: More consecutive lags
- CADiS displaces two subarrays: Increases unique lags and reduces mutual coupling

$\breve{M} = M/p$ with $2 \le p \le M$





S. Qin, Y. D. Zhang, and M. G. Amin, "Generalized coprime array configurations for direction-of-arrival estimation," IEEE Trans. Signal Processing, 2015.

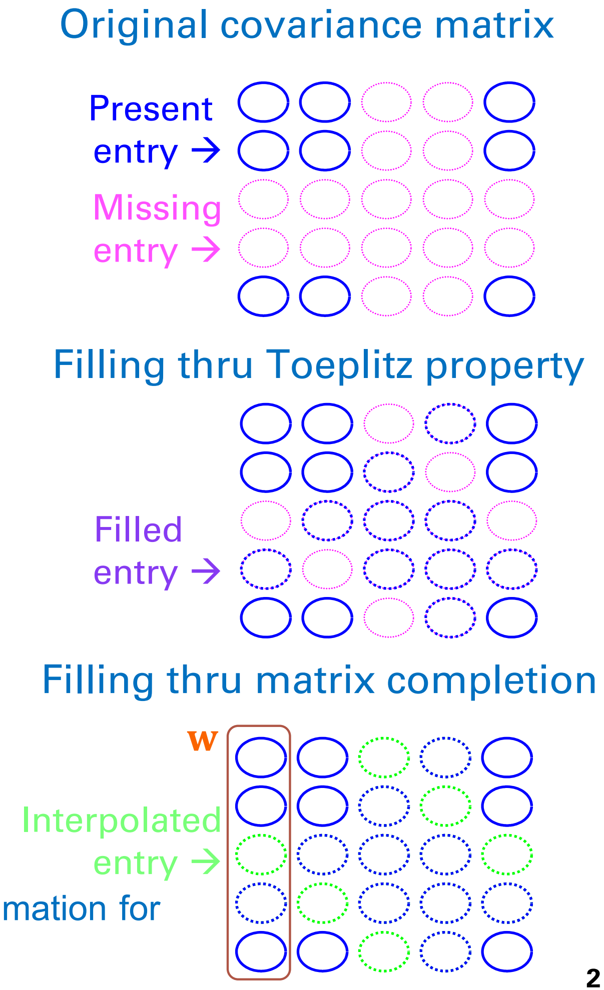
Sparse array and sparsity-based DOA estimation

Array interpolation for sparse arrays: Missing entries in the covariance matrix can be interpolated through matrix completion exploiting the Toeplitz property.

$$\begin{aligned} \min_{\mathbf{w}} \quad & \operatorname{rank}(\mathcal{T}(\mathbf{w})) \\ \text{s.t.} \quad & \left\| \mathcal{T}(\mathbf{w}) \circ \mathbf{B} - \widehat{\mathbf{R}} \right\|_F^2 \le \delta \\ & \mathcal{T}(\mathbf{w}) \succcurlyeq 0 \end{aligned}$$

$\mathcal{T}(\mathbf{w})$: Hermitian and Toeplitz matrix with $\mathbf{w}$ as the first column
$\mathbf{B}$: Binary matrix with 1 in observed entries
$\widehat{\mathbf{R}}$: Observed covariance matrix with missing entries

- $\operatorname{rank}(\mathcal{T}(\mathbf{w}))$ is nonconvex and can be relaxed as nuclear norm $\|\mathcal{T}(\mathbf{w})\|_* = \operatorname{trace}(\sqrt{\mathcal{T}^{\mathrm{H}}(\mathbf{w})\mathcal{T}(\mathbf{w})})$
- Supports gridless DOA estimation using, e.g., MUSIC
- More robust to noise and number of samples



C. Zhou, Y. Gu, X. Fan, Z. Shi, G. Mao, and Y. D. Zhang, "Direction-of-arrival estimation for coprime array via virtual array interpolation," IEEE Trans. Signal Processing, 2018.

Sparse array and sparsity-based DOA estimation

Optimized redundancy-free sparse array design

- Array interpolation supports optimized array design that
  - Redundancy-free to achieve the highest DOFs: Non-zero lags appear at most once
  - Allow holes in lags: Further add DOFs and reduce mutual coupling
- N-element linear array with positions at $0 = p_1 d < p_2 d < \cdots < p_N d,\ d = \lambda/2$

$$
\begin{aligned}
\min_{p_{n \in Z^+}, \forall n} \quad & p_N \\
\text{subject to} \quad & p_N \geq A_{\text{desired}} \\
& p_i + \beta \leq p_{i+1}, \qquad i = 1, \cdots, N-1 \\
& p_i - p_j \neq p_k - p_l, \qquad i > j, k > l, j \neq l
\end{aligned}
$$

A_{desired}: Desired array aperture

$\beta \geq 1$: Minimum spacing between two adjacent antennas

  - Can be solved using mixed-integer programming

A. Ahmed and Y. D. Zhang, "Generalized non-redundant sparse array designs," IEEE Trans. Signal Processing, 2021.

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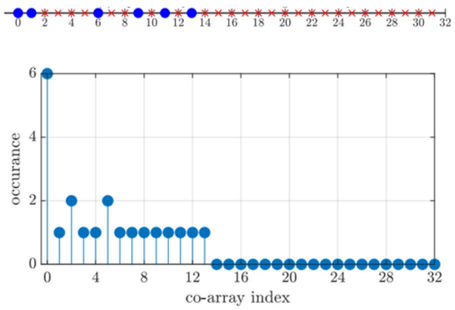
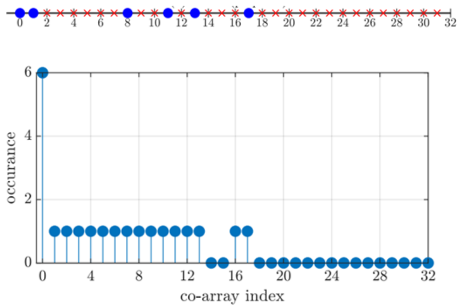
Sparse array and sparsity-based DOA estimation

Example: 6-antenna linear array (desired array aperture $22d$; min. spacing $2d$)

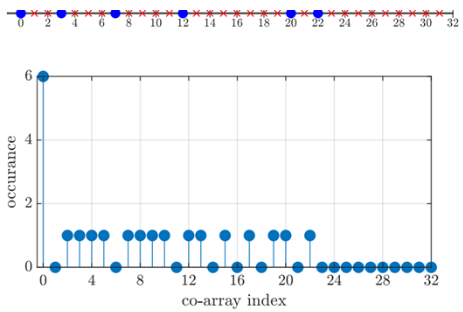
Minimum-redundancy array

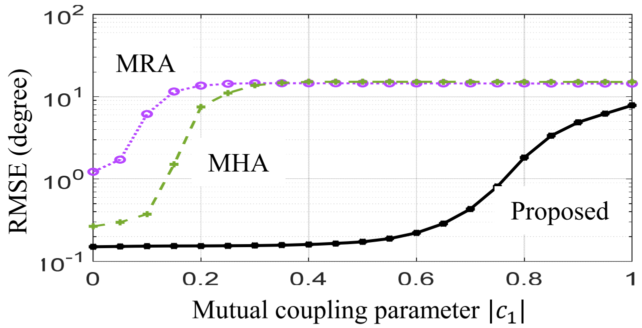


Minimum hole array



Proposed





RMSE for two closely spaces source case

A. Ahmed and Y. D. Zhang, "Generalized non-redundant sparse array designs," IEEE Trans. Signal Processing, 2021.

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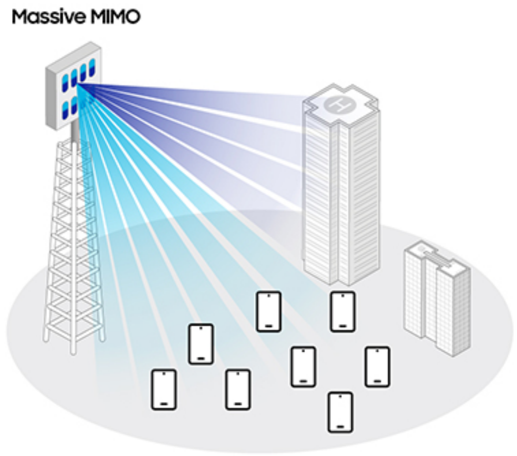
Sparse array and sparsity-based DOA estimation

- Massive MIMO can reduce the number of RF chains using compressed sampling:

$$\boldsymbol{y}(t) = \boldsymbol{\Phi}\boldsymbol{x}(t) = \boldsymbol{\Phi}\mathbf{A}\boldsymbol{s}(t) + \boldsymbol{\Phi}\boldsymbol{n}(t)$$

$\boldsymbol{\Phi} \in \mathcal{C}^{M\times N}$: compressed sampling matrix ($M \ll N$)

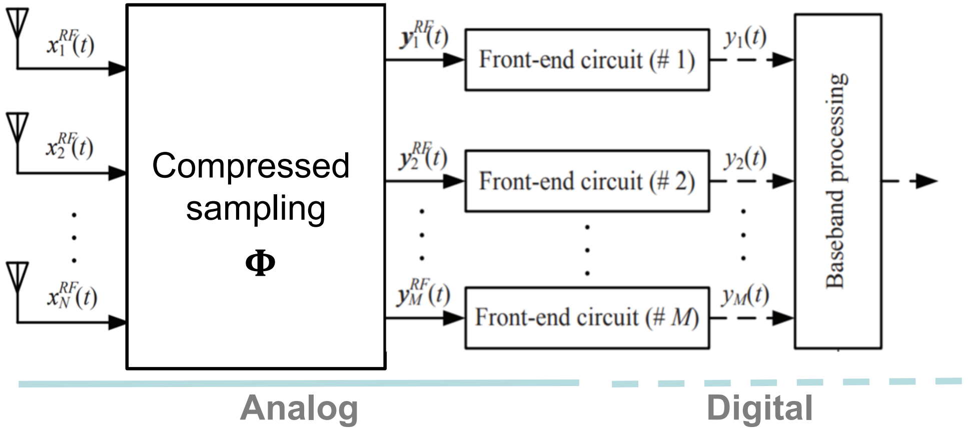


- Analog compression matrix $\boldsymbol{\Phi}$ is optimized by maximizing the mutual information between signal direction θ and compressed output $\boldsymbol{y}$:

$$I(\boldsymbol{y};\theta) = h(\boldsymbol{y}) - h(\boldsymbol{y}|\theta)$$

$h(\boldsymbol{y})$: Differential entropy of vector $\boldsymbol{y}$

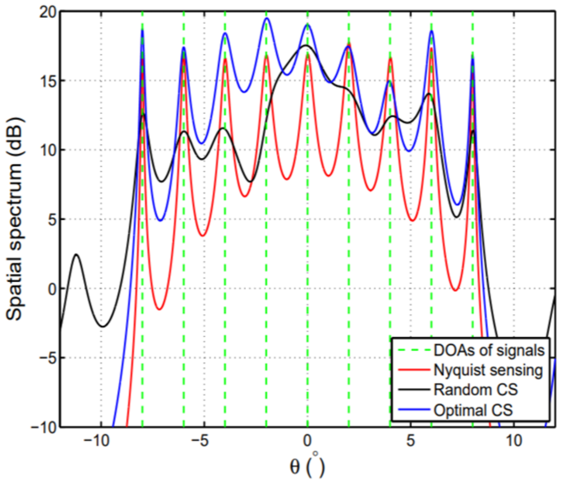
$h(\boldsymbol{y}|\theta)$: Conditional differential entropy of $\boldsymbol{y}$ given θ



Y. Gu and Y. D. Zhang, "Compressive sampling optimization for user signal parameter estimation in massive MIMO systems," Digital Signal Processing, 2019.

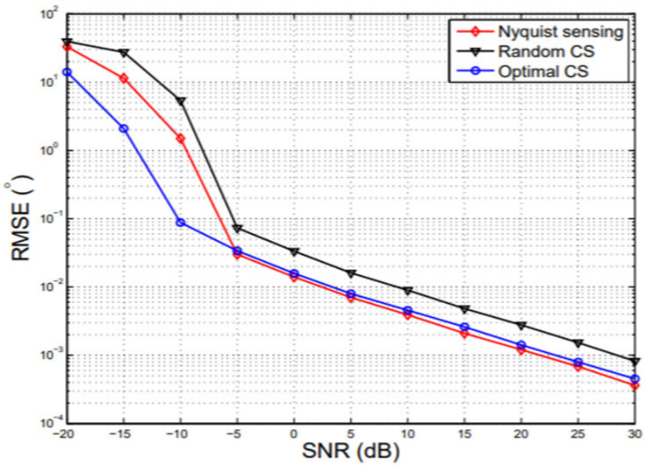
Sparse array and sparsity-based DOA estimation

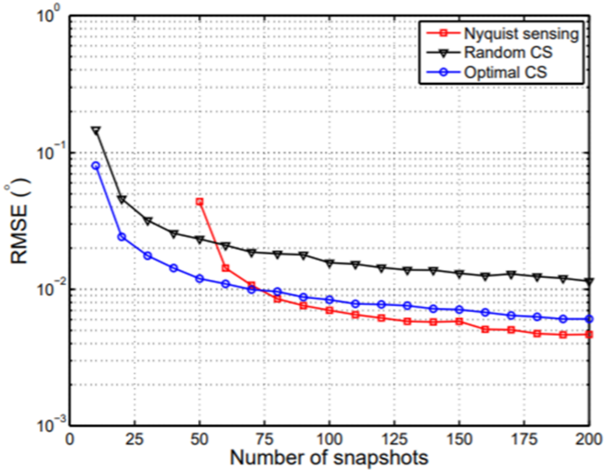
- 50-element ULA with $\lambda/2$ spacing
- 10 RF chains $(N/M = 5)$
- 9 sources from $[-8^{o}{:}2{:}8^{o}]$ with 20 dB input SNR
- 100 snapshots



Root mean square error (RMSE) of DOA estimates

- Insignificant SNR loss (Random CS suffers 7 dB SNR loss)
- Clear advantage in low SNR and small number of snapshots





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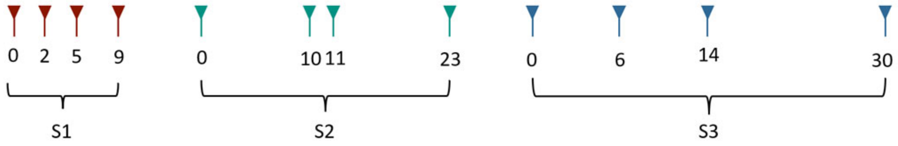
Robust DOA estimation exploiting machine learning

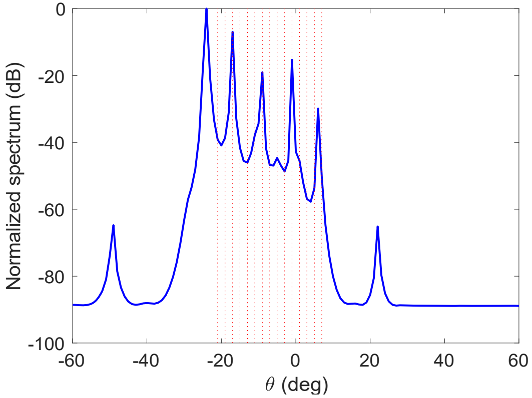
Deep learning can be used to provide robust DOA and channel estimation.

Consider a partially calibrated distributed array using non-coherent processing:

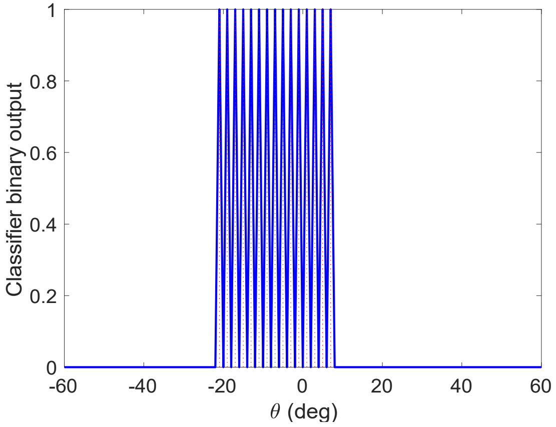


- A neural network is trained to perform DOA estimation with antenna gain/phase errors
- As a result, it provides robust DOA estimation in the presence of such errors

M. S. R. Pavel, M. W. T. S. Chowdhury, Y. D. Zhang, D. Shen, and G. Chen, "Machine learning-based direction-of-arrival estimation exploiting distributed sparse arrays," Asilomar Conference on Signals, Systems, and Computers, 2021.



MUSIC pseudo-spectrum



Angular indices from DL

Remarks: Radar-communication coexistence

- Coexistence of different wireless systems introduces mutual interference.
- We considered robust beamforming to enable effective interference cancellation and desired signal preservation.
- DOA estimation, long considered as a radar task, has become an important part of communications to perform robust beamforming and channel estimation.
- Sparse array designs and sparsity-based processing provide great potentials to enhance sensing and communication capability and performance with reduced complexity.
- Machine learning methods can be trained to be robust to environment changes and calibration errors.

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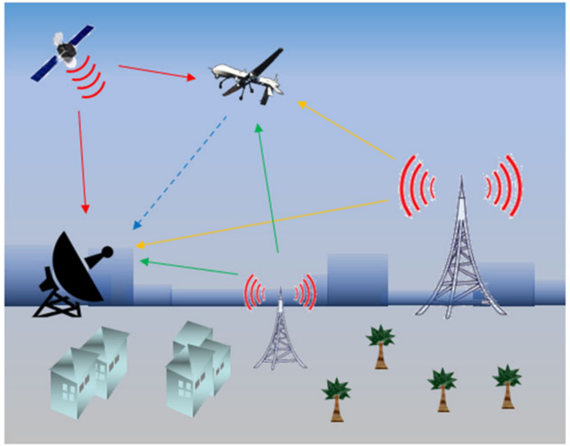
III. Passive radar

Exploits signals of opportunity that are designed for other applications such as broadcasting, wireless communications, and satellite navigation.

Pros:

- No dedicated spectrum needed for radar
- Low-cost implementation
- Multimodes and multistatic diversity
- Silent electromagnetic operations



Cons:

- Narrow signal bandwidth
- Waveforms not optimized for radar sensing
- Third-party illuminators

H. Li, Y. D. Zhang, and B. Himed, "Signal Processing for Passive Radar," tutorial given at 2019 IEEE Radar Conference, https://ieeexplore.ieee.org/document/8835736

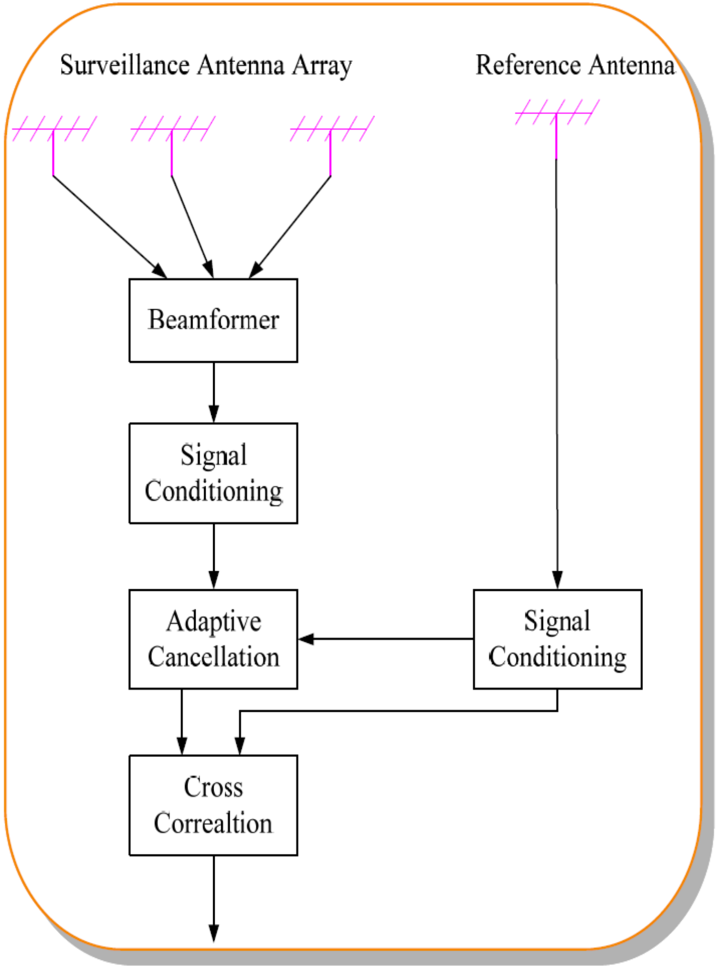
Passive radar

Illuminators	Frequency	Modulation, Bandwidth	Typical EIRP
Analog FM Radio	~ 100 MHz	FM, 50 kHz (Composite signal)	Up to 250 kW (UK) 4500 FM Tx ≥ 5kW (US)
Digital Audio Broadcast	~220 MHz	COFDM, 220 kHz	10kW
Cellular Phone (GSM)	900 MHz, 1.8 GHz	GMSK, FDM/TDMA/FDD, 200 kHz	100W
Cellular Phone (3G)	~2 GHz	TD-CDMA , 3.84 MHz TD-SCDMA, 1.28 MHz	100W
Analog UHF TV	~550 MHz	VSB AM (vision), 64μs Repetition Rate; FM (sound), 5.5 MHz	1 MW (UK)
Digital TV	~750 MHz	DVB-T(C-OFDM), Europe/Australia ISDB-T (OFDM, 2D Interleaving), Japan, S. America ATSC (8VSB), USA DTMB(TDF-OFDM), China 6MHz	8 kW, (WKTV-DT 29 : ERP=708kW)
DBS TV, Satellite Radio	~11-12GHz ~2.33 GHz		52dBW

Passive radar

- **Surveillance channel**
  - Receiver channel to monitor target activity
  - Interference suppression

- **Reference channel**
  - Dedicated receiver channel to monitor the source signal
  - Provide a reference for cross correlation
  - Also used to cancel directed source signal observed in surveillance channel



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Passive radar

Sparsity and group sparsity-based processing is attractive in passive radar

- Low bandwidth
- Availability of multiple sources of opportunity
- Sparse scene or motion parameters

We have demonstrated the usefulness of sparsity and group sparsity-based techniques for

- Radar imaging
- Moving target tracking
- Clutter suppression

Tutorial and review article:

H. Li, Y. D. Zhang, and B. Himed, “Signal Processing for Passive Radar,” tutorial given at 2019 IEEE Radar Conference, https://ieeexplore.ieee.org/document/8835736

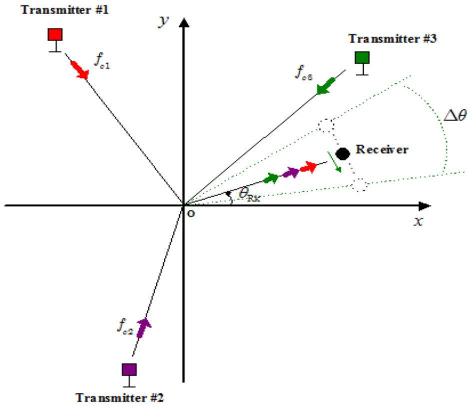
Y. D. Zhang, M. G. Amin, and B. Himed, "Structure-aware sparse reconstruction and applications to passive multi-static radar," IEEE Aerospace and Electronic Systems Magazine, 2017.

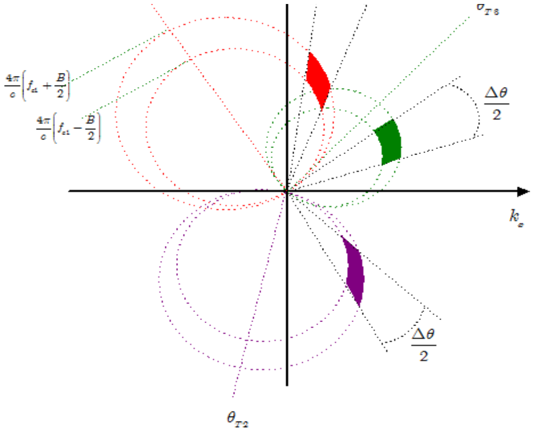
Passive radar: SAR imaging

- In synthetic aperture radar (SAR), the image is obtained as the Fourier transform of the wavenumber-domain observations.
- In passive radar-based SAR, the observed wavenumber regions are randomly sampled, thus are sparse and discontinuous.
- Conventional Fourier-based techniques (back-projection) do not provide high quality SAR imaging:
  - Narrow bandwidth results in low range resolution
  - Random sampling yields sidelobes and noise
  - Overall problem is ill-posed

X. Mao, Y. D. Zhang, and M. G. Amin, "Low-complexity sparse reconstruction for high-resolution multi-static passive SAR imaging," EURASIP Journal on Advanced Signal Processing, 2014.

Example: 3 illuminators





Wavenumber-domain observations

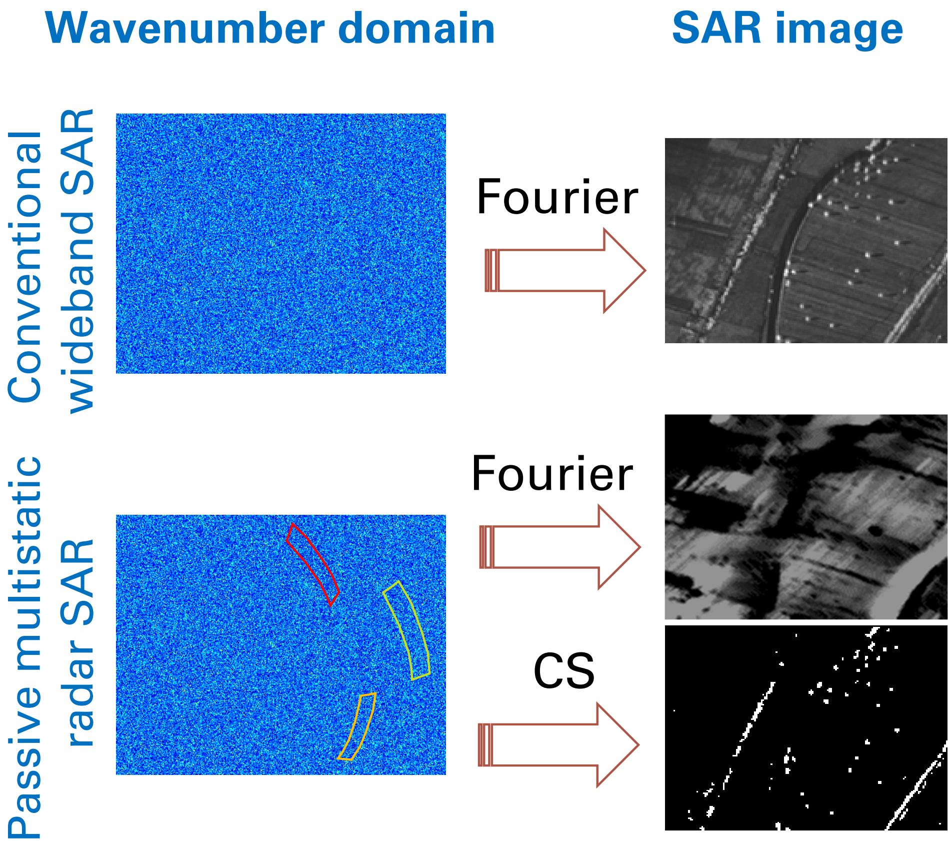
Passive radar: SAR imaging

In passive radar SAR:

- Cannot design dictionary matrix
- Limited flexibility of observations
- High coherence between closely spaced pixels

Group sparsity-based methods exploit multistatic observations to achieve high-resolution image:

- Improves both azimuth and range resolution
- Easily handle angle-dependence of the target reflectivity



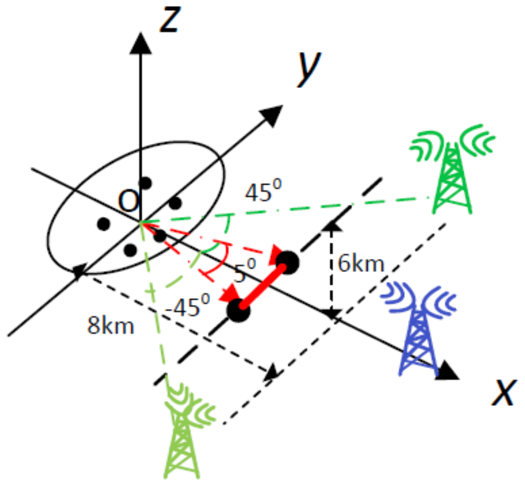
Q. Wu, Y. D. Zhang, M. G. Amin, and B. Himed, "High-resolution passive SAR imaging exploiting structured Bayesian compressive sensing," IEEE Journal of Selected Topics in Signal Processing, 2015.

Passive radar: SAR imaging

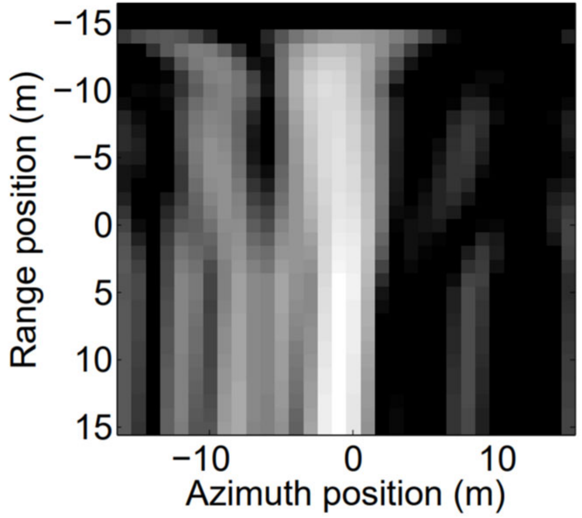
DVB-T signal with 7.8 MHz bandwidth: Range resolution of 20 m

3 illuminators at -45^o, 0^o, 45^o; Azimuth angle width: 5^o

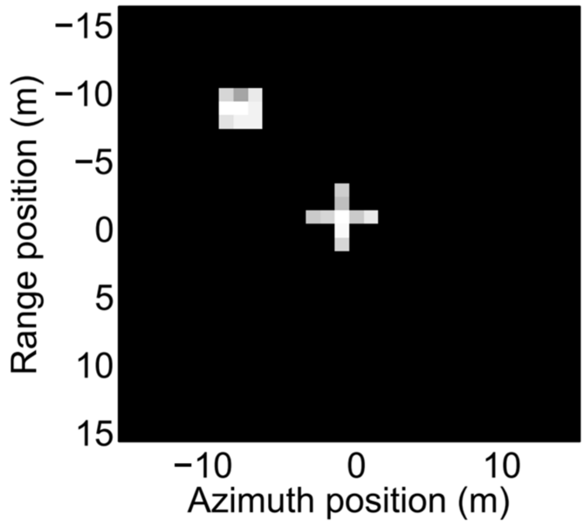
Complex Bayesian compressive sensing compared with backprojection



Multi-angle SAR



Backprojection result

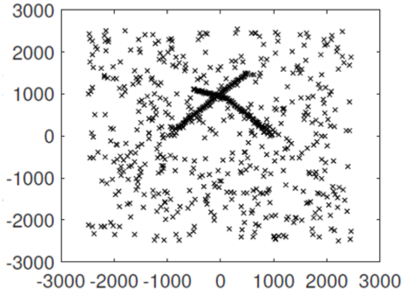


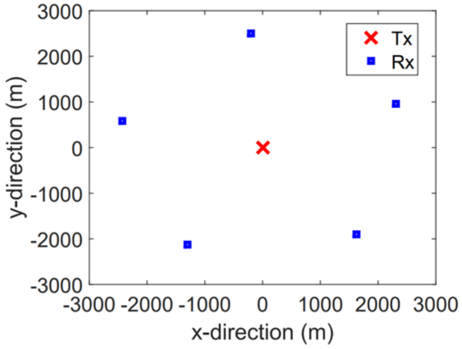
Compressive sensing result

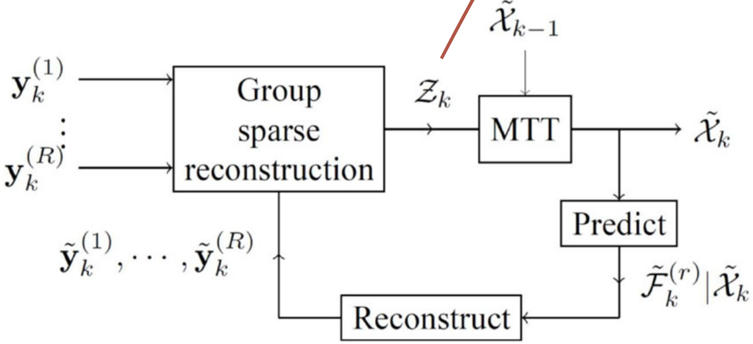
Q. Wu, Y. D. Zhang, M. G. Amin, and B. Himed, "High-resolution passive SAR imaging exploiting structured Bayesian compressive sensing," IEEE Journal of Selected Topics in Signal Processing, 2015.

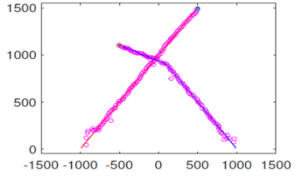
Passive radar: Target localization and tracking

- Consider a passive multistatic radar consisting of an illuminator and multiple receivers.
- We use Doppler-only observations that allow data fusion with very low data traffic.
- Observations include noise, clutter, and missed samples.
- Group sparse reconstruction followed by multi-target tracking (MTT) filter.









S. Subedi, Y. D. Zhang, M. G. Amin, and B. Himed, "Group sparsity based multi-target tracking in passive multi-static radar systems using Doppler-only measurements," IEEE Trans. Signal Processing, 2016.

S. Subedi, Y. D. Zhang, M. G. Amin, and B. Himed, "Cramer-Rao type bounds for sparsity-aware multi-sensor multi-target tracking," Signal Processing, 2018.

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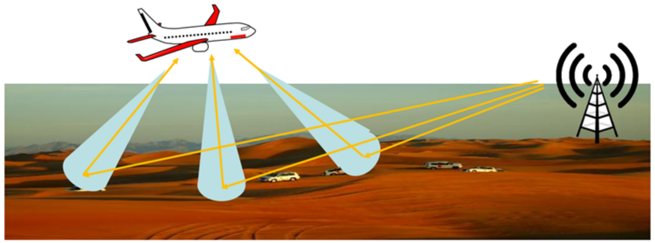
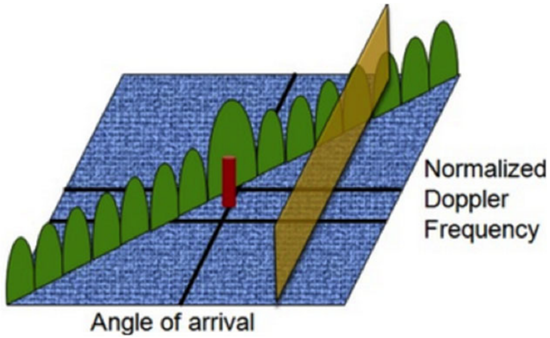
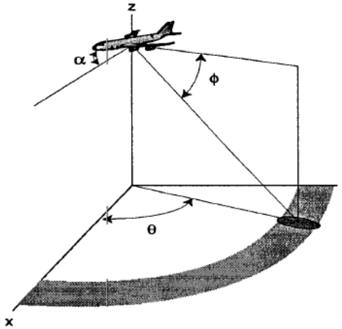
Passive radar: Space-time adaptive processing



- Space-time adaptive processing (STAP) is a key to detect slowly moving targets for airborne radar.
- Clutter Doppler in airborne radar:
  - Angle-dependent: Require joint space-time processing
  - Different to ground radars (clutter Doppler near zero)



- Estimation of clutter covariance matrix:
  - A high number of samples are required to ensure full rank of clutter covariance matrix
  - Cannot repeat time-domain samples because radar is moving
  - Use nearby range cells where signals are absent (assuming homogeneity)
  - Number of neighboring range cells are limited in passive radar due to its low range resolution



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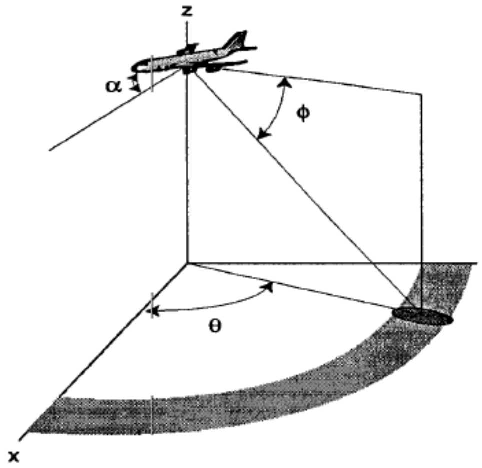
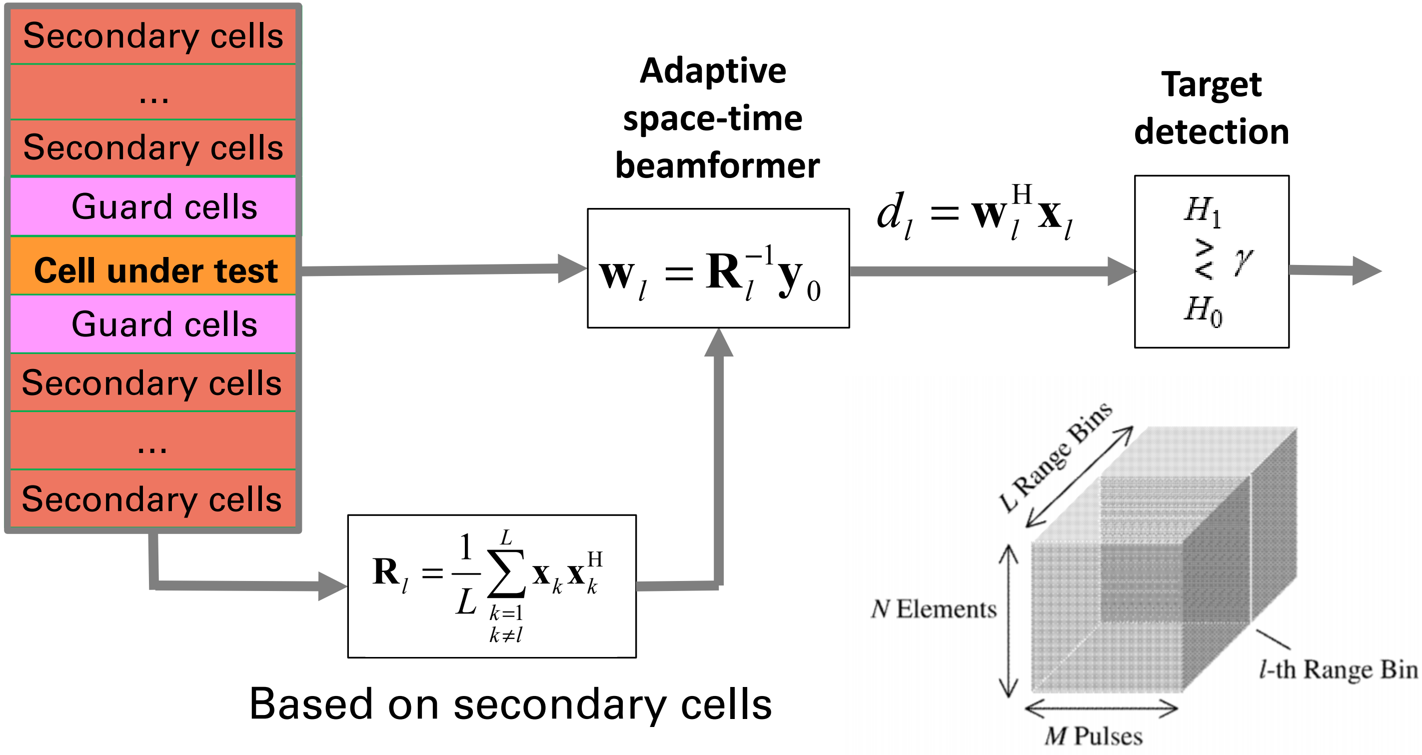
Passive radar: Space-time adaptive processing

- STAP weight vector is determined by $\mathbf{w} = \mathbf{R}^{-1}\mathbf{y}_0$, where $\mathbf{R}$ is the clutter covariance matrix, and $\boldsymbol{y}_0$ is the steering vector toward the target Doppler and angle.
- Feasible for wideband radar but not for passive radar with narrow signal bandwidth.





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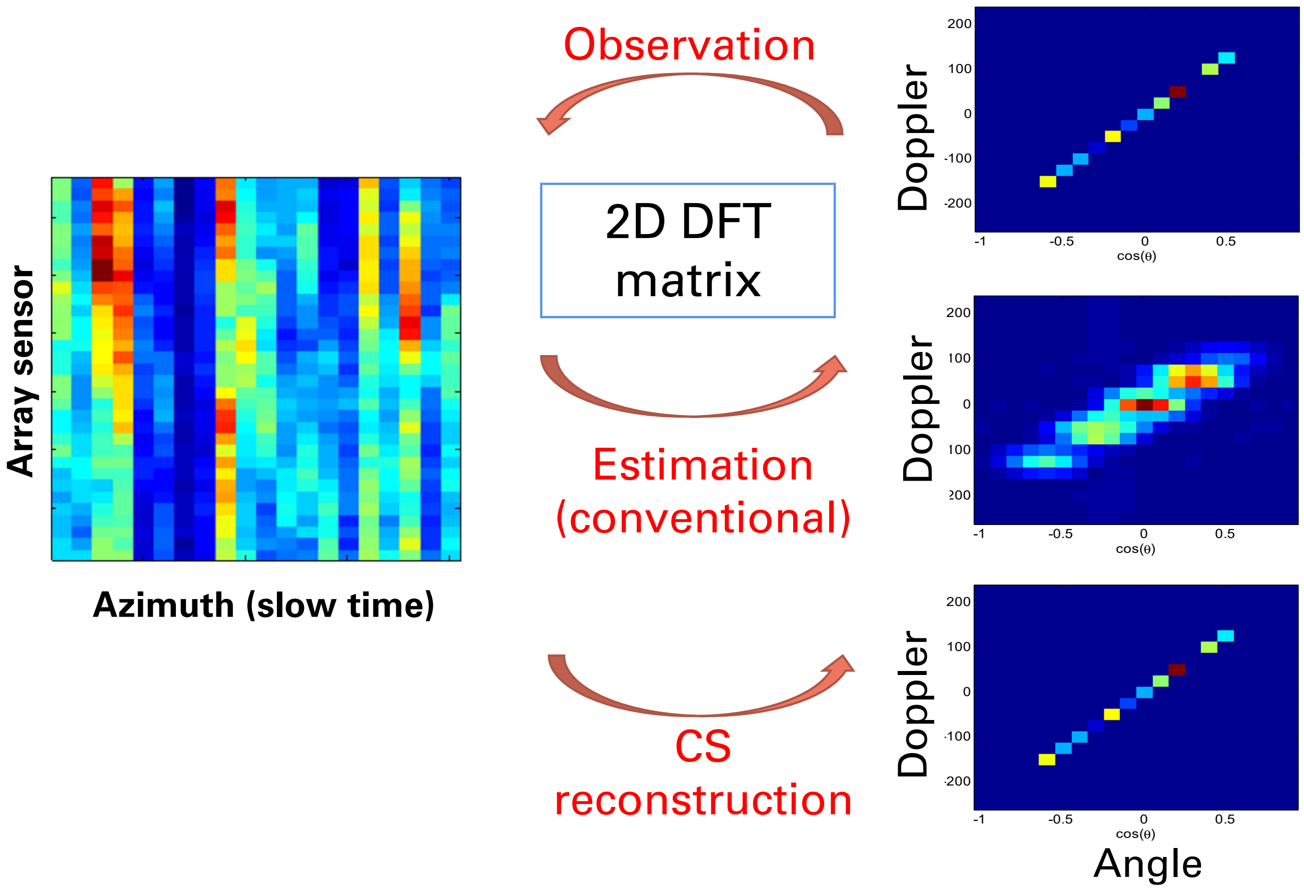
Passive radar: Space-time adaptive processing

Consider sparsity-based clutter reconstruction in angle-Doppler domain.



Clutter ground truth in angle-Doppler domain

Fourier-based estimation does not provide high-resolution clutter estimation

CS-based methods reconstruct clutter in angle-Doppler domain

Passive radar: Space-time adaptive processing

- Due to the sparsity of the clutter in angle-Doppler domain, clutter spectrum can be sparsely recovered using only one or few samples.
- Must ensure that target signal is removed in the clutter covariance matrix
- Proposed method:
  - Estimate clutter region from group sparsity-based clutter angle-Doppler profile estimation using secondary data
  - Clutter angle-Doppler profile estimation from cell under test data
  - Clutter covariance matrix constructed using

$$\hat{\mathbf{R}} = \sum_{m=1}^{M} |\hat{w}(\nu_m, \phi_m)|^2 \, \mathbf{h}(\nu_m, \phi_m)\mathbf{h}^{\mathrm{H}}(\nu_m, \phi_m) + \hat{\beta}\mathbf{I}_{NL}$$

$\hat{w}(\nu_m, \phi_m)$: Clutter coefficient of mth clutter component

$\mathbf{h}(\nu_m, \phi_m)$: Spatio-temporal signature of mth clutter

N: number of antennas

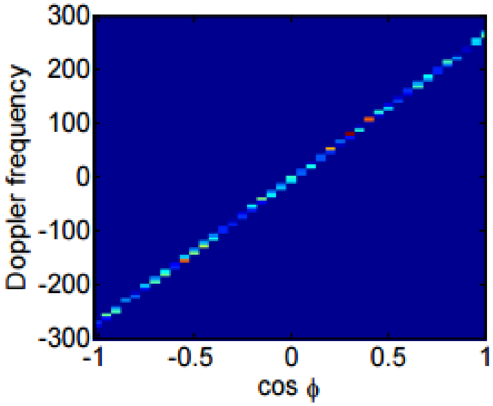
L: Number of azimuth samples

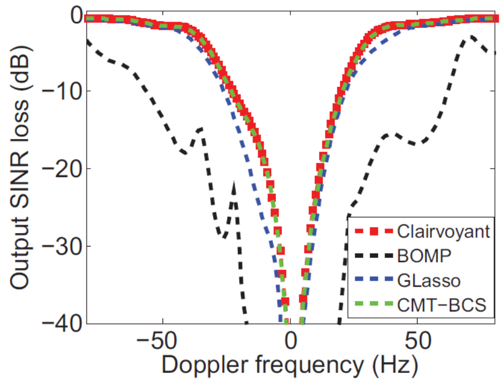
M: Number of clutter components

Q. Wu, Y. D. Zhang, M. G. Amin, and B. Himed, "Space-time adaptive processing and motion parameter estimation in multistatic passive radar using sparse Bayesian learning," IEEE Trans. Geoscience and Remote Sensing, 2016.

Passive radar: Space-time adaptive processing

- Carrier frequency 800 MHz with 20 MHz bandwidth
- 20-element uniform linear array with half-wavelength spacing
- Azimuth sampling frequency 600 Hz
- Simulation with 4 nearby secondary samples
- Clutter profile is discretized into 90 Doppler bins in −300-300 Hz and 40 angle bins in -180^o - 180^o
- Gaussian noise is added with clutter-to-noise ratio of 40 dB
- Insufficient samples to perform conventional sample matrix inversion-based approach





Q. Wu, Y. D. Zhang, M. G. Amin, and B. Himed, "Space-time adaptive processing and motion parameter estimation in multistatic passive radar using sparse Bayesian learning," IEEE Trans. Geoscience and Remote Sensing, 2016.

Remarks: Passive radar

- Passive radars provide a green solution for sensing: No emission, low-cost
- Attractive in various applications
  - Spectrum is not available
  - Covert and difficulty of jamming
  - Dense deployment (low-cost, no interference) for, e.g., drone detection
- More interests from defense and homeland security
- Concerns remain regarding performance and operational guarantee

Concluding remarks

We considered spectrum sharing:

- **Joint radar-communications:** Signaling in radar-centric systems
- **Radar-communication coexistence**: Robust beamforming and DOA estimation
- **Passive radar:** Sparsity-based processing for target imaging, localization, and STAP for clutter suppression

Future directions:

- Sensing and communication function will be more closely integrated: Multi-function radar; UAV network; automotive radar and V2X
- Array processing exploiting convex and mixed-integer optimization, sparsity-based processing, information-theoretical learning, and machine learning will pay critical roles

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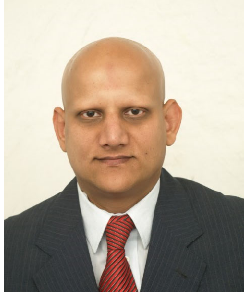

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