Compressive Measurements Exploiting Coprime Frequencies for Direction Finding

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***Abstract*—Sparse arrays are considered as an effective solution to reduce the system complexity for direction-of-arrival (DOA) estimation as they can resolve more sources than the number of sensors. Inspired by such techniques, the coprime frequency-based array structure was recently proposed to further reduce the number of required sensors. Multiple sensing signals with mutually coprime carrier frequencies are transmitted, and the reflected signals are used to estimate the target DOAs. The cross-correlation obtained from each pair of frequencies provides additional degrees of freedom (DOFs). On the other hand, a compression matrix can be designed to reduce the dimension of the received signal vector, thereby effectively reducing the number of front-end circuit chains. In this paper, we use the compressive measurements of the received signal vector obtained from a coprime frequency-based array structure to estimate the signal DOAs. The proposed scheme can obtain an increased number of DOFs and high estimation accuracy because of the coarray operation and the large array aperture of the coprime frequency-based array structure. Meanwhile, the system complexity is also significantly reduced. The effectiveness of the proposed scheme is validated using simulation results.**

***Index Terms*—Compressive sensing, coprime array, coprime frequency, DOA estimation, group sparsity**

I. INTRODUCTION

Direction-of-arrival (DOA) estimation is an important field in array signal processing [1], [2]. When a uniform linear array (ULA) is used for DOA estimation, the number of degrees of freedom (DOFs) depends on the number of physical sensors. On the other hand, the estimation accuracy is strongly affected by the array aperture. Hence, in order to achieve a high estimation accuracy, a large number of sensors is required, resulting in high system complexity and cost. To address this issue, sparse array structures have been proposed under the coarray framework, where the auto-correlation information of the received signal vector is utilized so that an N-sensor sparse array can estimate $\mathcal{O}(N^2)$ sources. In addition to classical sparse array configurations, such as the minimum redundancy array [3], several new sparse array structures are recently developed for systematical design using the nested and coprime array schemes [4]–[8]. The performance of DOA estimation exploiting sparse arrays has been analyzed in [9]–[11]. To further reduce the number of physical sensors, the coprime frequency-based array structure was proposed in [12] and was further extended and analyzed in [13]–[15]. The main concept of the coprime frequency-based array structure is to transmit two or more continuous-wave sensing signals, whose carrier frequencies have a mutually coprime relationship, and then use the received reflection signals at all frequencies to estimate the signal DOAs. In so doing, the number of physical sensors required to estimate the same number of sources is reduced by a factor of K, i.e., the number of coprime frequencies. The corresponding Cramér-Rao bound was derived in [16]. Note that multiple frequencies can also be used to fill in the holes in the coarray of coprime arrays [17].

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Indicated by the compressive sensing theory, signals with certain sparse properties can be successfully recovered through sub-Nyquist sampling so as to reduce the system complexity [18]. To this end, a compression matrix $\mathbf{\Phi} \in \mathbb{C}^{M \times L}$ is used to reduce the dimension of the received signal vector [19]–[23], where M is the number of front-end circuit chains and L is the number of sensors satisfying $M < L$. DOA estimation using compressive measurements based on the coprime array was proposed in [24], which can estimate more sources than the number of array sensors. Furthermore, a generalized DOA estimation scheme using the compressive measurements based on sparse arrays is considered in [25]. It is shown that, for a given number of circuit chains, using compressive measurements based on sparse arrays can achieve a higher number of DOFs than using classical sparse arrays.

In this paper, we propose the use of compressive measurements based on a sparse ULA exploiting coprime frequencies to estimate the signal DOAs. The number of circuit chains is effectively reduced by using the proposed scheme. Meanwhile, a higher number of DOFs is achieved compared with the classical coprime array structures given the same number of circuit chains. In addition, improved DOA estimation accuracy is achieved because a larger array aperture is achieved through the combined use of a sparse ULA, coprime frequencies, and a compression matrix. To summarize, the proposed scheme achieves a lower system complexity compared with that associated with the classical coprime structure given the same number of sensors, whereas it achieves a higher number of DOFs and improved DOA estimation accuracy compared with the classical coprime structure given the same number of circuit chains. Numerical simulations results validate the effectiveness of the proposed scheme.

For the convenience of presentation, we use random compression matrices which are commonly used in the literature. It is noted, however, that the use of random compressive matrices generally lead to an information loss, depending on the ratio of M/L [26]. On the other hand, when certain prior knowledge about the signals, such as their DOA distribution, is available, the compression matrix can be optimized to minimize such information loss through, e.g., information-theoretic approaches [22], [23], [25].

Notations: We use lower-case letters (e.g., a), lower-case bold letters (e.g., $\mathbf{a}$), and upper-case bold letters (e.g., $\mathbf{A}$) to represent scalars, vectors, and matrices, respectively. The superscripts $(\cdot)^*$, $(\cdot)^T$ and $(\cdot)^H$ denote the complex conjugate, the transpose and the complex conjugate transpose, respectively. vec$(\cdot)$ represents the vectorization operation, and $\mathbb{E}[\cdot]$ denotes the expectation operation. The diagonal matrix whose diagonal entries are given in $\mathbf{a}$ is expressed by diag$(\mathbf{a})$. $j=\sqrt{-1}$ is the unit imaginary number, and $\mathbf{I}_L$ stands for the $L\times L$ identity matrix. In addition, $\otimes$ and $\circ$ are used to represent the Kronecker product and Khatri-Rao product.

II. System Model

For simplicity, only two frequencies are considered in this paper. As demonstrated in [12], [14], two signals with carrier frequencies $f_1 = M_1 f_0$ and $f_2 = M_2 f_0$ are transmitted to estimate the directions of the spatial targets, where M_1 and M_2 are a pair of coprime integers and f_0 is the unit frequency. Then, the signals reflected by the targets are received by an L-sensor ULA. Consider Q far-field targets in the spatial domain, and use the ULA with inter-element spacing $d_0 = \lambda_0/2 = c/(2f_0)$, where λ_0 is the wavelength corresponding to f_0 and c is the speed of light. Note that the inter-element spacing $\lambda_0/2$ equals to $M_1\lambda_1/2$ and $M_2\lambda_2/2$, respectively, when evaluated at carrier frequencies f_1 and f_2. The received baseband signal vector at the k-th frequency is expressed as [14]

$$\begin{aligned}\mathbf{x}^{(k)}(t) &= \sum_{q=1}^{Q}\rho_q^{(k)}(t)\mathbf{a}^{(k)}(\theta_q) + \mathbf{n}^{(k)}(t)\\ &= \mathbf{A}^{(k)}\mathbf{s}^{(k)}(t) + \mathbf{n}^{(k)}(t), \qquad k=1,2,\end{aligned} \tag{1}$$

where $\mathbf{s}^{(k)}(t) = [\rho_1^{(k)}(t),\cdots,\rho_Q^{(k)}(t)]^T$. Note that $\rho_q^{(k)}(t)$ is the complex envelope of the q-th target corresponding to f_k with $q=1,2,\cdots,Q$, which depends on f_k because the propagation phase delay varies with the frequency. For two frequencies f_1 and f_2, the relationship between $\rho_q^{(1)}(t)$ and $\rho_q^{(2)}(t)$ can be described by introducing an additional phase which is random and unknown. In the above expression, $\mathbf{n}^{(k)}(t)$ is the complex white Gaussian noise at frequency f_k with mean $\mathbf{0}$ and covariance matrix $\sigma_{nk}^2\mathbf{I}_L$, and $\mathbf{A}^{(k)} = [\mathbf{a}^{(k)}(\theta_1),\cdots,\mathbf{a}^{(k)}(\theta_Q)]$ is the array manifold matrix at frequency f_k, where $\mathbf{a}^{(k)}(\theta_q)$ denotes the steering vector with respect to the q-th target and frequency f_k, expressed as

$$\mathbf{a}^{(k)}(\theta_q) = \left[1, e^{-jM_k\pi\sin(\theta_q)},\cdots,e^{-j(L-1)M_k\pi\sin(\theta_q)}\right]^T. \tag{2}$$

The dimension of the received signal vector can be reduced by introducing a compression matrix $\mathbf{\Phi}\in\mathbb{C}^{M\times L}$ with $M<L$. The compressive measurement vector corresponding to frequency f_k, denoted as $\mathbf{y}^{(k)}(t)$, is expressed as

$$\mathbf{y}^{(k)}(t) = \mathbf{\Phi}\left(\mathbf{A}^{(k)}\mathbf{s}^{(k)}(t) + \mathbf{n}^{(k)}(t)\right). \tag{3}$$

We choose $\mathbf{\Phi}$ to be row-orthonormal, i.e., $\mathbf{\Phi\Phi}^H = \mathbf{I}_M$, so as to guarantee that the noise covariance matrix after compression remains unchanged. It is assumed that the noise elements observed at different frequencies are independent. Then, the covariance matrices of the self-lags (between the received signal corresponding to the same frequency) and cross-lags (between the received signals corresponding to different frequencies) are respectively expressed as

$$\begin{aligned}\mathbf{R}_{\mathbf{yy}}^{(k,k)} &= \mathbb{E}\left[\mathbf{y}^{(k)}(t)(\mathbf{y}^{(k)}(t))^H\right]\\ &= \mathbf{\Phi A}^{(k)}\mathbf{R}_{\mathbf{ss}}^{(k,k)}(\mathbf{A}^{(k)})^H\mathbf{\Phi}^H + \sigma_{nk}^2\mathbf{\Phi\Phi}^H,\end{aligned} \tag{4}$$

$$\begin{aligned}\mathbf{R}_{\mathbf{yy}}^{(i,k)} &= \mathbb{E}\left[\mathbf{y}^{(i)}(t)(\mathbf{y}^{(k)}(t))^H\right]\\ &= \mathbf{\Phi A}^{(i)}\mathbf{R}_{\mathbf{ss}}^{(i,k)}(\mathbf{A}^{(k)})^H\mathbf{\Phi}^H,\end{aligned} \tag{5}$$

where $\mathbf{R}_{\mathbf{ss}}^{(k,k)} = \text{diag}([\sigma_{k1}^2,\cdots,\sigma_{kQ}^2])$ and $\mathbf{R}_{\mathbf{ss}}^{(i,k)} = \text{diag}([\sigma_1^{(i,k)},\cdots,\sigma_Q^{(i,k)}])$ are the covariance matrices of the targets corresponding to the self- and cross-lags, respectively. Note that $\sigma_q^{(i,k)}$ for $i\neq k$ is usually a complex number. In practice, the covariance matrix is estimated from the receive data, expressed as

$$\hat{\mathbf{R}}_{\mathbf{yy}}^{(k,k)} = \frac{1}{T}\sum_{t=1}^{T}\mathbf{y}^{(k)}[t](\mathbf{y}^{(k)}[t])^H, \tag{6}$$

$$\hat{\mathbf{R}}_{\mathbf{yy}}^{(i,k)} = \frac{1}{T}\sum_{t=1}^{T}\mathbf{y}^{(i)}[t](\mathbf{y}^{(k)}[t])^H, \tag{7}$$

where T is the number of snapshots.

III. Proposed DOA Estimation Approach

As demonstrated in [14], the number of DOFs depends on the number of unique cross-lags. The use of self-lags does not contribute additional DOFs, but can help suppress spurious peaks, thus improving the DOA estimation accuracy. Note that, according to (4) and (5), using the self-lags will also induce additional noise, thus may compromise the DOA estimation accuracy in a low signal-to-noise ratio (SNR) scenario. On the other hand, the additional information offered by the self-lags may improve the DOA estimation accuracy in the high SNR region. Thus, in this section, we describe both DOA estimation approaches, respectively exploiting only cross-lags and both self- and cross-lags. Note that $\mathbf{R}_{\mathbf{yy}}^{(k,i)}$ has a Hermitian relationship with $\mathbf{R}_{\mathbf{yy}}^{(i,k)}$. Thus, exploiting both matrices instead of one of them does not improve the performance.

A. Using Cross-lags Only

For notation simplicity, denote $(\mathbf{\Phi}^*\otimes\mathbf{\Phi})$ as $\mathbf{\Psi}\in\mathbb{C}^{M^2\times L^2}$. Vectorizing (5) and utilizing the property vec$(\mathbf{XYZ}) = (\mathbf{Z}^T\otimes$

$\mathbf{X})\text{vec}(\mathbf{Y})$ [27], we have

$$\begin{aligned}\mathbf{r}_{\mathbf{yy}}^{(i,k)} &= \text{vec}\left(\mathbf{R}_{\mathbf{yy}}^{(i,k)}\right) \\ &= \mathbf{\Psi}\left((\mathbf{A}^{(k)})^* \otimes \mathbf{A}^{(i)}\right)\text{vec}(\mathbf{R}_{\mathbf{ss}}^{(i,k)}) \\ &= \mathbf{\Psi}\left((\mathbf{A}^{(k)})^* \circ \mathbf{A}^{(i)}\right)\mathbf{p}_{\mathbf{s}}^{(i,k)},\end{aligned} \tag{8}$$

where $\mathbf{p}_{\mathbf{s}}^{(i,k)} = [\sigma_1^{(i,k)}, \cdots, \sigma_Q^{(i,k)}]^T$. Discretize the spatial domain into a grid of DOAs. Denote the discretized array manifold at frequency f_k as $\mathbf{A}_{\text{grid}}^{(k)}$ and the sparse vector to be estimated as $\mathbf{b}^\circ$. Let $\mathbf{A}_{\text{SM}}^{(i,k)}$ be the sensing matrix such that $\mathbf{A}_{\text{SM}}^{(i,k)} = \mathbf{\Psi}((\mathbf{A}_{\text{grid}}^{(k)})^* \circ \mathbf{A}_{\text{grid}}^{(i)})$. Then, (8) is rewritten as

$$\mathbf{r}_{\mathbf{yy}}^{(i,k)} = \mathbf{A}_{\text{SM}}^{(i,k)}\mathbf{b}^\circ. \tag{9}$$

As such, the DOA estimation problem becomes a standard compressive sensing problem, and vector $\mathbf{b}^\circ$ can be recovered by optimizing the following problem

$$\min_{\mathbf{b}^\circ}\|\mathbf{b}^\circ\|_0 \qquad \text{s. t.} \ \left\|\mathbf{r}_{\mathbf{yy}}^{(i,k)} - \mathbf{A}_{\text{SM}}^{(i,k)}\mathbf{b}^\circ\right\|_2 \leq \varepsilon, \tag{10}$$

where $\|\cdot\|_p$ represents the l_p norm, and ε is the tolerance which is typically determined based on the error between the sampled covariance matrix and the theoretical covariance matrix.

Equation (10) can be solved by a number of compressive sensing methods. In this paper, the LASSO algorithm is used, which reformulates the above constrained optimization problem as

$$\hat{\mathbf{b}}^\circ = \arg\min_{\mathbf{b}^\circ}\left[\frac{1}{2}\left\|\mathbf{r}_{\mathbf{yy}}^{(i,k)} - \mathbf{A}_{\text{SM}}^{(i,k)}\mathbf{b}^\circ\right\|_2 + \eta\|\mathbf{b}^\circ\|_1\right], \tag{11}$$

where η is the regularization parameter. Then, the estimated DOAs are obtained as the positions corresponding to the non-zero entries in $\hat{\mathbf{b}}^\circ$.

B. Using Both Self-lags and Cross-lags

By using both the self- and the cross-lags, the estimation is essentially a multiple measurement vector problem. Vectorizing (4) yields

$$\begin{aligned}&\mathbf{r}_{\mathbf{yy}}^{(k,k)} = \text{vec}\left(\mathbf{R}_{\mathbf{yy}}^{(k,k)}\right) \\ &= \mathbf{\Psi}\left[\left((\mathbf{A}^{(k)})^* \otimes \mathbf{A}^{(k)}\right)\text{vec}(\mathbf{R}_{\mathbf{ss}}^{(k,k)}) + \sigma_{nk}^2\text{vec}(\mathbf{I}_L)\right] \\ &= \mathbf{\Psi}\left[\left((\mathbf{A}^{(k)})^* \circ \mathbf{A}^{(k)}\right)\mathbf{p}_{\mathbf{s}}^{(k,k)} + \sigma_{nk}^2\text{vec}(\mathbf{I}_L)\right],\end{aligned} \tag{12}$$

where $\mathbf{p}_{\mathbf{s}}^{(k,k)}=[\sigma_{k1}^2, \cdots, \sigma_{kQ}^2]^T$. Stack the vectorized covariance matrices and group them to different frequency pairs as

$$\begin{bmatrix}\mathbf{r}_{\mathbf{yy}}^{(1,1)} \\ \mathbf{r}_{\mathbf{yy}}^{(1,2)} \\ \mathbf{r}_{\mathbf{yy}}^{(2,2)}\end{bmatrix} = \begin{bmatrix}\mathbf{\Psi}\left((\mathbf{A}^{(1)})^* \circ \mathbf{A}^{(1)}\right)\mathbf{p}_{\mathbf{s}}^{(1,1)} \\ \mathbf{\Psi}\left((\mathbf{A}^{(2)})^* \circ \mathbf{A}^{(1)}\right)\mathbf{p}_{\mathbf{s}}^{(1,2)} \\ \mathbf{\Psi}\left((\mathbf{A}^{(2)})^* \circ \mathbf{A}^{(2)}\right)\mathbf{p}_{\mathbf{s}}^{(2,2)}\end{bmatrix} + \begin{bmatrix}\sigma_{n1}^2\text{vec}(\mathbf{I}_L) \\ \mathbf{0} \\ \sigma_{n2}^2\text{vec}(\mathbf{I}_L)\end{bmatrix}, \tag{13}$$

which can be compactly expressed as

$$\mathbf{r} = \mathbf{A}_{\text{GS}}\mathbf{p}_{\mathbf{s}} + \mathbf{B}\mathbf{p}_{\mathbf{n}}, \tag{14}$$

where $\mathbf{r} = [(\mathbf{r}_{\mathbf{yy}}^{(1,1)})^T, (\mathbf{r}_{\mathbf{yy}}^{(1,2)})^T, (\mathbf{r}_{\mathbf{yy}}^{(2,2)})^T]^T$ is the compressive measurement vector, and $\mathbf{A}_{\text{GS}} = \text{diag}(\mathbf{A}_{\text{GS}}^{(1,1)}, \mathbf{A}_{\text{GS}}^{(1,2)}, \mathbf{A}_{\text{GS}}^{(2,2)})$ is the corresponding array manifold matrix with $\mathbf{A}_{\text{GS}}^{(i,k)} = \mathbf{\Psi}\left((\mathbf{A}^{(k)})^* \circ \mathbf{A}^{(i)}\right)$. In addition, $\mathbf{p}_{\mathbf{s}} = [(\mathbf{p}_{\mathbf{s}}^{(1,1)})^T, (\mathbf{p}_{\mathbf{s}}^{(1,2)})^T, (\mathbf{p}_{\mathbf{s}}^{(2,2)})^T]^T$, $\mathbf{B} = \mathbf{I}_3 \otimes \text{vec}(\mathbf{I}_L)$, and $\mathbf{p}_{\mathbf{n}} = [\sigma_{n1}^2, 0, \sigma_{n2}^2]^T$.

Denote the sensing matrix corresponding to the self-lags at frequency f_k as $\mathbf{A}_{\text{SM}}^{(k,k)} = \mathbf{\Psi}((\mathbf{A}_{\text{grid}}^{(k)})^* \circ \mathbf{A}_{\text{grid}}^{(k)})$. Let $\mathbf{A}_{\text{SM}}^{(\text{GS})} = \text{diag}(\mathbf{A}_{\text{SM}}^{(1,1)}, \mathbf{A}_{\text{SM}}^{(1,2)}, \mathbf{A}_{\text{SM}}^{(2,2)})$. Then, (14) can be rewritten as

$$\mathbf{r} = \mathbf{V}\mathbf{b}_{\text{GS}}^\circ, \tag{15}$$

where $\mathbf{V} = [\mathbf{A}_{\text{SM}}^{(\text{GS})}, \mathbf{B}]$, and $\mathbf{b}_{\text{GS}}^\circ = [\mathbf{b}_{11}^{\circ T}, \mathbf{b}_{12}^{\circ T}, \mathbf{b}_{22}^{\circ T}, \mathbf{p}_{\mathbf{n}}^T]^T$ with $\mathbf{b}_{ik}^\circ$ being the spatial spectrum corresponding to $\mathbf{r}_{\mathbf{yy}}^{(i,k)}$. It is clear that $\mathbf{b}_{11}^\circ$, $\mathbf{b}_{12}^\circ$, and $\mathbf{b}_{22}^\circ$ have the same sparsity support, because their common non-zero entries correspond to the target DOAs. Take the l_2 norm with respect to each row in $[\mathbf{b}_{11}^\circ, \mathbf{b}_{12}^\circ, \mathbf{b}_{22}^\circ]$, and denote the result as $\mathbf{b}_0$. Then, we have a sparse column vector $\boldsymbol{\xi}$ defined as

$$\boldsymbol{\xi} = [\mathbf{b}_0^T, \mathbf{p}_{\mathbf{n}}^T]^T. \tag{16}$$

Thus, $\mathbf{b}_{\text{GS}}^\circ$ can be recovered by solving the following problem

$$\min_{\mathbf{b}_{\text{GS}}^\circ}\|\boldsymbol{\xi}\|_0 \qquad \text{s. t.} \ \|\mathbf{r} - \mathbf{V}\mathbf{b}_{\text{GS}}^\circ\|_2 \leq \varepsilon_{\text{GS}}, \tag{17}$$

where ε_{GS} is the tolerance. (17) can be solved using the group LASSO algorithm which relax this problem as the following:

$$\hat{\mathbf{b}}_{\text{GS}}^\circ = \arg\min_{\mathbf{b}_{\text{GS}}^\circ}\frac{1}{2}\|\mathbf{r} - \mathbf{V}\mathbf{b}_{\text{GS}}^\circ\|_2 + \eta_{\text{GS}}\|\boldsymbol{\xi}\|_1, \tag{18}$$

where η_{GS} is the regularization parameter. The estimated DOAs are the positions corresponding to the non-zero entries in $\hat{\boldsymbol{\xi}} = g(\hat{\mathbf{b}}_{\text{GS}}^\circ)$.

IV. Simulations Results

Throughout this section, we consider a ULA with 16 sensors as the receive array. The number of front-end circuit chains is set as 8, thus yielding the compression ratio to be 2. Each entry in the compression matrix $\mathbf{\Phi}$ is randomly selected from the complex Gaussian distribution $\mathcal{CN}(0, 1)$ and does not change through all the simulations. To show the superiority of the proposed structure, the classical coprime frequency based structure [14] exploiting an 8-element ULA is also considered for comparison. As such, both array structures have the same number of circuit chains. Two coprime frequencies with $M_1 = 2$ and $M_2 = 3$ are used for both structures. The target DOAs are assumed to be uniformly distributed between $-60°$ and $60°$. LASSO and group LASSO are performed to estimated the DOAs using only cross-lags and using both self- and cross-lags, respectively, where the *lasso* function in MATLAB R2015b is exploited. Note that both self- and cross-lags are utilized for classical coprime frequency based structure [14]. The searching grid is uniformly distributed between $-90°$ and $90°$ with a step size of $0.1°$.

A. Spatial Spectrum

Given the same number of circuit chains, the coarray of the proposed scheme consists of more lags than the classical coprime frequency based structure. Therefore, the proposed

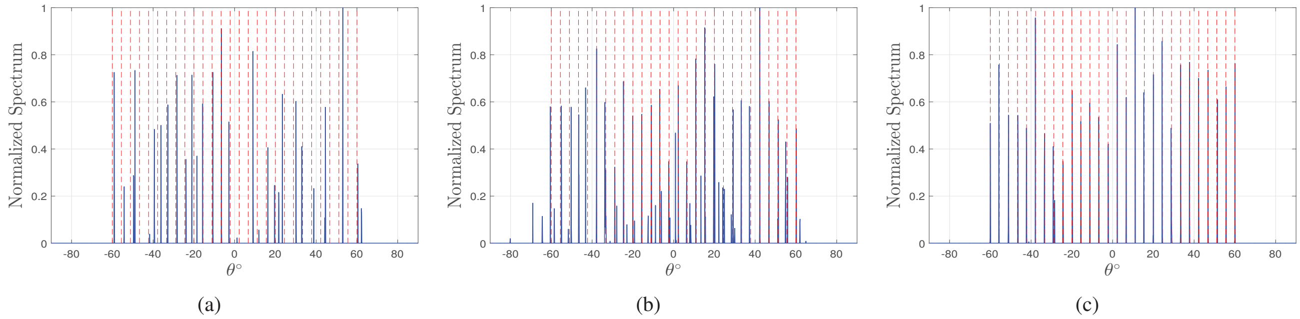

Fig. 1. The spatial spectra obtained from (a) the classical coprime frequency based structure, (b) the proposed scheme using only cross-lags, and (c) the proposed scheme using both self- and cross-lags.

structure is expected to achieve a higher number of DOFs. In Fig. 1, we compare the normalized spatial spectra obtained from the classical coprime frequency based array structure and the proposed compressive measurement array structure, where 28 uncorrelated far-field targets are considered, the input SNR is 20 dB, and $2,000$ snapshots are used. For the proposed scheme, the regularization parameters in the cross-lags only and both self- and cross-lags scenarios, i.e., η and η_{GS}, are respectively set as 0.1 and 17.5. As to the classical coprime frequency based structure, η_{GS} is set as 2.5. These parameters are chosen to optimize the respective performance for each individual algorithm.

As indicated by the CRB [16], for the underlying classical coprime frequency based array structure, the maximum number of DOFs is 23. Therefore, it cannot correctly estimate all the 28 sources, as clearly demonstrated in the spectrum depicted in Fig. 1(a). For the proposed scheme, when only cross-lags are exploited, many spurious peaks are generated, although the actual DOAs are correctly estimated in Fig. 1(b). On the other hand, when the self-lags are also used, they help eliminate the spurious peaks as shown in Fig. 1(c). This is consistent with the analysis made in Section III.

B. DOA Estimation Accuracy

Next, we examine the DOA estimation accuracy of the proposed scheme. Since the ULA with a larger aperture is used in the proposed scheme, the DOA estimation accuracy is expected to be higher than that obtained from the classical coprime frequency based structure. To enable comparison of the DOA estimation accuracy between the two array structures, 14 uncorrelated far-field targets are considered so that both array structures have enough DOFs. 500 Monte-Carlo trials are conducted to compute the root mean square error (RMSE), which is defined as

$$\text{RMSE} = \sqrt{\frac{1}{IQ}\sum_{i=1}^{I}\sum_{q=1}^{Q}\left(\theta_q^{(i)} - \hat{\theta}_q^{(i)}\right)^2}, \tag{19}$$

where I is the number of Monte-Carlo trials. $\theta_q^{(i)}$ and $\hat{\theta}_q^{(i)}$ denote the actual and estimated value of the q-th target in the i-th trial, respectively.

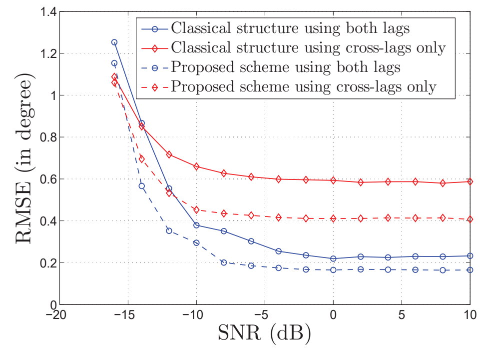

Fig. 2. The RMSE versus input SNR.

The RMSE results versus SNR are plotted in Fig. 2 where 500 snapshots are used, and the SNR varies between -20 dB and 10 dB. The regularization parameter η_{GS} is set as 2.5 and 8 for group LASSO in classical coprime frequency structure and the proposed scheme, respectively, whereas η is chosen to be 0.2 for LASSO. The floor observed in the RMSE performance is mainly caused by the off-grid effect. It is clear that the proposed structure obtains a higher estimation accuracy as expected. As we demonstrated before, cross-lags do not include the autocorrelation of the noise term, and thus the corresponding DOA estimation is more robust to noise in the low SNR region. On the other hand, in the high SNR region, the self-lags provide helpful information to improve the DOA estimation performance, particularly in suppressing the spurious peaks.

V. Conclusion

In this paper, we proposed a novel DOA estimation scheme exploiting a coprime frequency based structure, where the dimension of the received signal vector is compressed before feeding into the front-end circuit chains. Given a fixed number of circuit chains, the proposed scheme obtains a higher number of DOFs and a better estimation accuracy because of the extended receive array aperture. When the same number of sensors are used, the number of circuit chains is significantly reduced in the proposed scheme, thus having a lower system complexity than the classical coprime frequency based structure. Numerical simulation results validate the effectiveness of the proposed structure.

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