DOA Estimation Exploiting Distributed Array with Arbitrary Subarray Orientations

Yimin D. Zhang [1], Md Waqeeb T. S. Chowdhury [1], Yanwu Ding [2], Dan Shen [3], Khanh Pham [4], Erik Blasch [5], and Genshe Chen [3]

[1] Department of Electrical and Computer Engineering, Temple University, Philadelphia, PA 19122
[2] Department of Electrical and Computer Engineering, Wichita State University, Wichita, KS 67260
[3] Intelligent Fusion Technology, Inc., Germantown, MD 20876
[4] Air Force Research Laboratory, Kirtland Air Force Base, NM 87117
[5] Air Force Research Laboratory, Wright-Patterson Air Force Base, OH 45433

***Abstract* − This paper considers a two-dimensional direction-of-arrival (DOA) estimation problem from a collaborative, distributed antenna array where each subarray is a distributed sensing node that is arbitrarily oriented. While the relative locations of the subarrays are not precisely known, it is assumed that the configuration of each subarray is locally calibrated whereas the cross-covariance matrix between a pair of distributed nodes includes an unknown phase difference. Without explicitly estimating such unknown phase difference, subspace-based DOA estimation methods fail to coherently utilize the subarrays to locate the DOAs of the impinging signals. We propose a group sparsity-based approach to achieve accurate DOA estimation that is resilient to unknown phase disparities between subarrays. Simulation results clearly illustrate the effectiveness of the group sparsity-based approach using group LASSO, and the superiority over subspace-based methods, such as the MUSIC algorithm, is demonstrated.**

Keywords: Direction-of-arrival estimation, distributed array, sparse array, group sparsity, unmanned aerial vehicle, cube satellite network.

I. INTRODUCTION

Distributed sensing and source/target localization using distributed arrays have attracted significant interests in recent years [1 − 8]. A collaborative sensing system comprising distributed sensing nodes enables small platforms' coordination, such as sensor networks, unmanned aerial vehicle (UAV) swarms, and cube satellite (CubeSat) systems [9 − 13]. In particular, a distributed array exploiting multiple closely spaced subarrays provides an effective alternative to large-scale arrays when they cannot be feasibly built on a single platform.

To jointly perform directions-of-arrival (DOA) estimation based on a distributed array, non-coherent processing of partly-calibrated subarrays is studied in [1, 5]. In [6], collinear subarrays are considered, and covariance matrix completion is used to achieve a full aperture of the distributed array. Parallel sparse arrays are considered in [14] to achieve two-dimensional (2-D) DOA estimation. Mixed-precision data using full-precision self-subarray covariance matrices and one-bit data-based cross-subarray covariance matrices are developed in [7, 8] to achieve full-aperture exploitation and enhanced DOA estimation performance in a communication traffic-limited distributed array system. Direction finding using a partly-calibrated arrays in the presence of position errors is considered in [15].

Most work reported in the literature considers subarrays that are placed in either a collinear or a parallel fashion. Although such assumptions make the analyses simpler, in many practical applications, the subarrays may be placed in arbitrary orientations. In such scenarios, the signal models deviate from those developed for collinear and parallel subarrays. On the other hand, subarrays with different orientations form a 2-D distributed array aperture, thereby enabling 2-D DOA estimation of the scene.

In this paper, we consider a distributed array in which multiple linear subarrays are placed in a plane with arbitrary orientations. The signal model is developed with the consideration of the unknown phase differences between the signals received by these subarrays. The presence of such unknown phase differences impedes the direction application of subspace-based DOA estimation methods, such as MUltiple Signal Classification (MUSIC), for coherent processing across the entire array without explicitly estimating these phase differences [16]. In other words, the signals received at the subarrays cannot be coherently combined based on traditional subspace-based DOA estimation methods.

It is interesting to note that such a DOA estimation problem with unknown phase differences between subarrays is analogous to that encountered in the DOA estimation based on multi-frequency and wideband arrays, where the phase differences between the observed signals corresponding to different frequencies is unknown [17−20]. For such problems, group

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sparsity-based methods, which utilize the shared sparse patterns of the nonzero entries without phase alignment requirement, are known to provide near-coherent signal processing capabilities [21]. In this paper, we use the group Least Absolute Shrinkage and Selection Operator (LASSO) [22, 23] approach to produce robust DOA estimation estimates. Simulation results are provided to verify that, unlike MUSIC which is sensitive to the unknown phase differences between the subarrays, the group LASSO-based approach enables robust DOA estimation without relying on the knowledge of the phase differences.

Notations: We use lower-case (upper-case) bold characters to describe vectors (matrices). In particular, $\boldsymbol{I}_N$ denotes the $N \times N$ identity matrix and $\mathbf{0}$ stands for a vector or matrix with all elements to be zero. $(\cdot)^*$ represents complex conjugate, $(\cdot)^{\mathrm{T}}$ and $(\cdot)^{\mathrm{H}}$ respectively denote the transpose and conjugate transpose of a matrix or a vector. $\mathrm{diag}(\cdot)$ and $\mathrm{bdiag}(\cdot)$ respectively form diagonal and block diagonal matrices, and $\mathrm{vec}(\cdot)$ forms a vector from a matrix. $||\cdot||_2$ and $||\cdot||_{1,2}$ represent the l_2 norm and mixed $l_{1,2}$ norm of a vector, respectively. $\mathbb{E}(\cdot)$ performs statistical expectation and $\otimes$ performs Kronecker product. For a matrix $\boldsymbol{A}$, the element in the pth row and qth column is denoted as $\boldsymbol{A}(p,q)$.

II. Signal Model

Consider K arbitrarily oriented linear subarrays, and an example of a distributed array comprising two sparse subarrays is shown in Fig. 1 on a half-wavelength grid. Each subarray can be mounted on a small platform, such as a UAV or a CubeSat. The arrays are located at the same altitude and all antennas are placed in the same x-y plane. Each subarray is a linear array consisting of $M_k \geq 2$ sensors which may either be uniformly or sparsely placed [24–27]. The total number of antennas in the distributed array is $M = \sum_{k=1}^{K} M_k$. The configuration of each subarray is calibrated and known, whereas the relative positions of the subarrays are not precisely known.

The nodes are closely located such that the paths from the signals to these nodes are considered parallel, i.e., the subarrays observe the same DOAs of the scene. The position of the mth antenna in the kth subarray is denoted as (x_{km}, y_{km}). We take the first antenna of the first node as the global reference of the array, i.e., $(x_{11}, y_{11}) = (0,0)$. The orientation of the kth nodes is denoted as β_k with respect to the x-axis and is assumed to be known for all k. Without loss of generality, we assume $\beta_1 = 0$.

When L uncorrelated and far-field signals impinge on the arrays from directions (θ_l, ϕ_l), $l = 1, \cdots, L$, where θ_l and ϕ_l are respectively the elevation and azimuth angles of the lth signal, the baseband signal vector received at the subarray of the kth node is expressed as

$$\boldsymbol{x}^{(k)}(t) = \sum_{l=1}^{L} s_l(t)\, \mathrm{e}^{-j2\pi\kappa d_l^{(k)}} \boldsymbol{a}^{(k)}(\theta_l, \phi_l) + \boldsymbol{n}^{(k)}(t), \tag{1}$$

where $s_l(t)$ is the waveform of the lth signal, $\kappa = f_c/c$ is the wavenumber with f_c denoting the carrier frequency and c the propagation velocity of electromagnetic waves in free space,

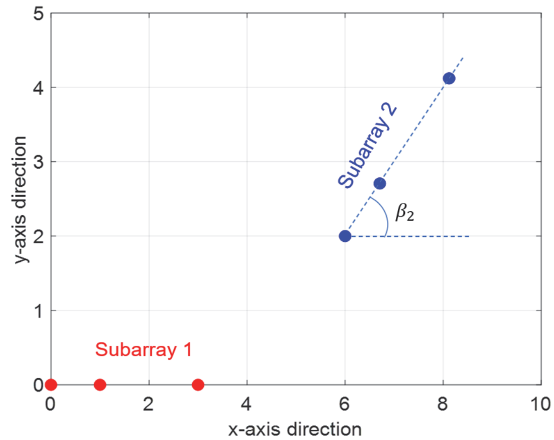

Figure 1. Geometry of a distributed array comprising two 3-element subarrays plotted on a half-wavelength grid.

$d_l^{(k)}$ is the unknown distance between the lth signal and the reference antenna of the kth node, $\boldsymbol{a}^{(k)}(\theta_l, \phi_l)$ is the steering vector of the subarray at the kth node towards the lth signal, and $\boldsymbol{n}^{(k)}(t) \sim \mathcal{CN}\left(\mathbf{0}, \sigma_n^2 \boldsymbol{I}_{M_k}\right)$ is an additive complex Gaussian noise vector. The steering vector is expressed as

$$\boldsymbol{a}^{(k)}(\theta_l, \phi_l) = \left[1, e^{-j2\pi\kappa p_2^{(k)} u_l^{(k)}}, \cdots, e^{-j2\pi\kappa p_{M_k}^{(k)} u_l^{(k)}}\right]^{\mathrm{T}}, \tag{2}$$

where $p_m^{(k)} = \|x_{km} - x_{k1}, y_{km} - y_{k1}\|_2$ is the distance between the mth sensor of the kth node to its local reference at (x_{k1}, y_{k1}), and $u_l^{(k)} = \cos(\theta_l)\sin(\phi_l - \beta_k)$.

Equation (1) can be written as

$$\boldsymbol{x}^{(k)}(t) = \boldsymbol{A}^{(k)}(\boldsymbol{\theta}, \boldsymbol{\phi}) \boldsymbol{V}^{(k)} \boldsymbol{s}(t) + \boldsymbol{n}^{(k)}(t), \tag{3}$$

where

$$\boldsymbol{A}^{(k)}(\boldsymbol{\theta}, \boldsymbol{\phi}) = \left[\boldsymbol{a}^{(k)}(\theta_1, \phi_1), \ldots, \boldsymbol{a}^{(k)}(\theta_L, \phi_L)\right] \tag{4}$$

is the subarray manifold matrix with $(\boldsymbol{\theta}, \boldsymbol{\phi}) = \{(\theta_1, \phi_1), \cdots, (\theta_L, \phi_L)\}$,

$$\boldsymbol{V}^{(k)} = \mathrm{diag}\left[\mathrm{e}^{-j2\pi\kappa d_1^{(k)}}, \cdots, \mathrm{e}^{-j2\pi\kappa d_L^{(k)}}\right], \tag{5}$$

and $\boldsymbol{s}(t) = [s_1(t), \cdots, s_L(t)]^{\mathrm{T}}$.

Cascading the data vectors across all subarrays yields the total received data vector across the entire distributed array as

$$\boldsymbol{x}(t) = \left[\left(\boldsymbol{x}^{(1)}(t)\right)^{\mathrm{T}}, \cdots, \left(\boldsymbol{x}^{(K)}(t)\right)^{\mathrm{T}}\right]^{\mathrm{T}} = \boldsymbol{A}(\boldsymbol{\theta}, \boldsymbol{\phi}) \boldsymbol{V} \boldsymbol{s}(t) + \boldsymbol{n}(t), \tag{6}$$

where the effective array manifold, the phase-difference term, and the noise across the entire distributed array are respectively given as

$$\boldsymbol{A}(\boldsymbol{\theta}, \boldsymbol{\phi}) = \left[\left(\boldsymbol{A}^{(1)}(\boldsymbol{\theta}, \boldsymbol{\phi})\right)^{\mathrm{T}}, \cdots, \left(\boldsymbol{A}^{(K)}(\boldsymbol{\theta}, \boldsymbol{\phi})\right)^{\mathrm{T}}\right]^{\mathrm{T}}, \tag{7}$$

$$\boldsymbol{V} = \mathrm{bdiag}\left[\boldsymbol{V}^{(1)}, \cdots, \boldsymbol{V}^{(K)}\right], \tag{8}$$

and

$$\boldsymbol{n}(t)=\left[\left(\boldsymbol{n}^{(1)}(t)\right)^{\mathrm{T}},\cdots,\left(\boldsymbol{n}^{(K)}(t)\right)^{\mathrm{T}}\right]^{\mathrm{T}}. \tag{9}$$

III. Array Covariance Matrix

The covariance matrix of the array data vector, $\boldsymbol{x}(t)$, given in (6), is expressed as

$$\boldsymbol{R}=\mathbb{E}[\boldsymbol{x}(t)\boldsymbol{x}^{\mathrm{H}}(t)]=\begin{bmatrix}\boldsymbol{R}_{11} & \boldsymbol{R}_{12} & \cdots & \boldsymbol{R}_{1K}\\ \boldsymbol{R}_{21} & \boldsymbol{R}_{22} & \cdots & \boldsymbol{R}_{2K}\\ \vdots & \vdots & \vdots & \vdots\\ \boldsymbol{R}_{K1} & \boldsymbol{R}_{K2} & \cdots & \boldsymbol{R}_{KK}\end{bmatrix} \tag{10}$$

where

$$\boldsymbol{R}_{uv}=\mathbb{E}\left[\boldsymbol{x}^{(u)}(t)\left[\boldsymbol{x}^{(v)}(t)\right]^{\mathrm{H}}\right],\ \ u,v\in 1,\cdots,K. \tag{11}$$

In (11), the cases of $u=v$ correspond to the self-subarray covariance matrices of the linear subarrays which are well known. On the other hand, when $u\neq v$, the cross-covariance matrix between two subarrays is obtained as,

$$\begin{aligned}\boldsymbol{R}_{uv}&=\mathbb{E}\left[\boldsymbol{x}^{(u)}(t)\left[\boldsymbol{x}^{(v)}(t)\right]^{\mathrm{H}}\right]\\ &=\boldsymbol{A}^{(u)}(\boldsymbol{\theta},\boldsymbol{\phi})\boldsymbol{V}^{(u)}\boldsymbol{R}_{ss}\left[\boldsymbol{V}^{(v)}\right]^{\mathrm{H}}\left[\boldsymbol{A}^{(v)}(\boldsymbol{\theta},\boldsymbol{\phi})\right]^{\mathrm{H}}\\ &=\boldsymbol{A}^{(u)}(\boldsymbol{\theta},\boldsymbol{\phi})\boldsymbol{R}_{ss}^{(uv)}\left[\boldsymbol{A}^{(v)}(\boldsymbol{\theta},\boldsymbol{\phi})\right]^{\mathrm{H}},\end{aligned} \tag{12}$$

where $\boldsymbol{R}_{ss}=\mathbb{E}[\boldsymbol{s}(t)\boldsymbol{s}^{\mathrm{H}}(t)]=\mathrm{diag}[\rho_1,\cdots,\rho_L]$ is the source covariance matrix with $\rho_l=\mathbb{E}[|s_l(t)|^2]$ being the source power of the lth signal. In addition,

$$\boldsymbol{R}_{ss}^{(uv)}=\boldsymbol{V}^{(u)}\boldsymbol{R}_{ss}\left[\boldsymbol{V}^{(v)}\right]^{\mathrm{H}}=\mathrm{diag}\left[\rho_1^{(uv)},\cdots,\rho_L^{(uv)}\right] \tag{13}$$

is the source cross-correlation matrix between the two subarrays, where $\rho_l^{(uv)}=\rho_l\mathrm{e}^{-j2\pi\kappa\left[d_l^{(u)}-d_l^{(v)}\right]}$, $l=1,\cdots,L$. Note that the phase difference between the two subarrays $\rho_l^{(uv)}$ is unknown and, in general, takes a complex value. Also notice that $\boldsymbol{R}_{21}=\boldsymbol{R}_{12}^{\mathrm{H}}$.

IV. Group Sparsity-Based DOA Estimation

When applying the MUSIC method to coherently exploit the measurements available to the K subarrays, we need to know their phase differences in order to form the steering vector corresponding to a direction to be sensed.

As mentioned earlier, the underlying DOA estimation problem is analogous to that encountered in multi-frequency sparse arrays, where the phase differences between the signals corresponding to different frequencies is unknown. However, unlike [18−20] where the virtual arrays obtained from multiple frequencies are collocated, the subarrays in the underlying problems are separated and with different orientations.

To understand the difference coarrays obtained from the two subarrays u and v, we vectorize $\boldsymbol{R}_{uv}$ in (11) and obtain [19]

$$\boldsymbol{z}_{uv}=\mathrm{vec}(\boldsymbol{R}_{uv})=\widetilde{\boldsymbol{A}}_{uv}\boldsymbol{b}_{uv}, \tag{14}$$

where $\widetilde{\boldsymbol{A}}_{uv}=[\widetilde{\boldsymbol{a}}_{uv}(\theta_1,\phi_1),\widetilde{\boldsymbol{a}}_{uv}(\theta_2,\phi_2),\cdots,\widetilde{\boldsymbol{a}}_{uv}(\theta_L,\phi_L)]$ with $\widetilde{\boldsymbol{a}}_{uv}(\theta_l,\phi_l)=\left[\boldsymbol{a}^{(u)}(\theta_l,\phi_l)\right]^*\otimes\boldsymbol{a}^{(v)}(\theta_l,\phi_l)$, and $\boldsymbol{b}_{uv}=\left[\rho_1^{(uv)},\cdots,\rho_L^{(uv)}\right]^{\mathrm{T}}$. To employ the group LASSO for 2-D DOA estimation, K^2 optimization vectors $\boldsymbol{b}_{uv}^{\mathrm{o}}$ with $u,v\in[1,K]$, each of size $G\times 1$, are initialized, where G is the total number of search grids for the DOAs. Let $\boldsymbol{B}_{uv}$ denote the dictionary matrix corresponding to difference corray between the uth and v th subarrays. The group sparsity problem can now be formulated as [11, 20]

$$\widehat{\boldsymbol{b}}^{\mathrm{o}}=\arg\min_{\boldsymbol{b}^{\mathrm{o}}}\sum_{u=1}^{K}\sum_{v=1}^{K}\|\boldsymbol{z}_{uv}-\boldsymbol{B}_{uv}\boldsymbol{b}_{uv}^{\mathrm{o}}\|_2+\xi\|\boldsymbol{b}_{uv}^{\mathrm{o}}\|_{1,2}, \tag{15}$$

where $\boldsymbol{b}^{\mathrm{o}}=[(\boldsymbol{b}_{11}^{\mathrm{o}})^{\mathrm{T}},\cdots,(\boldsymbol{b}_{KK}^{\mathrm{o}})^{\mathrm{T}}]^{\mathrm{T}}$, ξ is a regularization parameter, and the mixed $l_{1,2}$ norm of vector $\boldsymbol{b}_{uv}^{\mathrm{o}}$ is defined as

$$\|\boldsymbol{b}_{uv}^{\mathrm{o}}\|_{1,2}=\sum_{a=1}^{G}\left(\sum_{i=1}^{K^2}\boldsymbol{\mathcal{B}}(a,i)\boldsymbol{\mathcal{B}}^*(a,i)\right)^{\frac{1}{2}}, \tag{16}$$

with $\boldsymbol{\mathcal{B}}$ denoting the matrix constructed by concatenating all the vectors $\boldsymbol{b}_{uv}^{\mathrm{o}}$. The DOA estimates are given as the summed coefficients on the G-point grid, i.e.,

$$\widehat{\boldsymbol{b}}=\sum_{u=1}^{K}\sum_{v=1}^{K}\left|\widehat{\boldsymbol{b}}_{uv}^{\mathrm{o}}\right|. \tag{17}$$

V. Simulation Results

We assume the array geometry as depicted in Fig. 1. There are $K=2$ sparse subarrays with $M_1=M_2=3$. Take the orientation of the first node as the reference with $\beta_1=0°$, and the orientation of the second subarray is chosen as $\beta_2=45°$. Denoting a half-wavelength to be unity, the sensor locations of the first subarray are given as $(x_{11},y_{11})=(0,0)$, $(x_{12},y_{12})=(1,0)$, and $(x_{13},y_{13})=(3,0)$, and those of the second subarray are given as $(x_{21},y_{21})=(6,2)$, $(x_{22},y_{22})=(6.707,2.707)$, and $(x_{23},y_{23})=(8.121,\ 4.121)$. $L=6$ uncorrelated signals are considered, and their elevation and azimuth angles are randomly chosen from $\theta_l\in[0°,90°]$ and $\phi_l\in[-90°,90°]$, respectively, and are given as $(\theta_1,\phi_1)=(54°,-40°)$, $(\theta_2,\phi_2)=(45°,-24°)$, $(\theta_3,\phi_3)=(53°,-8°)$, $(\theta_4,\phi_4)=(78°,8°)$, $(\theta_5,\phi_5)=(37°,24°)$, and $(\theta_6,\phi_6)=(49°,40°)$. It is noted that, in this case, the number of signals is higher than the number of degrees of freedom of the physical array, and DOA estimation is carried out by exploiting the difference coarray. We use 1,000 data samples, and the input signal-to-noise ratio (SNR) is 0 dB. The regularization parameter for the group LASSO is chosen to be $\xi=20$.

We first show the 2-D DOA estimation performance when the MUSIC algorithm is used and the phase difference between the two subarrays is precisely known. In this case, as shown in Fig. 2, all the DOAs of the 6 signals are detected successfully. In practice, however, the phase difference between the two subarrays is random and unavailable. When an inaccurate phase difference is assumed, MUSIC fails to locate the signal DOAs, as depicted in Fig. 3.

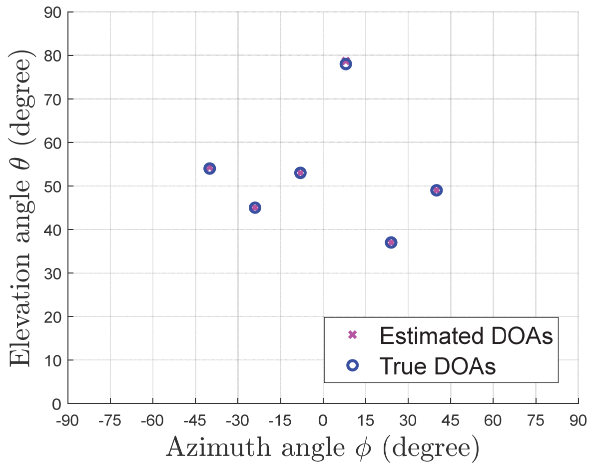

Figure 2. 2-D DOA estimation using MUSIC assuming precise knowledge of the phase difference between subarrays.

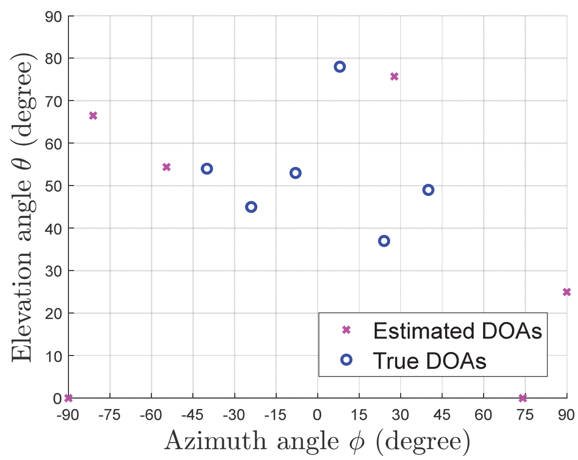

Figure 3. 2-D DOA estimation using MUSIC when an inaccurate phase difference between subarrays is assumed.

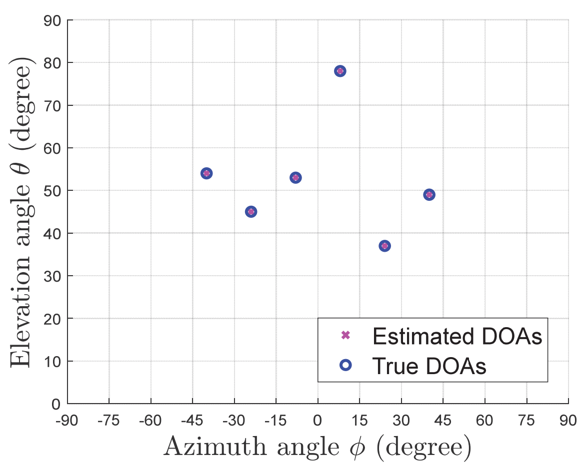

Figure 4. 2-D DOA estimation using group LASSO without knowledge of the phase difference between subarrays.

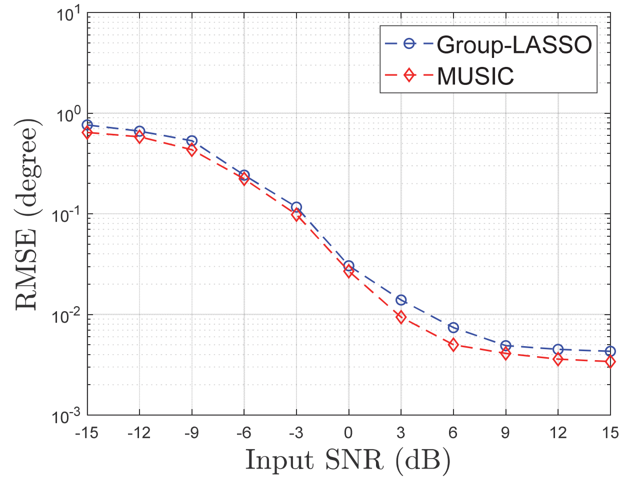

Figure 5. Comparison of RMSE performance of DOA estimation versus the input SNR for group LASSO and MUSIC with knowledge of the phase difference between subarrays.

Fig. 4 shows the 2-D DOA estimation results using the group LASSO when the phase difference between the two subarrays is unavailable. It is observed that all the sources are successfully detected, clearly depicting the robustness of the group sparsity-based approach to the unknown phase differences between distributed subarrays.

To analyze the DOA estimation performance of the group LASSO-based approach, we compare the root mean-square error (RMSE) results of both the group LASSO and MUSIC methods. The RMSE of estimated DOAs is defined as

$$\text{RMSE} = \sqrt{\frac{1}{NL}\sum_{n=1}^{N}\sum_{l=1}^{L}\left[\left(\hat{\theta}_{l,n}-\theta_l\right)^2+\left(\hat{\phi}_{l,n}-\phi_l\right)^2\right]}, \quad (18)$$

where N is the number of Monte Carlo trials and $(\hat{\theta}_{l,n}, \hat{\phi}_{l,n})$ represents the estimated elevation and azimuth DOAs of the lth signal obtained in the nth trial.

In Fig. 5, the RMSE results obtained from both group LASSO and MUSIC are shown with respect to the input SNR, where the MUSIC algorithm assumes precise knowledge of the phase difference between the two subarrays. Each RMSE result is computed from 100 Monte Carlo trials. The input SNR varies between -15 and 15 dB with an interval of 3 dB and the number of data samples assumed is 1,000. It is note that, when the information of the phase difference between the subarrays is not available, the MUSIC algorithm does not resolve the signals and, as a result, its RMSE values are not included for comparison.

It can be observed that the performance of the group LASSO is very close to that of the 2-D MUSIC algorithm, verifying that group LASSO achieves near-coherent processing capability without requiring the knowledge of the phase difference between the two subarrays. As such, it removes the barriers of requiring the precise phase difference between the subarrays as in the MUSIC algorithm, thereby enabling practical implementation for effective 2-D DOA estimation using distributed arrays.

VI. Conclusion

We considered a 2-D DOA estimation problem using a distributed array, in which the subarrays have arbitrary orientations. Such a configuration of the distributed array differs from the well-studied collinearly or parallelly placed distributed arrays. The analysis of the array covariance matrix was provided with a focus on the impact of random phase differences in the cross-covariance matrix between a pair of subarrays. The group LASSO is used to perform group sparsity-based DOA estimation which achieves near-coherent processing capability without requiring the knowledge of the phase differences between subarrays. Simulation results verified the effectiveness of the proposed approach to achieve accurate 2-D DOA estimation.

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