Multi-User Dual-Function Radar-Communications Exploiting Sidelobe Control and Waveform Diversity

Ammar Ahmed [†], Yimin D. Zhang [†], and Braham Himed [‡]

[†] Department of Electrical and Computer Engineering
Temple University, Philadelphia, PA 19122, USA

[‡] RF Technology Branch, Air Force Research Laboratory
AFRL/RYMD, Dayton, OH 45433, USA

***Abstract*—Dual-function radar-communications technology has attracted significant attention in recent years. In this paper, we propose a novel technique for embedding communication information in radar waveforms by exploiting sidelobe control and waveform diversity. The proposed strategy can serve multiple communication receivers located in the sidelobe region. In addition to information broadcasting, the proposed approach enables multi-user access, i.e., transmitting distinct information streams to the communication receivers located in different directions. The developed technique also serves as a generalized mathematical framework for the existing sidelobe control-based techniques.**

***keywords:* Spectrum sharing, dual-function radar-communications, co-existence, waveform diversity, sidelobe control**

I. Introduction

Spectrum sharing has become increasingly important since the past decade due to ongoing congestion of spectral resources [1–4]. Higher data rates in wireless communications require the expansion of existing frequency allocations. Moreover, new technical advances which benefit end consumers demand new allocation of spectral resources [5]. Significant research efforts have been made in the direction of cognitive radio [6] in order to effectively manage the existing frequency usage. Recently, co-existence of multiple platforms within the same frequency bands has mollified the spectral congestion by simultaneously sharing the same spectral resources for multiple applications [7–12]. Dual-function radar-communications (DFRC) is an important example of such platforms which performs the secondary communication operation in addition to the primary radar function while utilizing the same spectral resources [13–22].

In DFRC systems, radar and communication waveforms are transmitted from the same antenna array. A secondary communication function is activated by embedding communication symbols in radar waveforms such that the radar capabilities are not compromised. The popular techniques which enable such radar-embedded communications include waveform diversity-based method [13], sidelobe amplitude modulation (AM) method [14], multi-waveform amplitude shift keying (ASK) method [15, 17, 18], and phase shift keying (PSK) method [20]. In the waveform diversity-based method [13], a dictionary of waveforms is used such that each waveform represents one communication symbol. One radar waveform is selected during each radar pulse to embed communication information. One radar waveform is transmitted during each radar pulse and the communication receiver can reliably receive the transmitted information. The work on sidelobe AM method [14] employs multiple beamforming weight vectors corresponding to different sidelobe levels at the communication receivers. Each sidelobe level represents a unique communication information. The multi-waveform ASK-based method [15, 17] extends this concept and exploits multiple orthogonal radar waveforms, where each waveform is associated with an adaptive beamformer implementing two sidelobe levels. The communication receiver decodes the transmitted waveform as well as the sidelobe level to determine the transmitted information. Although this method exploits waveform diversity, only two sidelobe levels are used. The method in [18] increases the effective signal-to-noise ratio (SNR) at the communication receiver by using only one beamforming weight vector which corresponds to the maximum allowable sidelobe level. In PSK-based information embedding [20], a dictionary of phase symbols is used such that the phase of each symbol carries unique communication information. The communication receiver detects the corresponding phase symbol in the radar waveform to decode the transmitted information. Note that the PSK-based-method [20] and the multi-waveform sidelobe control-based methods [15, 17, 18] can only broadcast information, i.e., they are not designed to send distinct information to different communication receivers.

In this paper, we propose a novel DFRC strategy which exploits sidelobe control and waveform diversity for embedding communication information in the radar waveform. Unlike the existing multi-waveform sidelobe control-based schemes which only broadcast the same communication information to all communication receivers, the proposed approach enables transmission of distinct communication streams to different receivers while utilizing the same hardware resources as employed by the existing ASK-based techniques. In this context, the proposed technique can provide higher flexibility and overall throughput compared to the conventional approaches

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where multiplexing of communication information would be required to transmit different information streams to each receiver. Moreover, the proposed technique serves as the generalized mathematical model for the existing sidelobe ASK-based DFRC schemes.

II. Existing ASK-based Schemes

Consider a DFRC system equipped with an M-element linear transmit antenna array of an arbitrary shape. The objective of existing ASK-based schemes is to send information symbols to the communication receivers located in the sidelobe regions without introducing perturbation to the primary radar operation. This implies that the magnitude of the radar beampattern must not vary during each transmitted pulse. In order to realize this objective, different beamforming vectors are synthesized to allow different sidelobe levels at the receiver side while keeping the radar main beam at a constant amplitude. The following optimization can be used for this purpose [17, 21]:

$$\begin{aligned} \min_{\mathbf{u}_l} \max_{\theta_i} & \left| e^{j\varphi(\theta_i)} - \mathbf{u}_l^{\mathrm{H}} \mathbf{a}(\theta_i) \right|, \quad \theta_i \in \Theta, \\ \text{subject to } & \left| \mathbf{u}_l^{\mathrm{H}} \mathbf{a}(\theta_p) \right| \leq \varepsilon, \quad \theta_p \in \bar{\Theta}, \\ & \mathbf{u}_l^{\mathrm{H}} \mathbf{a}(\theta_r) = \delta_l, \quad 1 \leq l \leq L, \quad 1 \leq r \leq R, \end{aligned} \tag{1}$$

where Θ is the set of angles at which the radar main beam operates, and $\bar{\Theta}$ is the complement set of Θ representing the sidelobe region. In addition, $\mathbf{a}(\theta)$ is the response vector of the transmit antenna array at angle θ, $\varphi(\theta)$ is the phase profile of user's choice, $\mathbf{u}_l$ is the desired beamforming vector which achieves the sidelobe level δ_l at the communication receiver located at angle θ_r, and R is the total number of communication receivers located in the sidelobe region. In addition, L denotes the total number of allowable sidelobe levels, and $(.)^{\mathrm{H}}$ represents the Hermitian operator.

In the following, we summarize the three sidelobe-based DFRC schemes [14, 15, 17, 18]. We denote P as the total power transmitted by the antenna array and $\psi_1(t), \psi_2(t), \ldots, \psi_{\hat{K}}(t)$ be the $\hat{K}$ possible radar waveforms orthogonal to each other. Here, t is the fast time and each radar waveform is normalized to unit average power for the period of radar pulse.

A. Sidelobe AM-based Scheme

In the sidelobe AM-based scheme [14], L beamforming weight vectors are generated by using the optimization in Eq. (1) such that each vector results in a unique sidelobe level in the direction of the communication receivers. The signal transmitted from the DFRC platform during each radar pulse uses one of the L available beampatterns as follows:

$$\mathbf{s}(t, \tau) = \sqrt{P} \mathbf{U} \mathbf{b}(\tau) \psi(t), \tag{2}$$

where τ is the slow time (i.e., pulse index), $\psi(t)$ is a fixed waveform selected from the $\hat{K}$ possible radar waveforms, $\mathbf{b}(\tau) = [b_1(\tau), b_2(\tau), \ldots, b_L(\tau)]^{\mathrm{T}}$ is a binary selection vector which selects one beamforming weight vector $\mathbf{u}_l$ corresponding to the desired sidelobe level δ_l from the columns of the beamforming dictionary matrix $\mathbf{U} = [\mathbf{u}_1^*, \mathbf{u}_2^*, \ldots, \mathbf{u}_L^*]$ with $(.)^*$ and $(.)^{\mathrm{T}}$ representing the conjugate and transpose operators, respectively. During each radar pulse, only one element in $\mathbf{b}(\tau)$ is equal to one and the rest of the $L-1$ elements are equal to zero. As such, the fixed radar waveform carries $\log_2 L$ bits of information in each pulse.

B. Multi-waveform ASK-based Scheme

In the multi-waveform ASK-based scheme [15, 17], K ($\leq \hat{K}$) orthogonal radar waveforms are utilized while exploiting only two beamforming weight vectors. These weight vectors $\mathbf{u}_{\text{low}}$ and $\mathbf{u}_{\text{high}}$ correspond to the two sidelobe levels of δ_{low} and δ_{high}, where $\delta_{\text{low}} < \delta_{\text{high}}$, at the directions of all communication receivers. The transmitted signal for this scheme is given as:

$$\mathbf{s}(t, \tau) = \sqrt{\frac{P}{K}} \sum_{k=1}^{K} \left(b_k(\tau) \mathbf{u}_{\text{low}}^* + (1 - b_k(\tau)) \mathbf{u}_{\text{high}}^* \right) \psi_k(t). \tag{3}$$

The value of each coefficient $b_k(\tau)$ is either 0 or 1 during each radar pulse. These coefficients select the desired beamforming weight vector for each of the transmitted waveforms $\psi_1(t), \psi_2(t), \ldots, \psi_K(t)$, thereby carrying one bit of information for each of these waveforms. During each radar pulse, one radar waveform is transmitted with a power of $\delta_{\text{low}} P/K$ or $\delta_{\text{high}} P/K$ towards each communication receiver for the detection of embedded information at these receivers. Note that the same communication symbols are broadcast to all the receivers in this scheme.

C. Efficient Multi-waveform ASK-based Scheme

In this ASK-based scheme [18], only one beamforming weight vector $\mathbf{u}_{\text{high}}$ is employed which corresponds to the highest allowable sidelobe level at the communication receivers, resulting in the highest possible SNR for communication information. During each radar pulse, $\hat{K}-1$ bits are transmitted such that K bits are equal to 1 and the remaining $\hat{K}-K-1$ bits are equal to zero. This is achieved by transmitting $\hat{K}$ distinct orthogonal waveforms. The transmitted signal is given as:

$$\begin{aligned} \mathbf{s}(t, \tau) = & \sqrt{\frac{P}{K}} \sum_{k=1}^{\hat{K}-1} \left(b_k(\tau) \mathbf{u}_{\text{high}}^* \psi_k(t) \right) \\ & + \sqrt{P} \prod_{k=1}^{\hat{K}-1} (1 - b_k(\tau)) \mathbf{u}_{\text{high}}^* \psi_{\hat{K}}(t). \end{aligned} \tag{4}$$

Each of the coefficients $b_k(\tau)$ for $1 \leq k \leq \hat{K}-1$ is either 0 or 1 such that only K ($\leq \hat{K}-1$) coefficients are equal to 1 and the rest of them are equal to 0. The second term in the above equation expresses the case that, when all coefficients of $b_k(\tau)$ are equal to 0, the reference orthogonal waveform $\psi_{\hat{K}}(t)$ is transmitted. The same information is broadcast to all the communication receivers because the transmission is formulated to achieve the same sidelobe level at each communication receiver.

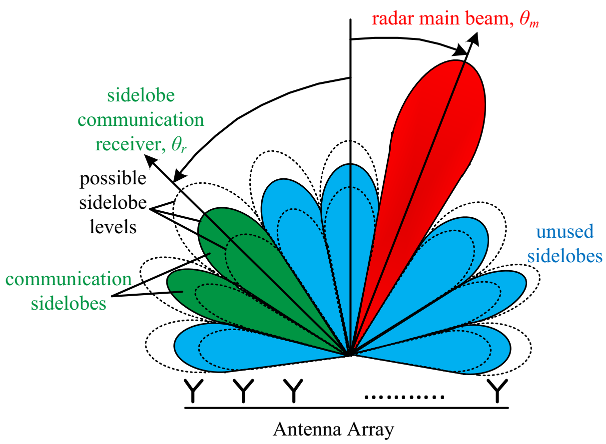

Fig. 1. The proposed DFRC approach exploiting sidelobe ASK and waveform diversity

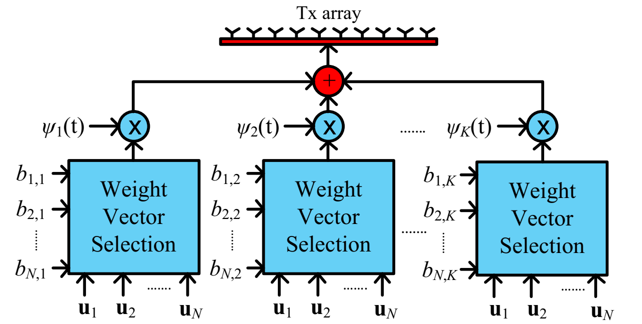

Fig. 2. The proposed DFRC approach exploiting sidelobe ASK and waveform diversity for the case of sidelobe-only communication.

III. Proposed Information Embedding Strategy

A. Proposed Information Embedding

We have observed that the existing ASK-based DFRC approaches [14, 15, 17, 18, 21] can only broadcast the same information to all the communication receivers. In our proposed approach, we show that different receivers located in the sidelobe region can be served with distinct individual communication information. Information embedding is achieved by sidelobe ASK and waveform diversity to feed distinct communication streams to multiple communication users located in different sidelobe directions. Moreover, we will show that the proposed scheme serves as the generalized mathematical formulation of all the existing ASK-based DFRC techniques. Fig. 1 shows the basic principle of the proposed approach.

In order to simultaneously transmit distinct information to different communication receivers, different sidelobe levels must be achieved at different communication receivers during each radar pulse. These sidelobe levels vary with the communication users but are kept time-invariant during each radar pulse which constitutes the symbol period. In order to enable different sidelobe levels at the communication receivers, Eq. (1) is modified as:

$$\begin{aligned} &\min_{\mathbf{u}_n} \max_{\theta_i} \left| e^{j\varphi(\theta_i)} - \mathbf{u}_n^{\mathrm{H}}\mathbf{a}(\theta_i) \right|, \quad \theta_i \in \Theta, \\ &\text{subject to } \left| \mathbf{u}_n^{\mathrm{H}}\mathbf{a}(\theta_p) \right| \le \varepsilon, \quad \theta_p \in \bar{\Theta}, \\ &\qquad \mathbf{u}_n^{\mathrm{H}}\mathbf{a}(\theta_r) = \delta_r, \quad 1 \le n \le N, \quad 1 \le r \le R. \end{aligned} \tag{5}$$

Here, $\mathbf{u}_n$ is the nth beamforming weight vector resulting in the sidelobe level δ_r at the rth communication receiver. Each δ_r can take any of the L allowable sidelobe levels. It can be noted from Eq. (5) that it is possible to transmit radar waveforms with same (broadcasting) or different (multi-user access) sidelobe levels at all communication receivers at the same time. In this case, the required number of beamforming weight vectors to support L distinct sidelobes in each of the R user directions is $N = L^R$. Note that a higher value of N does not require a higher number of antennas or compromise the detection performance at each communication receiver.

The signal transmitted from the antenna array is given as:

$$\mathbf{s}(t,\tau) = \sqrt{\frac{P}{K}} \sum_{k=1}^{K} (\mathbf{U}\mathbf{b}_k(\tau)\psi_k(t)), \tag{6}$$

where $\mathbf{U} = [\mathbf{u}_1^*, \mathbf{u}_2^*, \ldots, \mathbf{u}_N^*]$ is the beamforming dictionary matrix which includes N beamforming weight vectors optimized using Eq. (5), and $\mathbf{b}_k(\tau) = [b_{1,k}(\tau), b_{2,k}(\tau), \ldots, b_{N,k}(\tau)]^{\mathrm{T}}$ is an $N \times 1$ selection vector which selects the desired beamforming weight vector from the dictionary $\mathbf{U}$ for each transmitted waveform $\psi_k(t)$. All the elements in $\mathbf{b}_k(\tau)$ are 0 except only one element which is equal to 1. We utilize K $(\le \hat{K})$ orthogonal waveforms during each radar pulse and it is possible to use different values of K for each pulse. All the individual sidelobe levels at each communication receiver obtained by the weight vectors present in $\mathbf{U}$ generate different sidelobe levels in the direction of communication users, thus enabling us to transmit distinct communication streams to each receiver. Since the transmission scheme allows L unique sidelobe levels at each receiver, a transmitted waveform carries $\log_2 L$ bits of distinct information for each receiver. The proposed signaling strategy is outlined in Fig. 2.

Eq. (6) can be reformulated in the following compact form:

$$\mathbf{s}(t,\tau) = \sqrt{\frac{P}{K}} \mathbf{U}\mathbf{B}(\tau)\boldsymbol{\psi}(t), \tag{7}$$

where

$$\begin{aligned} \mathbf{B}(\tau) &= [\ \mathbf{b}_1(\tau) \quad \mathbf{b}_2(\tau) \quad \cdots \quad \mathbf{b}_K(\tau)\ ], \\ \boldsymbol{\psi}(t) &= [\ \psi_1(t) \quad \psi_2(t) \quad \cdots \quad \psi_K(t)\ ]^{\mathrm{T}}. \end{aligned} \tag{8}$$

We can observe that the proposed multi-waveform sidelobe ASK-based signaling strategy treats any of the existing ASK-based techniques discussed in [14, 15, 17, 18] as a special case. Table I summarizes the parameters which can be changed in Eq. (6) to yield these existing ASK-based signaling methods. This implies that the proposed signaling scheme represents a generalized framework for the class of ASK-based signaling techniques.

B. Sum Rate Analysis

In this section, we evaluate the sum of the maximum number of bits during one radar pulse which can be transmitted using the proposed technique to the sidelobe communication receivers located at distinct angles. Consider that R receivers utilize L (> 1) possible sidelobe levels such that a fixed number of K waveforms are used during each radar pulse. According to Eq. (5), for each radar waveform we can transmit $\log_2 L$ bits of information to each communication receiver. Thus, the maximum number of bits which can be transmitted during one radar pulse to all the R receivers is $RK \log_2 L$. Note that the information streams for different receivers can be independent of each other in the proposed scheme. We can calculate the maximum data rate for the case with a varying number of waveforms K, where $K \leq \hat{K}$, during each radar pulse in a similar manner.

On the other hand, we can extract that the maximum possible number of bits which can be broadcast to each communication receiver located in the sidelobe region during one radar pulse is $\log_2 L$, K $(\leq \hat{K})$, and $\hat{K} - 1$ for [14], [15, 17], and [18], respectively. It is clear that the the proposed technique outperforms the existing methods in terms of the overall throughput or sum data rate because it enables the utilization of multiple distinct sidelobe levels and waveform diversity for the transmission scheme.

C. Detection at the Communication Receiver

The signal $x_r(t, \tau)$ received at the rth communication receiver located in sidelobe region at angle $\theta_r \in \bar{\Theta}$ can be described as:

$$x_r(t, \tau) = \alpha_r(\tau) \left(\mathbf{a}_T^{\mathrm{T}}(\theta_r) \mathbf{s}(t, \tau) \right) + n_r(t), \quad (9)$$

where $\alpha_r(\tau)$ is the channel response towards the rth user, which is characterized as a constant during each radar pulse, and $n_r(t)$ is the zero-mean white Gaussian noise. Matched filtering the received signal (9) to each of the $\hat{K}$ possible waveforms at the rth communication receiver yields the following scalar:

$$\begin{aligned} y_{r,k}(\tau) &= \frac{1}{T} \int_0^T x_r(t, \tau) \psi_k(t) \, dt \\ &= \begin{cases} \sqrt{\frac{P}{K}} \alpha_r(\tau) \delta_r + n_{r,k}(\tau), & \text{if } \psi_k(t) \text{ was transmitted}, \\ n_{r,k}(\tau), & \text{otherwise}. \end{cases} \end{aligned} \quad (10)$$

TABLE I. PARAMETERS FOR CONVERTING THE PROPOSED SCHEME INTO EXISTING TECHNIQUES

Signalling Strategy	Parameters for Eq. (6)
Sidelobe AM [14]	$K = 1, N = L$
Multiwaveform ASK [15, 17]	$N = 2, \mathbf{u}_1 = \mathbf{u}_{\text{low}}, \mathbf{u}_2 = \mathbf{u}_{\text{high}}$, $b_{1,k}(\tau) = 1 - b_{2,k}(\tau)$, fixed $K (\leq \hat{K})$
Efficient Multi-waveform ASK [18]	$N = 1, \mathbf{u}_1 = \mathbf{u}_{\text{high}}$, varying K For all zeros: $K = 1, b_{1,K} = 1$. else: $2 \leq k \leq \hat{K}, b_{1,k} = 0$ or 1, $\sum_{k=2}^{\hat{K}} b_{1,k} = K$.

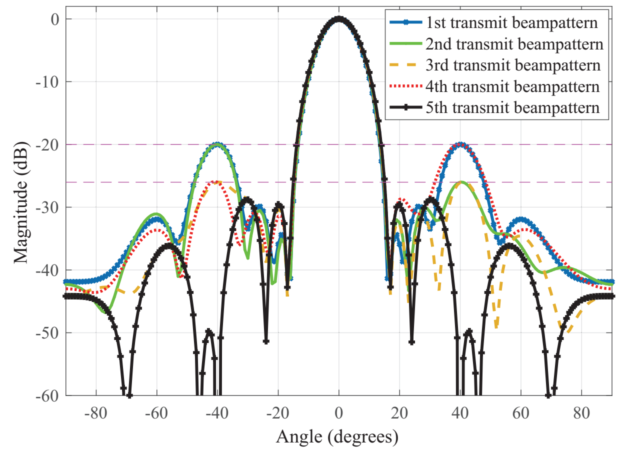

Fig. 3. Example beampatterns for the proposed technique (ULA with $M = 10$, $L = 3$, $R = 2$ at 40^{o} and -40^{o}).

where T is the pulse width and $n_{r,k}(\tau)$ is the noise received at the output of the kth matched filter. By analyzing $y_{r,k}(\tau)$ at the rth $(1 \leq r \leq R)$ receiver using all the radar waveforms (i.e., $1 \leq k \leq \hat{K}$), it is possible to determine the transmitted waveforms and the corresponding sidelobe level δ_r which decodes the embedded communication information.

IV. Simulation Results

We consider a uniform linear array (ULA) consisting of 10 transmit antennas. The primary function of the radar is assumed to form a main beam at 0^{o}. There are two communication receivers located in the sidelobe region at -40^{o} and 40^{o}, respectively. Three different sidelobe levels ($L = 3$) of 0 (i.e., $-\infty$ dB), 0.05 (i.e., -26 dB), and 0.1 (i.e., -20 dB) are considered for communications in the sidelobes. According to Eqs. (5) and (6), we need to design $L^R = 3^2 = 9$ beamforming weight vectors (beampatterns corresponding to 5 of such vectors are illustrated in Fig. 3). It can be observed that each beampattern has a unique set of sidelobe levels for the communication receivers. If one waveform is used, the five beampatterns can be perceived as dual-symbol pairs, respectively denoted as $(2, 2)$, $(2, 1)$, $(1, 1)$, $(1, 2)$, and $(0, 0)$, where the first symbol corresponds to the receiver at -40^{o} and the second symbol corresponds to the receiver at 40^{o}. Note that the symbols transmitted to each receiver can differ. The use of multiple waveforms further enhances the information rate for the given number of sidelobe levels.

On the other hand, the method described in [14] can utilize the first, third and fifth beampatterns only, the method in [15, 17] can only use the first and third beampatterns, whereas [18] only incorporates the first beampattern illustrated in Fig. 3. Such restricted selection of beampatterns limits the ability to transmit different information streams to the communication receivers located in different directions. Thus, the proposed technique exhibits superiority over these existing schemes.

V. Conclusion

In this paper, a novel multi-waveform ASK-based scheme exploiting sidelobe control and waveform diversity was proposed which enables the transmission of different communication information to different receivers during one radar pulse. Compared to the existing methods which broadcast the communication information in sidelobe-region directions, the proposed strategy enables multi-user access by transmitting distinct information in different directions. This is achieved by using distinct sidelobe levels in multiple directions serving different communication users. Compared to existing techniques, the proposed technique provides significant improvement in flexibility and sum rates. In addition, the proposed signaling strategy also serves as a generalized mathematical formulation for all the existing ASK-based DFRC methods.

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