ROBUST DOA ESTIMATION IN THE PRESENCE OF MIS-CALIBRATED SENSORS

Ben Wang[†], *Si Qin*[‡], *Yimin D. Zhang*[†], *and Moeness G. Amin*[‡]

[†] Department of Electrical and Computer Engineering, Temple University, Philadelphia, PA 19122, USA
[‡] Center for Advanced Communications, Villanova University, Villanova, PA 19085, USA

ABSTRACT

In this paper, we consider robust direction-of-arrival (DOA) estimation for an array that contains mis-calibrated sensors with unknown gain and phase uncertainties. We develop two robust DOA estimation algorithms based on the maximum correntropy criterion (MCC). In the first algorithm, adaptively optimized weighting factors are obtained and applied to each sensor to effectively mitigate the effect of calibration error and array manifold distortions, and the results are fed into sparse reconstruction methods for DOA estimation. In the second algorithm, we further estimate the gain and phase errors of the mis-calibrated sensors so that the entire array is fully calibrated for improved DOA estimation. The effectiveness of the proposed techniques is verified using simulation results.

Index Terms— Robust DOA estimation, gain-phase error, maximum correntropy criterion, array calibration

1. INTRODUCTION

Direction-of-arrival (DOA) estimation determines the spatial spectrum of the impinging electromagnetic waves on an antenna array. It finds variety of applications in radar, sonar, radio astronomy, and mobile communication systems [1]. Recently, sparse signal representation (SSR) framework has emerged and attracted significant interest in DOA estimation, mainly due to the key observation that the DOAs of signals are sparse in the entire spatial domain. The idea of utilizing SSR, which is intrinsically different from the subspace-based methods like MUSIC [2] and ESPRIT [3], provides a new sparse signal reconstruction perspective for DOA estimation, and has been well studied in various contexts. The ℓ_1-based singular value decomposition (L1-SVD) algorithm [4] mainly addresses the DOA estimation problem by directly representing the array output in time domain with an overcomplete basis from the array response vector. A number of methods have been developed to exploit the spatial sparsity of the signal arrivals so as to improve the resolution, increase the degrees of freedom, handle spatially distributed sources, and achieve robustness against coherent signals [5–8].

However, all these strategies assume exact knowledge of the gain and phase of all sensor arrays. In a practical operation, however, the actual sensor gain and phase characteristics may deviate or be perturbed from their assumed values due to the variation in the temperature and operational environment. Since the conventional SSR based algorithms are generally based on the minimum mean square error (MMSE) criterion, the corresponding quadratic operation amplifies the contribution of such perturbations, resulting in significant degradations of the DOA estimation accuracy.

In this paper, we consider robust DOA estimation for an array with mis-calibrated sensors, i.e., their gain and phase characteristics are unknown. More specifically, two robust DOA estimation algorithms are developed based on the maximum correntropy criterion (MCC) [9]. MCC is a useful tool to detect outliers and suppress their contributions, whereas it attains the same optimality as MMSE when no outliers are present. In [9], it was shown that correntropy induces a new metric which is equivalent to the ℓ_2-norm distance if the points are close, where it behaves similarly to the ℓ_1-norm distance as the points get further apart and eventually approaches the ℓ_0-norm as they are far apart. This geometric interpretation elucidates the robustness of correntropy for outlier (caused by manifold perturbations) rejection. The MCC has been demonstrated to achieve robust results when observations are corrupted by non-Gaussian noise or outliers. It has been successfully applied in various applications, such as radar localization [10], ellipse fitting problem [11], and DOA estimation [12].

Two robust DOA estimation algorithms are developed in this paper. In the first algorithm, adaptively optimized weighting factors are obtained and applied to each sensor to effectively mitigate the effect of calibration error and array manifold distortions. More specifically, the value of the weights will be close to unity when the corresponding sensor is properly calibrated, whereas the weights will be close to zero when the corresponding sensor is highly mis-calibrated. The weighted results are then fed into sparse reconstruction methods to achieve robust DOA estimation. In the second algorithm, we further estimate the gain and phase errors of the mis-calibrated sensors so that the entire array is fully calibrated for improved DOA estimation.

Notations: We use lower-case (upper-case) bold characters to denote vectors (matrices). In particular, $\mathbf{I}_N$ stands for the $N \times N$ identity matrix. $(\cdot)^*$ implies complex conjugation, whereas $(\cdot)^T$ and $(\cdot)^H$ respectively denote the transpose and conjugate transpose of a matrix or a vector. $\mathrm{diag}(\mathbf{x})$ denotes a diagonal matrix that uses the elements of $\mathbf{x}$ as its diagonal elements. $\otimes$ denotes the Kronecker product and $\mathrm{vec}(\mathbf{A})$ stands for stacking the columns of matrix $\mathbf{A}$ into a single-column vector. $\|\cdot\|_1$ and $\|\cdot\|_2$ denote the ℓ_1-norm and ℓ_2-norm, respectively. $\mathrm{phase}(a)$ returns the phase of the element a.

The work of S. Qin, Y. D. Zhang, and M. G. Amin is supported in part by the Office of Naval Research (ONR) under grant No. N00014-13-1-0061.

2. PROBLEM FORMULATION

2.1. Signal Model

Consider Q narrowband far-field signals impinging on an arbitrary linear array with M antennas. Denote p_m as the position of the mth antenna where $m = 1, \ldots, M$, and the first sensor is assumed as the reference, i.e., $p_1 = 0$. Assume that the signals are from angles $\Theta = [\theta_1, \ldots, \theta_Q]^T$, and their discretized baseband waveforms are expressed as $s_q(t)$, for $q = 1, \ldots, Q$, any $t = 1, \ldots, T$. Then, the $M \times 1$ array output vector with T snapshots can be modelled as

$$\mathbf{x}(t) = \sum_{q=1}^{Q} \mathbf{a}(\theta_q) s_q(t) + \mathbf{n}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t), \quad (1)$$

for $t = 1, \ldots, T$, where

$$\mathbf{a}(\theta_q) = \left[1, e^{j\frac{2\pi p_2}{\lambda}\sin(\theta_q)}, ..., e^{j\frac{2\pi p_M}{\lambda}\sin(\theta_q)}\right]^T \quad (2)$$

is the steering vector of the array corresponding to θ_q, $\mathbf{A} = [\mathbf{a}(\theta_1), ..., \mathbf{a}(\theta_Q)]$, and $\mathbf{s}(t) = [s_1(t), ..., s_Q(t)]^T$. In addition, $\mathbf{n}(t)$ is the noise vector. We introduce the following standard assumptions about the properties of the signals and noise:

(1) The signals $s_q(t)$, $q = 1, \ldots, Q$, are uncorrelated zero-mean random variables with $\mathrm{E}[s_q(t)s_p^*(t)] = \sigma_q^2 \delta_{q,p}$, $1 \leq q, p \leq Q$;

(2) The noise elements are independent and identically distributed (i.i.d.) random variables following the complex Gaussian distribution $\mathcal{CN}(0, \sigma_n^2 \mathbf{I}_M)$;

(3) The noise is statistically independent of all the signals.

2.2. Covariance Sparsity-Aware DOA Estimation

The $M \times M$ covariance matrix of $\mathbf{x}(t)$ is obtained as

$$\begin{aligned} \mathbf{R_{xx}} &= \mathrm{E}[\mathbf{x}(t)\mathbf{x}^H(t)] = \mathbf{A}\mathbf{R_{ss}}\mathbf{A}^H + \sigma_n^2 \mathbf{I}_M \\ &= \sum_{q=1}^{Q} \sigma_q^2 \mathbf{a}(\theta_q)\mathbf{a}^H(\theta_q) + \sigma_n^2 \mathbf{I}_M, \end{aligned} \quad (3)$$

where $\mathbf{R_{ss}} = \mathrm{E}[\mathbf{s}(t)\mathbf{s}^H(t)] = \mathrm{diag}([\sigma_1^2, ..., \sigma_Q^2])$ is the $Q \times Q$ source covariance matrix. In practice, the covariance matrix is estimated using the T available samples, i.e.,

$$\hat{\mathbf{R}}_{\mathbf{xx}} = \frac{1}{T}\sum_{t=1}^{T} \mathbf{x}(t)\mathbf{x}^H(t). \quad (4)$$

By vectorizing the matrix $\hat{\mathbf{R}}_{\mathbf{xx}}$, we obtain the following $M^2 \times 1$ vector

$$\mathbf{z} = \mathrm{vec}(\hat{\mathbf{R}}_{\mathbf{xx}}) = \tilde{\mathbf{A}}\mathbf{b} + \sigma_n^2 \tilde{\mathbf{I}} = \tilde{\mathbf{B}}\tilde{\mathbf{r}}, \quad (5)$$

where $\tilde{\mathbf{A}} = [\tilde{\mathbf{a}}(\theta_1), ..., \tilde{\mathbf{a}}(\theta_Q)]$, $\tilde{\mathbf{a}}(\theta_q) = \mathbf{a}^*(\theta_q) \otimes \mathbf{a}(\theta_q)$, $\mathbf{b} = [\sigma_1^2, ..., \sigma_Q^2]^T$, $\tilde{\mathbf{I}} = \mathrm{vec}(\mathbf{I}_M)$. In addition, $\tilde{\mathbf{B}} = [\tilde{\mathbf{A}}, \tilde{\mathbf{I}}]$ and $\tilde{\mathbf{r}} = [\mathbf{b}^T, \sigma_n^2]^T = [\sigma_1^2, ..., \sigma_Q^2, \sigma_n^2]^T$ are used for notational convenience. As such, we can regard $\mathbf{z}$ as a received signal vector from a single-snapshot signal vector $\mathbf{b}$, and the matrix $\tilde{\mathbf{A}}$ corresponds to the virtual array sensors. Note that the virtual vector $\mathbf{z}$ can be sparsely represented over the entire discretized angular grids as [13]

$$\hat{\mathbf{r}} = \arg\ \min ||\mathbf{r}||_1 \quad \mathrm{s.t.} \quad ||\mathbf{z} - \mathbf{B}\mathbf{r}||_2^2 < \epsilon, \quad (6)$$

where $\mathbf{B} = [\bar{\mathbf{A}}, \tilde{\mathbf{I}}]$. Herein, $\bar{\mathbf{A}}$ is defined as the collection of steering vectors $\tilde{\mathbf{a}}(\theta_g)$ over all possible grids θ_g, $g = 1, \ldots, G$, with $G \gg Q$, and $\mathbf{r}$ is the sparse vector whose non-zero entry positions correspond to the DOAs θ_q, $q = 1, \ldots, Q$. In addition, ϵ is a user-specific tolerance factor which reflects the discrepancies between the statistical expectation and the sample average in computing the covariance matrix. The sparse learning problem (6) can be solved within the SSR framework and various SSR methods (e.g., [14–19]) can be used for this purpose.

2.3. Problem Formulation

The signal model discussed in Sections 2.1 and 2.2 assume that all antennas are calibrated. In this section, we introduce the signal model in the presence of mis-calibrated antennas which are affected by unknown gain and phase distortions. Denote $\mathbb{M}$ as the set of mis-calibrated antennas with a cardinality $|\mathbb{M}| = M_0 < M$. Taking the gain-phase error into account, the model in (1) can be modified as

$$\mathbf{y}(t) = \mathbf{G}\mathbf{\Phi}\mathbf{A}\mathbf{s}(t) + \mathbf{n}(t) = \mathbf{\Gamma}\mathbf{A}\mathbf{s}(t) + \mathbf{n}(t), \quad (7)$$

where $\mathbf{G} = \mathrm{diag}([\rho_1, \ldots, \rho_M])$ contains gain error ρ_m, $m = 1, \ldots, M$, whereas the phase error ϕ_m, $m = 1, \ldots, M$, is included in $\mathbf{\Phi} = \mathrm{diag}([e^{j\phi_1}, \ldots, e^{j\phi_M}])$. In addition, $\mathbf{\Gamma}$ is a diagonal matrix whose diagonal entries can be expressed as

$$\mathbf{\Gamma}_{mm} = \gamma_m = \begin{cases} \rho_m e^{j\phi_m}, & m \in \mathbb{M}, \\ 1, & m \notin \mathbb{M}. \end{cases} \quad (8)$$

Denote $\hat{\mathbf{R}}_{\mathbf{yy}} = \frac{1}{T}\sum_{t=1}^{T} \mathbf{y}(t)\mathbf{y}^H(t)$ as the corresponding sample covariance matrix, whose expected value is

$$\mathbf{R_{yy}} = \mathbf{\Gamma}\mathbf{A}\mathbf{R_{ss}}\mathbf{A}^H\mathbf{\Gamma}^H + \sigma_n^2 \mathbf{I}_M, \quad (9)$$

and the sparse reconstruction problem is updated as

$$\hat{\mathbf{r}}^\circ = \arg\ \min ||\mathbf{r}^\circ||_1 \quad \mathrm{s.t.} \quad ||\tilde{\mathbf{z}}^\circ - \mathbf{B}\mathbf{r}^\circ||_2^2 < \epsilon, \quad (10)$$

with $\tilde{\mathbf{z}}^\circ = \mathrm{vec}(\hat{\mathbf{R}}_{\mathbf{yy}})$, and $\hat{\mathbf{r}}^{\mathbf{o}}$ is the estimated sparse vector with unknown gain-phase error. Note that the gain-phase error matrix $\mathbf{\Gamma}$ distorts the observation. In particular, the quadratic operation in (10) amplifies the contribution of such perturbations, yielding the incorrect estimates.

3. MAXIMUM CORRENTROPY CRITERION THEORY

In this section, the MCC theory is briefly reviewed [9]. Assume two arbitrary scalar random variables Z_1 and Z_2. The correntropy is a generalized similarity measure between Z_1 and Z_2 and is defined by

$$V_\sigma(Z_1, Z_2) = \mathrm{E}\left[\kappa_\sigma(Z_1 - Z_2)\right], \quad (11)$$

where $\kappa_\sigma(z) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{z^2}{2\sigma^2}\right)$ is a Gaussian kernel. The kernel size σ is, also referred to as bandwidth, a parameter that must be properly chosen based on density estimation, using, for example, Silverman's rule [20] or maximum likelihood [10]. In practice, the joint probability density function is unknown and only a finite number of data are available, leading to the sample estimator of correntropy

$$\hat{V}_\sigma(Z_1, Z_2) = \frac{1}{N}\sum_{n=1}^{N}\kappa_\sigma(z_{1_n} - z_{2_n}). \tag{12}$$

Note that the value of the correntropy is primarily dictated by the kernel function along the line $z_1 = z_2$, which implies that the correntropy is insensitive to outliers [9]. Such local criterion of similarity indicates that the MCC, given by,

$$\max_{\boldsymbol{\omega}} \frac{1}{N}\sum_{n=1}^{N}\kappa_\sigma(z_{1_n} - z_{2_n}), \tag{13}$$

achieves robust estimation result with outlier observations compared to the MMSE criterion in (10), where $\boldsymbol{\omega} = [\omega_1, \cdots, \omega_N]$ denotes a set of the adjustable parameters. By adjusting the values of $\boldsymbol{\omega}$, the optimal solution is achieved when (13) achieves the maximum value.

It should be noted that the kernel size acts as a zoom lens and adjusts the observation window which provides an effective mechanism to eliminate the detrimental effect of outliers. Thus, MCC has the advantage that it is a local criterion of similarity and it is useful when the observation noise is nonzero mean, non-Gaussian with outliers.

4. PROPOSED METHOD

In this section, we proposed a robust DOA estimation method under the gain-phase perturbations, based on the MCC and the re-calibration techniques.

4.1. Robust DOA Estimation

A. DOA estimation

Denote the estimation error as

$$e_m = |\tilde{z}_m^\circ - \omega_m \mathbf{b}_m^\circ \hat{\mathbf{r}}|, \tag{14}$$

with $m = 1, \cdots, M^2$, where $\tilde{z}_m^\circ$ and $\mathbf{b}_m$ denote the mth element of $\hat{\mathbf{z}}^\circ$ and the mth row of $\mathbf{B}$, respectively. In addition, $\hat{\mathbf{r}}^\circ$ is the estimate of $\mathbf{r}^\circ$. Under the MCC, the revised sparse reconstruction problem can be expressed as

$$\hat{\mathbf{r}}^\circ = \arg\ \min ||\mathbf{r}^\circ||_1 \quad \text{s.t.} \quad \frac{M^2}{\sum_{m=1}^{M^2}\kappa_\sigma(e_m)} < \tilde{\epsilon}, \tag{15}$$

where $\tilde{\epsilon}$ is a user-specific parameter. Using the results developed in [10], the above expression can be reformulated as

$$\hat{\mathbf{r}}^\circ = \arg\ \min ||\mathbf{r}^\circ||_1 \quad \text{s.t.} \quad \sum_{m=1}^{M^2}\omega_m e_m < \tilde{\epsilon}_1, \tag{16}$$

where $\tilde{\epsilon}_1$ is a user-specific parameter and the weight ω_m is given by

$$\omega_m = \kappa_\sigma(e_m). \tag{17}$$

It seems from (16) ω_m is a function of $\hat{\mathbf{r}}^\circ$, whereas $\hat{\mathbf{r}}^\circ$ depends on ω_m. This suggests an iterative algorithm that iterates between (16) and (17). The estimates of DOAs, $\hat{\theta}_q$, $q = 1, \ldots, Q$, can be obtained by positions of the high peaks of $\hat{\mathbf{r}}^\circ$ when the convergence is achieved. Note that $\omega_m \approx 1$ if e_m is small, whereas the value of ω_m will be significantly reduced when perturbed observations exist. As $\hat{\theta}_q$ converges to θ_q, the deficient antenna set $\mathbb{M}$ can be identified by the elements of weighting vector $\boldsymbol{\omega}$ with small values. Based on the deficient antenna positions and $\hat{\theta}_q$, we can further achieve the gain-phase error estimation as described in [21]. For the convenience of presentation, we assume that the first sensor is correctly calibrated, i.e., $\rho_1 = 1$ and $\phi_1 = 0$.

B. Gain error estimation

The eigen-decomposition of $\hat{\mathbf{R}}_{\mathbf{yy}}$ is given by

$$\hat{\mathbf{R}}_{\mathbf{yy}} = \mathbf{U}_s\mathbf{\Lambda}_s\mathbf{U}_s^H + \mathbf{U}_n\mathbf{\Lambda}_n\mathbf{U}_n^H, \tag{18}$$

where $\mathbf{U}_s \in \mathbb{C}^{M\times Q}$ and $\mathbf{U}_n \in \mathbb{C}^{M\times(M-Q)}$ contain eigenvectors corresponding to the signal and noise subspaces, respectively, and the associated eigenvalues are included in the diagonal matrices $\mathbf{\Lambda}_s = \text{diag}\{[\lambda_1, \lambda_2, \ldots, \lambda_Q]\}$ and $\mathbf{\Lambda}_n = \text{diag}\{[\lambda_{Q+1}, \lambda_{Q+2}, \ldots, \lambda_M]\}$. Then, the noise variance can be estimated as

$$\hat{\sigma}_n^2 = \frac{1}{M-Q}\sum_{m=Q+1}^{M}\lambda_m. \tag{19}$$

Subtracting the noise term from $\hat{\mathbf{R}}_{\mathbf{yy}}$, we have

$$\hat{\mathbf{R}}_{\mathbf{yy}_0} = \mathbf{\Gamma}\mathbf{A}\mathbf{R}_{\mathbf{ss}}\mathbf{A}^H\mathbf{\Gamma}^H. \tag{20}$$

Taking $\mathbf{r}$ as the main diagonal of $\hat{\mathbf{R}}_{\mathbf{yy}_0}$, it is easy to confirm that

$$\mathbf{r} = [\rho_1^2, \ldots, \rho_M^2]^T \cdot \sum_{q=1}^{Q}\sigma_q^2. \tag{21}$$

Then, the gain error can be estimated as

$$\hat{\rho}_m = \begin{cases} \sqrt{\dfrac{r_m}{r_1}}, & m \in \mathbb{M}, \\ 1, & m \notin \mathbb{M}. \end{cases} \tag{22}$$

C. Phase error estimation

In [2], it has shown that the noise eigenvectors are orthogonal to the space spanned by the column of $\mathbf{A}$. Hence, we need to minimize the following cost function

$$\begin{aligned} f_c(\mathbf{\Gamma}) &= \mathbf{\Gamma}^H\left\{\sum_{q=1}^{Q}\text{diag}\left(\mathbf{a}^H(\hat{\theta}_q)\right)\mathbf{U}_n\mathbf{U}_n^H\text{diag}\left(\mathbf{a}(\hat{\theta}_q)\right)\right\}\mathbf{\Gamma} \\ &= \mathbf{\Gamma}^H\mathbf{\Omega}\mathbf{\Gamma}, \end{aligned} \tag{23}$$

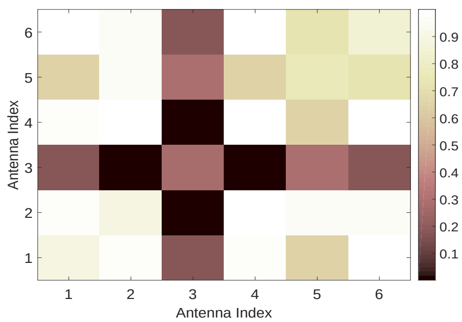

Fig. 1. Weighting factors of $\hat{\mathbf{R}}_{\mathbf{yy}}$.

under the constraints $\mathbf{\Gamma}^H\mathbf{c} = 1$ with $\mathbf{c} = [1, 0, \ldots, 0]^T$. The solution of this problem is given by

$$\boldsymbol{\xi} = \frac{\mathbf{\Omega}^{-1}\mathbf{c}}{\mathbf{c}^T\mathbf{\Omega}^{-1}\mathbf{c}}. \tag{24}$$

Then, the phase error can be estimated as

$$\hat{\phi}_m = \begin{cases} \text{phase}(\xi_m), & m \in \mathbb{M}, \\ 0, & m \notin \mathbb{M}. \end{cases} \tag{25}$$

D. Improved DOA estimation

Once the gain and phase errors of all sensors are estimated, the received data vector can be calibrated as

$$\bar{\mathbf{y}}(t) = \hat{\mathbf{\Phi}}^{-1}\hat{\mathbf{G}}^{-1}\mathbf{G}\mathbf{\Phi}\mathbf{A}\mathbf{s}(t) + \hat{\mathbf{\Phi}}^{-1}\hat{\mathbf{G}}^{-1}\mathbf{n}(t), \tag{26}$$

with the calibration matrices $\hat{\mathbf{G}} = \text{diag}(1/\hat{\rho}_1, \ldots, 1/\hat{\rho}_M])$ and $\hat{\mathbf{\Phi}} = \text{diag}([e^{-j\hat{\phi}_1}, \ldots, e^{-j\hat{\phi}_M}])$. As a result, an improved DOA estimates can be achieved by

$$\min ||\mathbf{r}^\circ||_1 \quad \text{s.t.} \quad ||\bar{\mathbf{z}} - \mathbf{B}\mathbf{r}^\circ||_2^2 < \epsilon, \tag{27}$$

where $\bar{\mathbf{z}} = \text{vec}(\hat{\mathbf{R}}_{\bar{\mathbf{y}}\bar{\mathbf{y}}}) = \text{vec}(\frac{1}{T}\sum_{t=1}^{T}\bar{\mathbf{y}}(t)\bar{\mathbf{y}}^H(t))$.

5. SIMULATION RESULTS

For illustrative purposes, we consider an $M = 6$ uniform linear array (ULA), where the interelement spacing is the half wavelength of carrier frequency. Assume that the 3rd and 5th sensors are mis-calibrated with errors $\rho_m \in [0.8, 1.2]$ and $\phi_m \in [0, \pi]$ when $m \in \mathbb{M} = \{3, 5\}$. $Q = 3$ far-field narrowband sources with identical powers are from $[-23°, 5°, 26°]$, respectively.

The proposed method is employed to achieve the robust DOA estimation. The covariance matrix is estimated by using 500 snapshots with a 0 dB SNR. The grid interval in the angular space is set to 0.2°. Fig. 1 shows the weighting factors of the covariance matrix $\hat{\mathbf{R}}_{\mathbf{yy}}$. Note that all entries corresponding to the 3rd and 5th sensors are significant reduced. As such, the deficient sensors can be detected.

In Fig. 2, we compare the proposed method with the conventional SSR based method. The MCC without calibration techniques is referred to as the MCC1, and the MCC2 represents the MCC with calibration technique. It is clear that the proposed method can achieve robust DOA estimation when the conventional method fails. In addition, the re-calibration techniques improve the estimation accuracy.

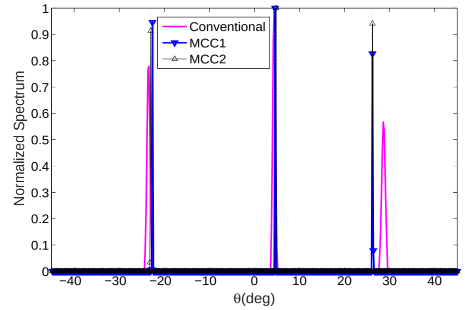

Fig. 2. Estimated spectra.

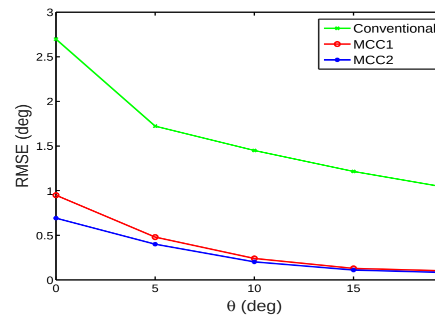

Fig. 3. RMSE of the DOA estimation versus SNR.

The estimation results of the gain and phase are summarized in Table 1. The mean and the standard deviance (STD) are obtained from 50 independent trials with SNR = 10 dB. In this experiment, the gain-phase errors for the 3rd and 5th sensors are set as $1.2e^{j2.0132}$ and $0.8e^{j0.3710}$. Table 1 verifies that the proposed method achieves accurate gain-phase error estimation.

Fig. 3 provides the RMSE results of the conventional SSR based method and the proposed method at different SNRs. At each SNR, the RMSE is calculated from 50 independent trials. Compared to the conventional method, the proposed method always achieves significant performance improvement. Utilizing the re-calibration algorithm, the performance of the MCC2 method further improves the estimated DOA performance as compared to the MCC1 method, especially in the important situatoin with a low input SNR.

Table 1. Gain and Phase Estimation Results

(a) Gain

Index	Actual	Mean	STD
ρ_3	1.2000	1.2116	0.0104
ρ_5	0.8000	0.7894	0.0156

(b) Phase (radian)

Index	Actual	Mean	STD
ϕ_3	2.0132	2.0171	0.0064
ϕ_5	0.3710	0.3782	0.0067

6. CONCLUSIONS

In this paper, two robust DOA estimation algorithms are developed in the sparse reconstruction framework for arrays with mis-calibrated sensors. Utilizing the MCC, the effect of perturbations of the mis-calibrated array sensors is effectively mitigated with the adaptively optimized weights. Further, by re-calibrating the mis-calibrated sensors, the DOA estimation performance is further improved. The effectiveness of these two methods are verified using simulation results.

7. REFERENCES

[1] H. L. Van Trees, *Detection, Estimation, and Modulation Theory, Part IV: Optimum Array Processing*. New York, NY: John Wiley, 2004.

[2] R. O. Schmidt, "Multiple emitter location and signal parameter estimation," *IEEE Trans. Antennas Propag.*, vol. 34, no. 3, pp. 276–280, 1986.

[3] R. Roy and T. Kailath, "ESPRIT – Estimation of signal parameters via rotation invariance techniques," *IEEE Trans. Acoust. Speech Signal Process.*, vol. 17, no. 7, pp. 984–995, 1989.

[4] D. Malioutov, M. Çetin, and A. S. Willsky, "A sparse signal reconstruction perspective for source localization with sensor arrays," *IEEE Trans. Signal Process.*, vol. 53, no. 8, pp. 3010–3022, 2005.

[5] M. Carlin, P. Rocca, G. Oliveri, F. Viani, and A. Massa, "Directions-of-arrival estimation through Bayesian compressive sensing strategies," *IEEE Trans. Antennas Propagat.*, vol. 61, no. 7, pp. 3828–3838, 2013.

[6] S. Qin, Y. D. Zhang, and M. G. Amin, "Generalized coprime array configurations for direction-of-arrival estimation," *IEEE Trans. Signal Process.*, vol. 63, no. 6, pp. 1377–1390, 2015.

[7] S. Qin, Y. D. Zhang, Q. Wu, and M. G. Amin, "DOA estimation of nonparametric spreading spatial spectrum based on Bayesian compressive sensing exploiting intra-task dependency," in *Proc. IEEE Int. Conf. Acoust. Speech Signal Process. (ICASSP)*, Brisbane, Australia, 2015, pp. 2399–2403.

[8] S. Qin, Y. D. Zhang, and M. G. Amin, "DOA estimation of mixed coherent and uncorrelated targets exploiting coprime MIMO radar," *Digital Signal Process.*, vol. 61, pp. 26–34, 2017.

[9] W. Liu, P. Pokharel, and J. C. Principe, "Correntropy: Properties and application in non-Gaussian signal processing," *IEEE Trans. Signal Process.*, vol. 55. no. 11, pp. 5286–5298, 2007.

[10] J. Liang, D. Wang, L. Su, B. Chen, H. Chen, and H. C. So, "Robust MIMO radar target localization via nonconvex optimization," *Signal Process.*, vol. 122, pp. 33–38, 2016.

[11] J. Liang, Y. Wang, and X. Zeng, "Robust elipse fitting via half-quadratic and semidefinite relaxation optimization," *IEEE Trans. Image Process.*, vol. 24, no. 11, pp. 4276–4286, 2015.

[12] B. Wang, Y. D. Zhang, and W. Wang, "Robust group compressive sensing for DOA estimation with partially distorted observations," *EURASIP J. Adv. Signal Process.*, vol. 2016, no. 2016:128, pp. 1–10, 2016.

[13] Y. D. Zhang, M. G. Amin, and B. Himed, "Sparsity-based DOA estimation using co-prime arrays," in *Proc. IEEE Int. Conf. Acoust. Speech Signal Process. (ICASSP)*, Vancouver, Canada, 2013, pp. 3967–3971.

[14] R. Tibshirani, "Regression shrinkage and selection via the lasso," *J. R. Stat. Soc.*, Ser. B, vol. 58, no. 1, pp. 267–288, 1996.

[15] S. S. Chen, D. L. Donoho, and M. A. Saunders, "Atomic decomposition by basis pursuit," *SIAM J. Sci. Comput.*, vol. 20, no. 1, pp. 33–61, 1998.

[16] J. A. Tropp and A. C. Gilbert, "Signal recovery from random measurements via orthogonal matching pursuit," *IEEE Trans. Inf. Theory*, vol. 53, no. 12, pp. 4655–4666, 2007.

[17] S. Ji, Y. Xue, and L. Carin, "Bayesian compressive sensing," *IEEE Trans. Signal Process.*, vol. 56, no. 6, pp. 2346–2356, 2008.

[18] Q. Wu, Y. D. Zhang, and M. G. Amin, "Complex multitask Bayesian compressive sensing," in *Proc. IEEE Int. Conf. Acoust. Speech Signal Process. (ICASSP)*, Florence, Italy, May 2014, pp. 3375–3379.

[19] Q. Wu, Y. D. Zhang, M. G. Amin, and B. Himed, "Multitask Bayesian compressive sensing exploiting intra-task dependency," *IEEE Signal Process. Lett.*, vol. 22, no. 4, pp. 430-434, 2015.

[20] B. W. Silverman, *Density Estimation for Statistics and Data Analysis*. London, U.K.: Chapman and Hall, 1986.

[21] S. Cao, Z. Ye, D. Xu, and X. Xu, "A Hadamard product based method for DOA estimation and gain-phase error calibration," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 49. no. 2, pp. 1224–1233, 2013.