Structured Decorrelation of Covariance Matrix for DOA Estimation of Coherent Signals

Saidur R. Pavel and Yimin D. Zhang
Department of Electrical and Computer Engineering
Temple University
Philadelphia, PA 19122, USA
{pavel.saidur, ydzhang}@temple.edu

Braham Himed
Distributed RF Sensing Branch
Air Force Research Laboratory
WPAFB, OH 45433, USA
braham.himed@us.af.mil

***Abstract*—This paper develops a computationally efficient approach for direction of arrival (DOA) estimation of coherent signals. The proposed method structurally reconstructs a decorrelated covariance matrix by exploiting a single row of the rank-deficient covariance matrix and its flipped and conjugated counterpart. The reconstruction produces a rank-restored covariance matrix suitable for DOA estimation. Compared to existing approaches, the proposed method offers a more computationally efficient and flexible solution for coherent signal DOA estimation while achieving more robust performance.**

***Index Terms*—Direction of arrival estimation, coherent signals, decorrelation, structured reconstruction.**

I. Introduction

Direction of arrival (DOA) estimation is an important area in array signal processing with applications in various fields, such as radar, sonar, wireless communication, radio astronomy [1]–[6]. Numerous methods have been developed for DOA estimation, among which maximum likelihood-based methods are effective regardless of the mutual coherence of the impinging signals [7]–[10]. However, these methods are computationally demanding due to the necessity of solving multi-dimensional optimization problems.

Alternatively, subspace-based methods, such as multiple signal classification (MUSIC) [11] and estimation of signal parameters via rotational invariance techniques (ESPRIT) [12], are widely used to achieve high-resolution DOA estimation with lower computational complexity. These methods exploit the eigenstructure properties of the covariance matrix of the received signals at the sensor array under the assumption that the impinging signals are uncorrelated, ensuring a full-rank covariance matrix. However, when the signals are fully or partially correlated, such as in scenarios involving multipath propagation, the covariance matrix becomes rank-deficient, making direct application of subspace-based DOA estimation methods infeasible. In such cases, a decorrelation strategy is necessary to restore the rank of the covariance matrix and enable effective DOA estimation.

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Several techniques have been developed to decorrelate the rank-deficient covariance matrix. The spatial smoothing technique [13]–[15] partitions the entire array into several overlapping subarrays, and the average of the subarray covariance matrices restores the rank, enabling subspace-based DOA estimation despite of signal coherence. However, this method reduces the number of degrees of freedom (DOFs) compared to the conventional approach. Particularly, when an N-antenna uniform linear array (ULA) is used, the maximum number of DOFs becomes $\lfloor \frac{N}{2} \rfloor$, where $\lfloor \cdot \rfloor$ denotes the floor function. To increase the number of DOFs, the forward-backward spatial smoothing (FBSS) technique was introduced in [16], which utilizes subarray covariance matrices along with their flipped and conjugated versions, termed as backward matrices, for averaging. This approach increases the number of DOFs to $\lfloor \frac{2}{3} N \rfloor$. However, this method is computationally more expensive than standard spatial smoothing, as FBSS allows for larger subarray sizes, requiring additional computation for matrix averaging.

A computationally efficient approach was developed in [17], which enables the detection of coherent signals without requiring spatial smoothing. This method constructs a Toeplitz matrix from a single row of the covariance matrix, effectively restoring its rank and making it suitable for DOA estimation of coherent signals. For an odd number of antennas, N, this method can detect up to $\frac{N-1}{2}$ signals. To further increase the number of DOFs using the arrangement approach, [18] introduced an approach that incorporates both the forward and backward vectors, achieving a DOF count similar to the FBSS approach. In [18], two methods were developed for constructing the forward and backward vectors: the eigenvector method (EVM) and the correlation vector method (CVM). The forward-backward covariance matrix, derived from the signal subspace using one of these methods, is then employed for DOA estimation via the ESPRIT algorithm. However, this approach requires prior knowledge of the number of sources to construct the decorrelated covariance matrix, which may not be available in practical scenarios. Additionally, the resulting forward-backward covariance matrix is non-square, limiting its compatibility with certain key DOA estimation algorithms, such as MUSIC.

In [19], a different approach was introduced, wherein a single row and a single column are used to construct a Toeplitz matrix. Multiple Toeplitz matrices, generated from different rows and columns, are averaged to obtain a decorrelated covariance matrix capable of detecting a mixture of uncorrelated and coherent signals. This method preserves the dimension of the decorrelated covariance matrix as the original one and can detect a larger number of mixed signals than $\lfloor \frac{2}{3}N \rfloor$ offered by the forward-backward approach. However, its performance degrades as the number of coherent signals increases.

In this paper, we propose a computationally efficient algorithm for the DOA estimation of coherent sources, offering a similar number of DOFs as FBSS but without the need for spatial smoothing. The proposed method structurally constructs a decorrelated matrix utilizing a single row of the original covariance matrix, along with its flipped and conjugated version. Unlike [18], this approach does not require prior knowledge of the number of sources to obtain the decorrelated covariance matrix. Additionally, it leverages both the signal and noise subspaces, enhancing the robustness of DOA estimation performance [20]. As the resulting matrix is square, it retains full-rank properties, and can be seamlessly integrated with existing DOA estimation algorithm to handle coherent signals.

Notations: We use bold lower-case (upper-case) letters to describe vectors (matrices). Specifically, $\boldsymbol{I}_L$ represents the identity matrix of size $L \times L$. $(\cdot)^{\mathrm{T}}$, $(\cdot)^*$ and $(\cdot)^{\mathrm{H}}$ respectively indicate the transpose, conjugate, and conjugate transpose (Hermitian) of a matrix or a vector. The symbol $j = \sqrt{-1}$ represents the unit imaginary number and $\mathbb{E}(\cdot)$ denotes the statistical expectation.

II. Signal Model

Consider K far-field narrowband signals, $\boldsymbol{s}(t) = [s_1(t), s_2(t), \cdots, s_K(t)]^{\mathrm{T}} \in \mathbb{C}^K$, impinging on a ULA consisting of N omnidirectional sensors with DOAs $\boldsymbol{\theta} = [\theta_1, \theta_2, \cdots, \theta_K]^{\mathrm{T}}$. We assume that the signals exhibit mutual coherence with each other, and denote signal waveform $s_1(t)$ as the reference. The other waveforms are its scaled versions by complex scalars α_k, i.e.,

$$s_k(t) = \alpha_k s_1(t), \tag{1}$$

for $k = 1, \cdots, K$ with $\alpha_1 = 1$. Accordingly, the array received signal vector at time t can be expressed as

$$\begin{aligned} \boldsymbol{x}(t) &= s_1(t) \sum_{k=1}^{K} \alpha_k \boldsymbol{a}(\theta_k) + \boldsymbol{n}(t) \\ &= \boldsymbol{A}\boldsymbol{s}(t) + \boldsymbol{n}(t), \end{aligned} \tag{2}$$

where

$$\boldsymbol{a}(\theta_k) = [1, e^{-j\frac{2\pi}{\lambda} d \sin\theta_k}, \cdots, e^{-j\frac{2\pi}{\lambda}(N-1)d\sin\theta_K}]^{\mathrm{T}} \in \mathbb{C}^N \tag{3}$$

denotes the steering vector corresponding to DOA θ_k, $\boldsymbol{A} = [\boldsymbol{a}(\theta_1), \boldsymbol{a}(\theta_2), \cdots, (a(\theta_K)] \in \mathbb{C}^{N \times K}$ is the array manifold matrix, and $\boldsymbol{n}(t) \sim \mathcal{CN}(\boldsymbol{0}, \sigma_n^2 \boldsymbol{I}_N)$ is the additive white Gaussian noise vector. Then, the covariance matrix of the array received signal is obtained as

$$\boldsymbol{R} = \mathbb{E}\left[\boldsymbol{x}(t)\boldsymbol{x}^{\mathrm{H}}(t)\right] = \boldsymbol{A}\boldsymbol{R}_s\boldsymbol{A}^{\mathrm{H}} + \sigma_n^2 \boldsymbol{I}_N, \tag{4}$$

where $\boldsymbol{R}_s$ is the source covariance matrix and is expressed as

$$\boldsymbol{R}_s = \sigma_s^2 \boldsymbol{\alpha}\boldsymbol{\alpha}^{\mathrm{H}}, \tag{5}$$

σ_s^2 is the signal power of the reference waveform $s_1(t)$, and $\boldsymbol{\alpha} = [\alpha_1, \alpha_2, \cdots, \alpha_K]^{\mathrm{T}}$ is a complex scaling vector for the coherent signals. The covariance matrix $\boldsymbol{R}$ can be expanded as

$$\boldsymbol{R} = \sigma_s^2 \sum_{k=1}^{K} \sum_{k'=1}^{K} \alpha_k^* \alpha_{k'} \boldsymbol{a}(\theta_{k'}) \boldsymbol{a}^{\mathrm{H}}(\theta_k) + \sigma_n^2 \boldsymbol{I}_N. \tag{6}$$

It is observed in Eq. (6) that the covariance matrix contains cross-correlations among the coherent sources, leading to rank deficiency. The (m, n)th element of the covariance matrix $\boldsymbol{R}$ can be expressed as

$$\begin{aligned} R(m,n) &= \sigma_s^2 \sum_{k'=1}^{K} \alpha_{k'} e^{-j\frac{2\pi}{\lambda} m d \sin\theta_{k'}} \sum_{k=1}^{K} \alpha_k^* e^{j\frac{2\pi}{\lambda} n d \sin\theta_k} \\ &\quad + \sigma_n^2 \delta_{m,n} \\ &= b_m \sum_{k=1}^{K} \alpha_k^* e^{j\frac{2\pi}{\lambda} n d \sin\theta_k} + \sigma_n^2 \delta_{m,n}, \end{aligned} \tag{7}$$

where $b_m = \sum_{k'=1}^{K} \alpha_{k'} e^{-j\frac{2\pi}{\lambda} m d \sin\theta_{k'}}$ depends on the row index m of the covariance matrix and $\delta_{m,n}$ is the Kronecker delta function.

III. Decorrelation of the Covariance Matrix

In this section, we present a decorrelation strategy for the rank-deficient covariance matrix of coherent signals. To achieve this, we use one row (say, the mth row) of the covariance matrix and obtain the forward and backward vectors, respectively expressed as

$$\begin{aligned} \boldsymbol{r}_f &= \boldsymbol{R}(m,:), \\ \boldsymbol{r}_b &= \boldsymbol{J}\boldsymbol{r}_f^*, \end{aligned} \tag{8}$$

where $\boldsymbol{J}$ is the exchange matrix, which has ones along its anti-diagonal elements and zeros elsewhere.

We extend the Toeplitz arrangement-based method developed in [17], which originally utilizes only the forward vectors, by incorporating both forward and backward vectors. The arrangement proposed in [17] is depicted as

$$\boldsymbol{D}_f = \begin{bmatrix} r_f(0) & r_f(1) & \cdots & r_f(\lfloor \frac{N}{2} \rfloor) \\ r_f(-1) & r_f(0) & \cdots & r_f(\lfloor \frac{N}{2} \rfloor - 1) \\ \cdots & \cdots & \ddots & \cdots \\ r_f(-\lfloor \frac{N}{2} \rfloor) & r_f(-\lfloor \frac{N}{2} \rfloor + 1) & \cdots & r_f(0) \end{bmatrix}. \tag{9}$$

In this arrangement, by indexing the elements of $\boldsymbol{r}_f$ from $-\lfloor \frac{N}{2} \rfloor$ to $\lfloor \frac{N}{2} \rfloor$, the first row of $\boldsymbol{D}_f$ comprises elements between $r_f(0)$ and $r_f(\lfloor \frac{N}{2} \rfloor)$. The second row is shifted one index to the left, ranging between $r_f(-1)$ and $r_f(\lfloor \frac{N}{2} \rfloor - 1)$. This pattern is repeated for the subsequent rows, resulting in the decorrelated matrix $\boldsymbol{D}_f$ having a dimension of $(\lfloor \frac{N}{2} \rfloor + 1) \times (\lfloor \frac{N}{2} \rfloor + 1)$.

Unlike in [17], where only the forward vector is utilized, the proposed method incorporates both forward and backward vectors to construct the decorrelated covariance matrix. This enables an increase in the dimension of the decorrelated matrix, thereby enhancing the number of DOFs. To enhance the dimension of the decorrelated matrix and hence the number of DOFs, the first row of the forward-backward decorrelated matrix starts p elements earlier. In so doing, although the number of rows constructed by the arrangement will be smaller than the number of columns, the backward vector can be utilized to fill up the remaining rows to make the decorrelated matrix in square shape. The forward-backward decorrelated matrix is constructed as

$$\boldsymbol{D}_{fb} = \begin{bmatrix} r_f(-p) & r_f(-p+1) & \cdots & r_f(\lfloor \frac{N}{2} \rfloor) \\ r_f(-p-1) & r_f(-p) & \cdots & r_f(\lfloor \frac{N}{2} \rfloor - 1) \\ \cdots & \cdots & \ddots & \cdots \\ r_f(-\lfloor \frac{N}{2} \rfloor) & r_f(-\lfloor \frac{N}{2} \rfloor + 1) & \cdots & r_f(p) \\ r_b(-\lfloor \frac{N}{2} \rfloor + 2p - 1) & r_b(-\lfloor \frac{N}{2} \rfloor + 2p) & \cdots & r_b(3p-1) \\ \cdots & \cdots & \ddots & \cdots \\ r_b(-\lfloor \frac{N}{2} \rfloor) & r_b(-\lfloor \frac{N}{2} \rfloor + 1) & \cdots & r_b(p) \end{bmatrix}. \tag{10}$$

The dimension of the forward-backward decorrelated matrix $\boldsymbol{D}_{fb}$ is $(\lfloor \frac{N}{2} \rfloor + 1 + p) \times (\lfloor \frac{N}{2} \rfloor + 1 + p)$. The (m', n')th element of $\boldsymbol{D}_{fb}$ can be obtained from $\boldsymbol{R}$ as

$$\boldsymbol{D}_{fb}(m', n') = \begin{cases} r(-m' + n' - p), & m' \leq \lfloor \frac{N}{2} \rfloor - p, \\ r_b(-m' + n' + p), & m' > \lfloor \frac{N}{2} \rfloor - p. \end{cases} \tag{11}$$

From Eqs. (11) and (7), element $D_{fb}(m', n')$ can be expanded as

$$D_{fb}(m', n') = b_m^{(fb)} \sum_{k=1}^{K} \gamma_k^{(fb)} e^{-j\pi(m'-n')\sin\theta_k} + \sigma_n^2 \delta_{\tilde{m}, m}, \tag{12}$$

where

$$b_m^{(fb)} = \begin{cases} b_m, & m' \leq \lfloor \frac{N}{2} \rfloor - p, \\ b_m^*, & m' > \lfloor \frac{N}{2} \rfloor - p, \end{cases} \tag{13}$$

$$\gamma_k^{(fb)} = \begin{cases} \gamma_k, & m' \leq \lfloor \frac{N}{2} \rfloor - p, \\ \gamma_k^*, & m' > \lfloor \frac{N}{2} \rfloor - p, \end{cases} \tag{14}$$

$$\tilde{m} = \begin{cases} -m' + n' - p, & m' \leq \lfloor \frac{N}{2} \rfloor - p, \\ -n' + m' - p, & m' > \lfloor \frac{N}{2} \rfloor - p, \end{cases} \tag{15}$$

and $\gamma_k = \alpha^* e^{-j\frac{2\pi}{\lambda} p d \sin\theta_k}$.

As a result, the decorrelated covariance matrix $\boldsymbol{D}_{fb}$ is formulated as

$$\boldsymbol{D}_{fb} = b_m^{(fb)} \sum_{k=1}^{K} \gamma^{(fb)} \boldsymbol{g}(\theta_k) \boldsymbol{g}^{\mathrm{H}}(\theta_k) + \boldsymbol{N}_{fb}, \tag{16}$$

where $\boldsymbol{g}(\theta_k) = [1, e^{-j\frac{2\pi}{\lambda} d \sin\theta_k}, \cdots, e^{-j\frac{2\pi}{\lambda}(\lfloor \frac{N}{2} \rfloor + p) d \sin\theta_K}]^{\mathrm{T}} \in \mathbb{C}^{\lfloor \frac{N}{2} \rfloor + p + 1}$ acts as the steering vector in the underlying forward-backward decorrelation scheme and $\boldsymbol{N}_{fb}$ is the noise term associated with the $\boldsymbol{D}_{fb}$. As evident from Eq. (16), the decorrelated matrix $\boldsymbol{D}_{fb}$ is a summation of K rank-one matrices, thus exhibiting a rank of K, i.e., the full rank is restored, provided that $\lfloor \frac{N}{2} \rfloor + p \geq K$, as detailed in the following section.

IV. Number of Degrees of Freedom Analysis

The decorrelated full-rank covariance matrix $\boldsymbol{D}_{fb} \in \mathbb{C}^{(\lfloor \frac{N}{2} \rfloor + p + 1) \times (\lfloor \frac{N}{2} \rfloor + p + 1)}$ can be utilized to estimate the DOAs of coherent signals using an subspace-based DOA estimation algorithm, such as MUSIC or ESPRIT. Given the dimension of the full-rank matrix $\boldsymbol{D}_{fb}$, the maximum number of DOFs is given by

$$\mathrm{DOF} = \left\lfloor \frac{N}{2} \right\rfloor + p. \tag{17}$$

From this expression, it is observed that an optimal value of p must be determined to maximize the number of DOFs.

First, let us consider the forward-only decorrelated matrix $\boldsymbol{D}_f$, which is constructed using the forward vector $\boldsymbol{r}_f$ as defined in Eq. (9). This amounts to $p = 0$ in (17). When N takes an odd number, $\boldsymbol{D}_f$ has $\frac{N+1}{2}$ rows and $\frac{N+1}{2}$ columns. As p increases, as shown in Eq. (10), the number of columns expands to $\frac{N+1}{2} + p$, while the number of rows derived from the forward vectors decreases to $\frac{N+1}{2} - p$. When the backward vectors $\boldsymbol{r}_b$ are used, the remaining rows are filled with the properly arranged elements of $\boldsymbol{r}_b$, ensuring that the resulting matrix is square with $\lfloor \frac{N+1}{2} \rfloor + p$ rows and $\lfloor \frac{N+1}{2} \rfloor + p$ columns.

To guarantee that $\boldsymbol{D}_{fb}$ retains a square matrix structure in this case, the following condition needs to be satisfied:

$$2\left(\left\lfloor \frac{N+1}{2} \right\rfloor - p\right) \geq \left\lfloor \frac{N+1}{2} \right\rfloor + p, \tag{18}$$

which implies

$$p \leq \left\lfloor \frac{N+1}{6} \right\rfloor. \tag{19}$$

Therefore, the optimal value of p for an odd number of N is

$$p = \left\lfloor \frac{N+1}{6} \right\rfloor. \tag{20}$$

When N takes an even number, it can be shown that, for $p = 1$, $\boldsymbol{D}_f$ has $\frac{N}{2} + 1$ rows while the number of columns is $\frac{N}{2}$. To guarantee that $\boldsymbol{D}_{fb}$ retains a square matrix structure in this case, $p = \lfloor \frac{N}{6} + \frac{2}{3} \rfloor$ becomes the optimum choice. As a result, in general, we have the following proposition regarding the number of DOFs.

Proposition: The number of DOFs is given as

$$\mathrm{DOF} = \begin{cases} \left\lfloor \frac{2}{3} N \right\rfloor - 1, & N = 6v \text{ or } 6v \pm 3, \\ \left\lfloor \frac{2}{3} N \right\rfloor, & \text{otherwise}, \end{cases} \tag{21}$$

where $v \geq 1$ is a positive integer.

Proof. Consider an odd integer $N \geq 3$ and express it as $N = 2u + 1$, where $u \geq 1$ is an integer. From Eq. (17), the number of DOFs is

$$\mathrm{DOF} = \frac{N+1}{2} + \left\lfloor \frac{N+1}{6} \right\rfloor - 1 = (u+1) + \left\lfloor \frac{u+1}{3} \right\rfloor - 1. \tag{22}$$

We consider three cases based on the divisibility of 3.

1) $u+1 \equiv 0 \pmod 3$: In this, we can write $u+1 = 3v$, where $v \geq 1$ is an integer. In this case, the number of DOFs form Eq. (22) becomes

$$\text{DOF} = 3v + v - 1 = 4v - 1, \tag{23}$$

which is the same as $\lfloor \frac{2}{3}N \rfloor$ since

$$\begin{aligned} \left\lfloor \frac{2}{3}N \right\rfloor &= \left\lfloor \frac{2}{3}(2u+1) \right\rfloor = \left\lfloor \frac{4u+2}{3} \right\rfloor \\ &= \left\lfloor 4v - \frac{2}{3} \right\rfloor = 4v - 1. \end{aligned} \tag{24}$$

This corresponds to $N = 2u+1 = 6v-1$.

2) $u+1 \equiv 1 \pmod 3$: In this, we can write $u+1 = 3v+1$. In this case, the number of DOFs form Eq. (22) becomes

$$\text{DOF} = 3v + 1 + v - 1 = 4v, \tag{25}$$

which is the same as $\lfloor \frac{2}{3}N \rfloor$ since

$$\left\lfloor \frac{2}{3}N \right\rfloor = \left\lfloor \frac{4u+2}{3} \right\rfloor = \left\lfloor 4v + \frac{2}{3} \right\rfloor = 4v. \tag{26}$$

This corresponds to $N = 2u+1 = 6v+1$.

3) $u+1 \equiv 2 \pmod 3$: In this, we can write $u+1 = 3v+2$. In this case, the number of DOFs form Eq. (22) becomes

$$\text{DOF} = 3v + 2 + v - 1 = 4v + 1, \tag{27}$$

which is the same as $\lfloor \frac{2}{3}N \rfloor - 1$ since

$$\left\lfloor \frac{2}{3}N \right\rfloor = \left\lfloor \frac{4u+2}{3} \right\rfloor = \lfloor 4v + 2 \rfloor = 4v + 2. \tag{28}$$

This corresponds to $N = 2u+1 = 6v+3$.

In a similar way, when N is even, it can be proven that the number of DOFs is $\lfloor \frac{2}{3}N \rfloor$ for $N = 6v+2$ or $6v+4$, which equals $\lfloor \frac{2}{3}N - 1 \rfloor = \frac{2}{3}N - 1$ when $N = 6v$. □

V. Computations Complexity Analysis

Assuming the availability of the coherent covariance matrix $\boldsymbol{R}$, the computation of the proposed forward-backward decorrelated covariance matrix involves the following steps:

1) Construction of forward and backward vectors $\boldsymbol{r}_f$ and $\boldsymbol{r}_b$: The forward vector $\boldsymbol{r}_f$ is obtained by selecting a row from $\boldsymbol{R}$, while the backward vector $\boldsymbol{r}_b$ is constructed by flipping and conjugating the elements of $\boldsymbol{r}_f$, without requiring any arithmetic operations.
2) Construction of $\boldsymbol{D}_{fb}$: The matrix $\boldsymbol{D}_{fb}$ is formed by arranging the elements of $\boldsymbol{r}_f$ and $\boldsymbol{r}_b$, without requiring additional arithmetic operations.

In comparison, FBSS requires the following steps:

1) To perform FBSS, L subarray covariance matrices of size $M \times M$ is formed, where $M < N$. The $\frac{L}{2}$ forward subarray covariance matrices can be obtained by slicing the original covariance matrix $\boldsymbol{R}$, while the backward subarray matrices are constructed by flipping and conjugating $\frac{L}{2}$ matrices, without any additional computational cost.

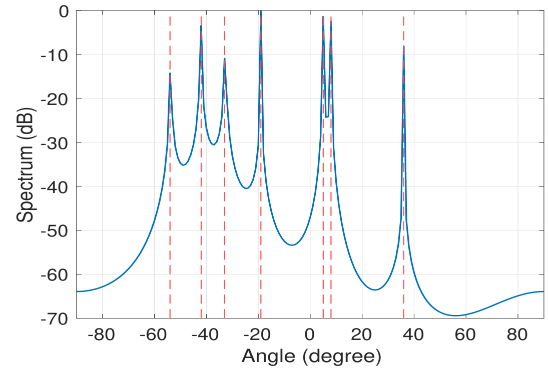


Fig. 1: Estimated MUSIC spectrum for 7 coherent signals.

2) The averaging operation for L forward and backward matrices incurs a computational complexity of $\mathcal{O}(LM^2)$.

As a result, the total computational complexity for FBSS is $\mathcal{O}(LM^2)$. Unlike the proposed approach, where the computational cost remains fixed, the values of L and M in FBSS depend on the number of sources to be detected, contributing to the overall complexity. Considering the maximum number of sources to be determined, i.e., $K = \frac{2}{3}N$, M should be at least $K+1$, and L should be at least $\frac{K}{2}$. Thus, the overall computational complexity is $\mathcal{O}(\frac{4}{27}N^3)$. Additionally, the CVM method described in [18], which is based on the arrangement of correlation elements, has a computational complexity similar to the proposed approach, while the EVM method, which requires an eigen-decomposition with a computational complexity of $\mathcal{O}(N^3)$, which is dominated by matrix multiplication.

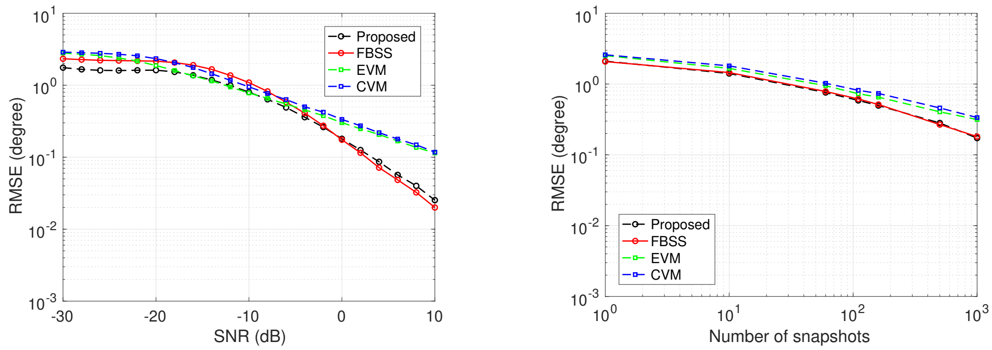
VI. Simulation Results

In this section, we present simulation results of the proposed method and compare its performance with FBSS [16], EVM, and CVM [17]. We consider a ULA consisting of 11 antennas. First, we examine the case where the number of coherent sources equals the number of DOFs, i.e., $K = \lfloor \frac{2}{3}N \rfloor = 7$. Fig. 1 considers 7 coherent signals with DOAs of $36°$, $-42°$, $8°$, $54°$, $5°$, $-33°$, and $-19°$. The input signal-to-noise ratio (SNR) of the reference signal is set to 20 dB, and $1,000$ snapshots are used. The magnitude of the scaling factor is chosen from a uniform distribution within the range of 0.5 to 2, while the phase is chosen between $30°$ and $60°$ Fig. 1 demonstrates that the proposed approach successfully detect all 7 sources, which represent the maximum number of DOFs achievable using an 11-element ULA.

Fig. 2 compares the performance of the considered approaches in terms of root mean-squared error (RMSE). The RMSE is defined as

$$\text{RMSE} = \sqrt{QK \sum_{q=1}^{Q} \sum_{k=1}^{K} (\theta_k - \hat{\theta}_{q,k})}, \tag{29}$$

where Q is the number of Monte Carlo trials, and θ_k and $\hat{\theta}_{q,k}$ represent the true and estimated DOAs for the qth trial



(a) RMSE versus input SNR

(b) RMSE versus number of snapshots

Fig. 2: RMSE performance comparison.

of the kth source, respectively. We consider 5 signals with DOAs $-25°$, $-14°$, $4°$, $-10°$, $16°$ for the RMSE computation. $5,000$ Monte-Carlo trials are performed. Fig. 2(a) illustrates the RMSE values against the input SNR, where the input SNR varies between -30 dB and 10 dB, with the number of snapshots fixed at 1,000. It is observed that the proposed approach and FBSS achieve lower RMSE values compared to EVM and CVM. Specially, at lower SNR levels, the proposed approach achieves the lowest RMSE among all methods. It is noted that the proposed method incurs a significantly lower computational complexity compared to the FBSS and EVM.

Fig. 2(b) depicts the RMSE values against the number of snapshots used. In this case, the number of snapshots varies from 10 to 1,000, while the input SNR is fixed at 0 dB. It is observed that the proposed approach and the FBSS outperform the other methods in this case as well.

VII. Conclusion

In this paper, we proposed an efficient approach to decorrelate the rank-deficient covariance matrix of an array data vector corresponding to coherent impinging signals. The method utilizes a single row of the coherent covariance matrix along with its flipped and conjugated version to structurally arrange and restore the rank of the covariance matrix. The proposed method is significantly more computationally efficient than FBSS and EVM-based approaches while providing robust performance. Moreover, unlike CVM and EVM, this method preserves the square matrix structure of the decorrelated covariance matrix, offering greater flexibility for different estimation algorithms.

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