Sparsity-Based Robust Beamforming in the Presence of Fast-Moving Interference Signals

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***Abstract*—In this paper, we present a robust time-varying adaptive beamformer that employs a novel strategy for the suppression of fast-moving interference sources. By exploiting the sparsity of the interference sources, the parameters, which include the direction-of-arrival and power of the desired and interference signals as well as the noise power, are estimated, and then the interference-plus-noise covariance matrix is reconstructed. The proposed time-varying adaptive beamformer is compared with the state-of-the-art beamformers for mobile interference sources that employ a wide-null approach. It is demonstrated that the proposed beamformer achieves near-optimal performance and outperforms the wide-null beamformers in terms of the output signal-to-interference-plus-noise ratio performance.**

Keywords: Direction-of-arrival estimation, interference-plus-noise covariance matrix, robust beamforming, low earth orbit (LEO) satellite, moving interference signal.

I. Introduction

In array signal processing, adaptive beamforming is one of the most widely used technologies that finds application in radar, sonar, wireless communications, radio astronomy, and medical imaging [1–3]. Adaptive beamformers can maximize the array gain along the direction of the desired signal while simultaneously suppressing the interference at the array output [4, 5]. The well-known minimum variance distortionless response (MVDR) beamformer requires that the steering vector of the desired signal and the interference-plus-noise covariance matrix (INCM) are available or can be accurately estimated. A number of algorithms have been proposed to improve robustness against signal steering vector mismatch and inaccuracy in INCM reconstruction [6–9].

Adaptive beamforming algorithms can be broadly classified into two major categories, namely, those based on the data covariance matrix without reconstructing the INCM (referred to as the non-reconstruction methods) and those based on the reconstruction of the INCM (referred to as the reconstruction methods). Examples of non-reconstruction methods include the minimum power distorionless response (MPDR) beamformer and those based on diagonal loading of the data covariance matrix and steering vector estimation [10, 11]. Since the data sample covariance matrix contains contribution from the desired signal, such beamformers are prone to self-nulling of the desired signal and cannot achieve near-optimal performance in terms of the signal-to-interference-plus-noise ratio (SINR) at the output of the beamformer. To avoid such sub-optimal performance, INCM reconstruction and desired signal steering vector estimation are two most important aspects. In [6, 9], the INCM is estimated from the steering vector and power of the interference signals, and an improved estimate of the desired signal steering vector is estimated. In [7, 8], the INCM is reconstructed using compressive sensing techniques.

When considering fast-moving interferers, the direction-of-arrival (DOA) of the interference signal varies rapidly with time, thus causing wide angular occupation of the interference signal over a coherent processing interval. In order to provide effective interference cancellation in this case, [15] proposed a null-broadening approach over the angular region that the moving interferers occupy by applying a transformation on the estimation of the INCM along the time period over which the motion is observed. In [16], the steering vector correlation matrix is constructed in the pre-defined angular sectors of the interference direction. This concept is recently applied in the minimum dispersion distortionless response (MDDR) beamformer to provide a null sector over a predefined range of DOA [17].

All these existing methods are underlying the concept that, as the interference moves, a broad angular sector is nullified. As such, a high number of array degrees-of-freedom (DOFs) is consumed for the cancellation of such spatially *widened* interference signals. This may be the only choice when an interference source moves randomly and cannot be tracked or predicted. However, when the trajectory of the interferers can be tracked or predicted, such information of the trajectory can be utilized to form time-varying nulling, thereby avoiding the unnecessary consumption of a high number of DOFs due to the motion of interferers.

Two interesting examples of such interference with predictable trajectory is automotive radar and satellites, particularly the low Earth orbit (LEO) satellites. Automotive radar has emerged as a key enabling technology for next-generation autonomous driving systems [18, 19]. As a result, a rapidly increasing number of automotive radars will be deployed and interference between different radar units is expected to be a serious problem [20]. Interference signals from radar units mounted on nearby vehicles are fast moving and their trajectory can, in general, be tracked. On the other hand, by utilizing hundreds to tens of thousands of satellites in the LEO orbits, mega satellite constellations deliver low-latency broadband data services anywhere on the planet and are considered an important means for providing 5G/6G Internet [21, 22]. For interference from LEO satellites, the trajectory and, thereby, the direction of the interference signals over time can be precisely characterized. In both applications, by taking the advantage and noticing the fact that each interference source is, at any time, impinging from a single direction, we can construct a time-varying adaptive beamformer which requires only a single DOF for each interference to be effectively cancelled.

In this paper, we propose a strategy for nullifying moving interference sources by deploying a time-varying beamformer that adjusts its weight vector as a function of time depending on the motion of the interference signals and creates deep and narrow nulls at each instance of time. In so doing, the data received over a longer coherent processing time can be utilized to provide an accurate estimate of the INCM estimate for effective interference cancellation, and the proposed approach does not consume unnecessary DOFs because of the widened nulls.

The rest of the paper is organized as follows. The system model of adaptive beamforming is described in Section II. In Section III, we present the sparsity-based parameter estimation to perform INCM reconstruction and implement robust beamforming for moving interference cancellation. Simulation results are provided in Section IV to demonstrate the effectiveness of the proposed method. Finally, conclusions are drawn in Section V.

Notations: We use lower-case and upper-case bold characters to denote vectors and matrices, respectively. In particular, $\mathbf{I}_N$ denotes the $N \times N$ identity matrix. $(.)^{\mathrm{T}}$ and $(.)^{\mathrm{H}}$ respectively represent the transpose and conjugate transpose (Hermitian) of a matrix or a vector. In addition, $||\cdot||_{\mathrm{F}}$ and $||\cdot||_1$ denote the Frobenius norm and ℓ_1 norm, respectively. $\mathbb{R}_+$ denotes the set of non-negative real numbers, and $\mathbb{C}^{M\times N}$ denotes the $M \times N$ complex space. $\mathbb{E}(\cdot)$ denotes statistical expectation.

II. System Model

Consider an arbitrary array consisting of N omni-directional sensors that receive one desired signal and L interference signals. All the signals are assumed to be stationary in this section, and we extend to moving interference signals in Section III. For simplicity of presentation, we consider a linear array dealing with narrowband signals in the azimuth directions, but generalization to a wideband signal model in both azimuth and elevation angles is straightforward. All signals are assumed to be located in the far field.

Denote the DOA and waveform of the desired signal as θ_0 and $s_0(t)$, and the waveforms of the uncorrelated interference signals are $s_1(t), \cdots, s_L(t)$ which impinge from distinct angles $\theta_1, \cdots, \theta_L$. The baseband signal vector $\mathbf{x}(t)$ received at the array is expressed as:

$$\mathbf{x}(t) = \mathbf{x}_{\mathrm{s}}(t) + \mathbf{x}_{\mathrm{i}}(t) + \mathbf{n}(t), \tag{1}$$

where

$$\begin{aligned} \mathbf{x}_{\mathrm{s}}(t) &= \mathbf{a}(\theta_0)s_0(t), \\ \mathbf{x}_{\mathrm{i}}(t) &= \sum_{l=1}^{L} \mathbf{a}(\theta_l)s_l(t), \end{aligned} \tag{2}$$

$\mathbf{a}(\theta_l)$ denotes the steering vector of the array corresponding to the signal impinging from angle θ_l, and $\mathbf{n}(t)$ denotes the additive circularly complex white Gaussian noise vector observed at the array. We can rewrite $\mathbf{x}(t)$ in a compact form as

$$\mathbf{x}(t) = \sum_{l=0}^{L} \mathbf{a}(\theta_l)s_l(t) + \mathbf{n}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t), \tag{3}$$

where $\mathbf{s}(t) = [s_0(t), s_1(t), \cdots, s_L(t)]^{\mathrm{T}}$ and $\mathbf{A} = [\mathbf{a}(\theta_0), \mathbf{a}(\theta_1), \ldots, \mathbf{a}(\theta_L)]$ is the array manifold matrix.

Where an adaptive weight vector $\mathbf{w} \in \mathbb{C}^{N\times 1}$ is used in the beamformer, the output of the beamformer is given as

$$\mathbf{y}(t) = \mathbf{w}^{\mathrm{H}}\mathbf{x}(t). \tag{4}$$

The weight vector of an MVDR beamformer is obtained by solving the following minimization problem:

$$\begin{aligned} &\underset{\mathbf{w}}{\text{minimize}} \quad \mathbf{w}^{\mathrm{H}}\mathbf{R}_{\mathrm{i+n}}\mathbf{w} \\ &\text{subject to} \quad \mathbf{w}^{\mathrm{H}}\mathbf{a}(\theta_0) = 1, \end{aligned} \tag{5}$$

where

$$\begin{aligned} \mathbf{R}_{\mathrm{i+n}} &= \mathbb{E}[(\mathbf{x}_{\mathrm{i}}(t) + \mathbf{n}(t))(\mathbf{x}_{\mathrm{i}}(t) + \mathbf{n}(t))^{\mathrm{H}}] \\ &= \sum_{l=1}^{L} \sigma_l^2 \mathbf{a}(\theta_l)\mathbf{a}^{\mathrm{H}}(\theta_l) + \sigma_{\mathrm{n}}^2\mathbf{I}_N \end{aligned} \tag{6}$$

denotes the INCM, and σ_{n}^2 denotes the noise power at each array element. The closed-form solution for the weight vector of problem (5) is given as

$$\mathbf{w}_{\mathrm{MVDR}} = \frac{\mathbf{R}_{\mathrm{i+n}}^{-1}\mathbf{a}(\theta_0)}{\mathbf{a}^{\mathrm{H}}(\theta_0)\mathbf{R}_{\mathrm{i+n}}^{-1}\mathbf{a}(\theta_0)}. \tag{7}$$

To avoid the estimation of the INCM, a common practice is to replace the INCM $\mathbf{R}_{\mathrm{i+n}}$ by the covariance matrix of the received data vector $\mathbf{x}(t)$, given as

$$\begin{aligned} \mathbf{R} &= \mathbb{E}[\mathbf{x}(t)\mathbf{x}^{\mathrm{H}}(t)] = \mathbf{A}\mathbf{S}\mathbf{A}^{\mathrm{H}} + \sigma_{\mathrm{n}}^2\mathbf{I}_N \\ &= \sum_{l=0}^{L} \sigma_l^2 \mathbf{a}(\theta_l)\mathbf{a}^{\mathrm{H}}(\theta_l) + \sigma_{\mathrm{n}}^2\mathbf{I}_N, \end{aligned} \tag{8}$$

where $\mathbf{S} = \mathbb{E}[\mathbf{s}(t)\mathbf{s}^{\mathrm{H}}(t)] = \mathrm{diag}([\sigma_0^2, \sigma_1^2, \sigma_2^2, \cdots, \sigma_L^2])$ is the source covariance matrix with σ_l^2 denoting the power of the lth source, $l = 0, 1, \ldots, L$. In practice, the data covariance matrix can be estimated using the K sampled data available at the array, i.e.,

$$\hat{\mathbf{R}} = \frac{1}{K}\sum_{t=1}^{K} \mathbf{x}(t)\mathbf{x}^{\mathrm{H}}(t). \tag{9}$$

When the data covariance matrix $\mathbf{R}$ is used in lieu of the INCM $\mathbf{R}_{\mathrm{i+n}}$, the resulting beamformer is called the MPDR beamformer, expressed as

$$\mathbf{w}_{\mathrm{MPDR}} = \frac{\mathbf{R}^{-1}\mathbf{a}(\theta_0)}{\mathbf{a}^{\mathrm{H}}(\theta_0)\mathbf{R}^{-1}\mathbf{a}(\theta_0)}, \tag{10}$$

III. Sparsity-Based INCM Reconstruction and Moving Interference suppression

In this section, we consider sparsity-based reconstruction of the INCM in the presence of moving interferers. It is achieved through the estimation of time-varying interference signal parameters, including their steering vectors and power, as well as the noise power. Once these parameters are estimated, the corresponding time-varying INCM can be constructed and be further processed to obtain a time-varying beamforming weight vector. Such beamformer is more effective when compared to broadened null patterns towards the moving interferers that treat the interferers as if they occupy a wide angular occupancy.

In Section III-A, we first describe the INCM reconstruction for stationary interferers [7, 8, 23, 24], and then extend the results to time-varying interferers in the Section III-B.

A. Sparsity-Based INCM Reconstruction for Stationary Interferers

Generally, signal arrivals are sparsely present in the space, and this property can be utilized to reconstruct the INCM [7, 8]. As discussed in Section I, the accuracy of the INCM reconstruction depends on the accuracy of the estimation of system parameters. We assume the system to have a single desired signal and L interference signals. Let M be the number of potential source locations forming predefined grids along the observed field. To estimate the parameters of both the desired signal and interferers, a sparsity-constrained covariance matrix fitting problem can be formulated as

$$\begin{aligned} \underset{\mathbf{p},\sigma_{\mathrm{n}}^2}{\text{minimize}} \quad & \left\|\hat{\mathbf{R}} - \mathbf{A}_g\mathbf{P}\mathbf{A}_g^{\mathrm{H}} - \sigma_n^2\mathbf{I}_N\right\|_{\mathrm{F}} \\ \text{subject to} \quad & \|\mathbf{p}\|_0 = L+1, \\ & \mathbf{p} \geq \mathbf{0}, \\ & \sigma_{\mathrm{n}}^2 \geq 0, \end{aligned} \tag{11}$$

where $\mathbf{p} \in \mathbb{R}_+^M$ denotes the sparse spatial spectrum distribution on the M sample grids along the observed field, $\mathbf{P} = \mathrm{diag}(\mathbf{p})$, and $\mathbf{A}_g \in \mathbb{C}^{N\times M}$ is the steering matrix of the N sensors along the M sample grids. The optimization problem in (11) finds the sparsest spatial spectrum distribution $\mathbf{p}$ and the noise power σ_{n}^2 such that the difference between the sample covariance matrix $\hat{\mathbf{R}}$ and the reconstructed covariance matrix $\mathbf{A}_g\mathbf{P}\mathbf{A}_g^{\mathrm{H}} + \sigma_{\mathrm{n}}^2\mathbf{I}_N$ is minimized. It can be observed that problem (11) is a non-convex problem due to the ℓ_0 norm in one of the constraints. Thus the problem can be reformulated into convex form through ℓ_1-norm relaxation as [25]

$$\begin{aligned} \underset{\mathbf{p},\sigma_{\mathrm{n}}^2}{\text{minimize}} \quad & \left\|\hat{\mathbf{R}} - \mathbf{A}_g\mathbf{P}\mathbf{A}_g^{\mathrm{H}} - \sigma_{\mathrm{n}}^2\mathbf{I}_N\right\|_{\mathrm{F}} + \gamma\|\mathbf{p}\|_1 \\ \text{subject to} \quad & \mathbf{p} \geq \mathbf{0}, \\ & \sigma_{\mathrm{n}}^2 \geq 0, \end{aligned} \tag{12}$$

where γ is the regularization parameter trading off between the sparsity of the spatial spectrum and the norm difference of covariance matrix fitting. This optimization problem can be solved using convex solvers, such as CVX [26]. Once the parameters are estimated, we can identify the array manifold corresponding to the interference signals as $\mathbf{A}_{\mathrm{i}} = [\mathbf{a}(\theta_1), \dots, \mathbf{a}(\theta_L)]$, and the interference signal powers as $\mathbf{S}_{\mathrm{i}} = \mathrm{diag}([\sigma_1^2, \sigma_2^2, \cdots, \sigma_L^2])$. Then, the INCM can be reconstructed as [8]

$$\hat{\mathbf{R}}_{\mathrm{i+n}} = \mathbf{A}_{\mathrm{i}}\mathbf{S}_{\mathrm{i}}\mathbf{A}_{\mathrm{i}}^{\mathrm{H}} + \sigma_{\mathrm{n}}^2\mathbf{I}_N. \tag{13}$$

B. Moving Interference Suppression

In this subsection, we consider the case where the interference signals have time-varying DOAs. We first describe the wide-null approach for the suppression of moving interference signals [27–29]. A time-varying beamformer is then considered whose weight vector is updated at every time instant and creates a sharp null at the exact locations of the interference.

Denote the angular variation of the lth interference signal as ω_l. For simplicity, ω_l is assumed to be constant over the processing time, but extension to a more general case with time-varying angular variation is straightforward. We also assume that the power of the interference signals and the noise power remain constant over the processing time period. Then, the instantaneous DOA of the interference signal over a time period of $0 \leq t \leq T$ is given as

$$\theta_l(t) = \theta_l(t=0) + \omega_l t, \quad 0 \leq t \leq T. \tag{14}$$

Thus the array manifold corresponding to the interference signals becomes time-varying and is denoted as $\mathbf{A}_{\mathrm{i}}(t)$. Hence, the corresponding INCM becomes

$$\hat{\mathbf{R}}_{\mathrm{i+n}} = \frac{1}{T}\int_0^T \mathbf{A}_{\mathrm{i}}(t)\mathbf{P}\mathbf{A}_{\mathrm{i}}^H(t)\,\mathrm{d}t + \sigma_{\mathrm{n}}^2\mathbf{I}_N. \tag{15}$$

Substituting (15) into eq. (7) yields the MVDR beamformer $\mathbf{w}_{\mathrm{MVDR}}$ which creates an array pattern with broadened nulls along the angular sector over which the interference signals traverse in processing time period. This approach requires a high number of DOFs than the number of interference signals.

On the other hand, if the time varying beamforming weight vector is formulated individually for every time instant, then the array pattern will create sharp nulls at each instant time and will update continuously over the time period T. And the time-varying INCM for the proposed strategy can be given as,

$$\hat{\mathbf{R}}_{\mathrm{i+n}}^{\mathrm{proposed}}(t) = \mathbf{A}(t)\mathbf{P}\mathbf{A}^H(t) + \sigma_{\mathrm{n}}^2\mathbf{I}_N. \tag{16}$$

It is seen that such a beamformer achieves better performance in terms of utilization of the DOFs and the output SINR.

IV. Simulation Results

We consider a uniform linear array (ULA) consisting of $N = 8$ omni-directional sensors. There are $L = 2$ interference signals and one desired signal impinging on the array from directions $[-12°, 0°, 10°]$. The second signal is considered to be the desired signal, whereas the other two signals are considered to be moving interference sources. The angular variation for the first interferer is considered to be $\omega_1 = 0.75$ degrees/second and that for the second interferer is $\omega_2 = 1.25$ degrees/second. For the simulation scenario we consider a time period of $T = 4$ seconds over which the interference sources are assumed to move with the defined angular variation ω. For both the scenarios, the number of snapshots available and the signal-to-noise ratio (SNR) are considered to be $K = 500$ and 5 dB respectively.

From Fig. 1 we see that the sparse reconstruction of the support for the DOAs of the signals has been estimated correctly. Furthermore for simulation the SNR of the desired signal, and the interference-to-noise-ration (INR) of the two interferers were set to 5 dB, 10 dB and 15 dB respectively which matches closely with the results obtained in Fig. 1. In addition, unit power noise was considered. The regularization parameter γ was chosen to be 0.5.

Fig. 2(a) depicts the beamforming scenario when the reconstructed noise plus interference covariance matrix is constructed over the total time frame using (15). We observe the broadened nulls at the directions of the two interference sources which are at $-12°$ and $10°$ respectively. It is to be noted that the width of the broad nulls depend on the angle sector that the two interference sources traverse within the considered time frame. Also for an array consisting of fewer number of antenna elements wide-null approach does not provide optimum performance as can be seen in Fig. 2(a).

In Fig. 2(b) we see that very sharp nulls are created along the time frame as each time frame has been considered separately for the construction of the INCM using (16). The nulls are sharp and deep due to the fact that the DOFs of the ULA are utilized more efficiently than the beam-null method. Also we observe that in such time adaptive beamforming the array gain is always maximum towards the desired signal at every instance of time.

Furthermore, Fig. 3 depicts the output SINR for input SNR ranging from -20 dB to 20 dB, and it was observed that the proposed method outperforms the state-of-the-art wide-null approach by a significant margin. The performance gain of antenna arrays with fewer elements is greater.

V. Conclusion

This paper proposes a new strategy for time-varying adaptive beamforming in moving interference scenario. The state of the art beamforming method for nullifying non-stationary mobile interference signal is to create a wide null along the angular sector corresponding to the motion of the interferer. It has been verified with simulation results that such a beamformer is not optimal in terms of output SINR and interference suppression. The proposed time adaptive beamforer is shown to be more robust and provides optimal performance compared to the wide null beamformer.

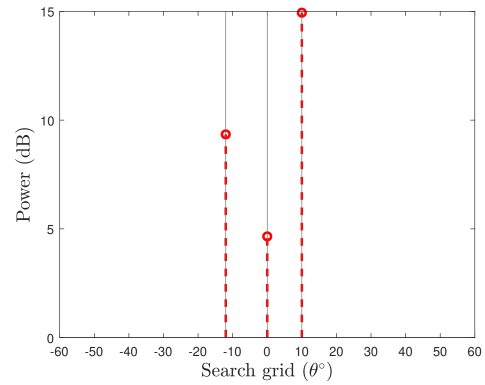
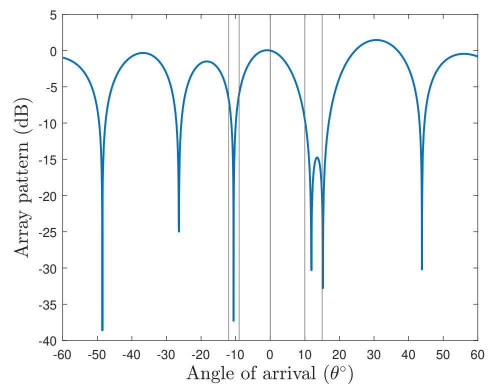
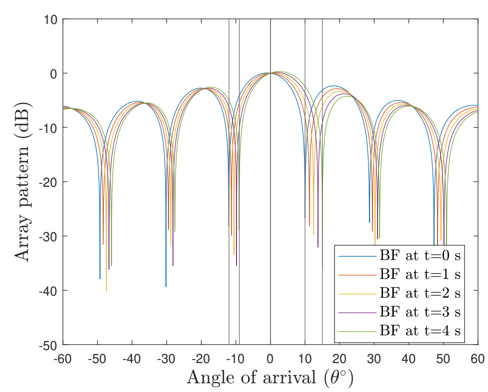


Fig. 1: Sparse spatial spectrum comparison.



(a) Beamforming with the INCM reconstructed over the time period.



(b) Beamforming with the INCM reconstructed at a time instant.

Fig. 2: Adaptive beamforming approaches to suppress the moving interference signals for LEO satellite scenario.

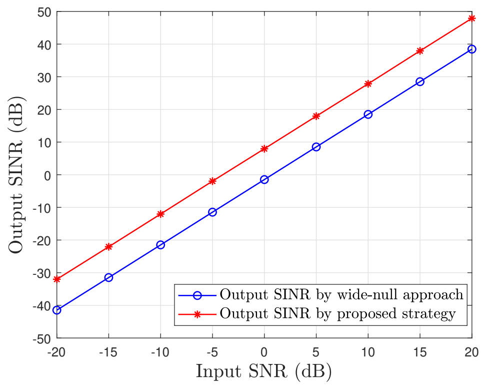


Fig. 3: Output SINR versus input SNR.

VI. References

[1] S. A. Vorobyov, “Principles of minimum variance robust adaptive beamforming design”, *Signal Process.*, vol. 93, no. 12, pp. 3264-3277, 2013.

[2] H. Cox, R. M. Zeskind, and M. H. Owen, “Robust adaptive beamforming,” *IEEE Trans. Acoust. Speech Signal Process.*, vol. 35, no. 10, pp. 1365-–1376, Oct. 1987.

[3] H. L. Van Trees, *Optimum Array Processing: Part IV of Detection, Estimation, and Modulation Theory.* Wiley, 2002.

[4] A. B. Gershman, N. D. Sidiropoulos, S. Shahbazpanahi, M. Bengtsson, and B. Ottersten, "Convex optimization-based beamforming," *IEEE Signal Process. Mag.*, vol. 27, no. 3, pp. 62–75, May 2010.

[5] J. Li and P. Stoica, *Robust Adaptive Beamforming*. Wiley, 2005.

[6] Y. Gu, and A. Leshem, “Robust adaptive beamforming based on interference covariance matrix reconstruction and steering vector estimation,” *IEEE Trans. Signal Process.*, vol. 60, no. 7, pp. 3881–3885, April 2012.

[7] K. Liu and Y. D. Zhang, “Coprime array-based robust beamforming using covariance matrix reconstruction technique," *IET Commun.*, vol. 12, no. 17, pp. 2206–2212, 2018.

[8] Y. Gu, N. A. Goodman, and Y. D. Zhang, “Adaptive beamforming via sparsity-based reconstruction of covariance matrix,” in A. De Maio, Y. C. Eldar, and A. Haimovich (eds.), *Compressed Sensing in Radar Signal Processing*, Cambridge Univ. Press, 2019.

[9] S. Zhang, Y. Gu, and Y. D. Zhang, “Robust astronomical imaging in the presence of radio frequency interference,” *J. Astronom. Instrument.*, vol. 8, no. 1, pp. 1–15, March 2019.

[10] J. Li, P. Stoica, and Z. Wang, “On robust Capon beamforming and diagonal loading,” *IEEE Trans. Signal Process.* vol. 51, no. 7, pp. 1702–1715, July 2003.

[11] A. Khabbazibasmenj, S. A. Vorobyov, and A. Hassanien, “Robust adaptive beamforming based on steering vector estimation with as little as possible prior information," *IEEE Trans. Signal Process.*, vol. 60, no. 6, pp. 2974–2987, June 2012.

[12] J. Zhuang and A. Manikas, “Interference cancellation beamforming robust to pointing errors,” *IET Signal Process.*, vol. 7, no. 2, pp. 120–127, 2013.

[13] F. Chen, F. Shen, and J. Song. “Robust adaptive beamforming using low-complexity correlation coefficient calculation algorithms,” *Electronics Lett.*, vol. 51, no. 6, pp. 443–445, 2015.

[14] F. Shen, F. Chen, and J. Song. “Robust adaptive beamforming based on steering vector estimation and covariance matrix reconstruction." *IEEE Commun. Lett.*, vol. 19, no. 9, pp 1636–1639, 2015.

[15] R. J. Mailloux, “Covariance matrix augmentation to produce adaptive array pattern troughs,” *Electronics Lett.*, vol. 31, no. 10, pp. 771–772, 1995.

[16] X. Mao, W. Li, Y. Li, Y. Sun, and Z. Zhai, “Robust adaptive beamforming against signal steering vector mismatch and jammer motion,” *Int. J. Antennas Propagat.*, vol. 2015, no. 780296, pp. 1–12, July 2015.

[17] L. Zhang, B. Li, L. Huang, T. Kirubarajan, and H. C. So, “Robust minimum dispersion distortionless response beamforming against fast-moving interferences,” *Signal Process.*, vol. 140, pp. 190–197, 2017.

[18] S. Sun, A. P. Petropulu, and H. V. Poor, “MIMO radar for advanced driver-assistance systems and autonomous driving: Advantages and challenges,” *IEEE Signal Process. Mag.*, vol. 37, no. 4, pp. 98—117, 2020.

[19] S. Sun and Y. D. Zhang, “4D automotive radar sensing for autonomous vehicles: A sparsity-oriented approach,” *IEEE Sel. Topics Signal Process.*, vol. 15, no. 4, pp. 879–891, June 2021.

[20] S. Alland, W. Stark, M. Ali, and M. Hegde, “Interference in automotive radar systems: Characteristics, mitigation techniques, and current and future research,” *IEEE Signal Process. Mag.*, vol. 36, no. 5, pp. 45–59, Sept. 2019.

[21] I. Leyva-Mayorga et al., “LEO Small-Satellite Constellations for 5G and Beyond-5G Communications,” *IEEE Access*, vol. 8, pp. 184955–184964, 2020.

[22] M. Neinavaie, J. Khalife, and Z. Z. Kassas, “Acquisition, Doppler tracking, and positioning with Starlink LEO satellites: First results,” *IEEE Trans. Aerosp. Electron. Syst.*, vol. 58, no. 2, pp. 1487–1491, April 2022.

[23] C. Zhou, Y. Gu, Y. D. Zhang, Z. Shi, T. Jin, and X. Wu, “Compressive sensing based coprime array direction-of-arrival estimation,” *IET Commun.*, vol. 11, no. 11, pp. 1719–1724, Aug. 2017.

[24] C. Zhou, Y. Gu, X. Fan, Z. Shi, G. Mao, and Y. D. Zhang, “Direction-of-arrival estimation for coprime array via virtual array interpolation,” *IEEE Trans. Signal Process.*, vol. 66, no. 22, pp. 5956–5971, Nov. 2018.

[25] S. S. Chen, D. L. Donoho, and M. A. Saunders, “Atomic decomposition by basis pursuit,” *SIAM Review*, vol. 43, no. 1, pp. 129–159, 2001.

[26] M. Grant, S. Boyd, and Y. Y. Ye, “CVX: Matlab Software for Disciplined Convex Programming Version 2.2,” Jan. 2020. Available: http://cvxr.com/cvx/

[27] A. Amar and M. A. Doron. “A linearly constrained minimum variance beamformer with a pre-specified suppression level over a pre-defined broad null sector," *Signal Process.*, vol. 109, pp. 165–171, 2015.

[28] J. Riba, J. Goldberg, and G. Vázquez. “Robust beamforming for interference rejection in mobile communications,” *IEEE Trans. Signal Process.*, vol. 45. no. 1, pp. 271–275, 1997.

[29] F. L. Liu, P. P. Chen, J. K. Wang, and L. Peng, “Null broadening and sidelobe control method based on multi-parametric quadratic programming,” *J. Northeastern Univ. (Natural Science)*, vol. 33, no. 11, pp. 1559-1562, 2012.