Double-IRS Aided Wireless Communications Exploiting Two-Dimensional Sparse Arrays

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***Abstract*—This paper investigates a double intelligent reflecting surface (IRS) hybrid network architecture, where the two surfaces are deployed to cooperatively assist communication between a multi-antenna base station and a single-antenna user over multipath channels. We decouple the double-IRS assisted user communication system by utilizing sparse active elements in the second IRS, where active beamforming at the base station and passive beamforming at the first IRS were simultaneously optimized using an alternating optimization method, and the user multipath channels are estimated by the second IRS. We optimize the reflection coefficients of the second IRS and analyze the feasible bit rate that the user can achieve. Simulation results are presented to validate the effectiveness of the proposed architecture and the benefits of the double-IRS in increasing spectral efficiency.**

Keywords: Intelligent reflecting surface, double IRS network, sparse array, structured matrix completion.

I. INTRODUCTION

The intelligent reflecting surface (IRS) is a viable emerging technology for 5G and beyond due to its ability to change the wireless channel between communicating nodes. IRS is a rectangular metasurface made up of a large number of passive reflecting elements, each of which can be digitally adjusted to cause a distinct amplitude change and/or phase shift in the incident signal, allowing broadcasters and receivers to cooperate to change the wireless links. In terms of channel estimation, wireless channel reconfiguration, and pilot overhead minimization, constructing an IRS with all passive elements except a few active elements can provide high-precision performance [1, 2]. Both passive and active IRSs can be used to build a chain or hybrid network to increase network capacity, particularly in the millimeter wave domain.

In [3–5], the authors implemented multi-IRS distributed networks where multiple users are served by a single IRS. However, the multi-IRS cooperation setup to serve the users was overlooked, limiting the potential for multi-IRS performance gain. A double IRS setup was discussed in [6, 7], where a joint passive beamforming design for the two IRSs was proposed, with the base station (BS) serving the user via the double reflection link. The inter-IRS link was assumed to have a line-of-sight (LoS) channel model, and they achieved a much higher gain than a single IRS scenario. To maximize the minimum signal-to-interference-plus-noise ratio (SINR) among all users in their uplink transmissions, the joint optimization of the (active) receive beamforming at the BS and the cooperative (passive) reflect beamforming at the two distributed IRSs was investigated in [8].However, none of the previous studies considered the practical scenario in which the user and the IRS have multiple LOS paths. Furthermore, because most BS and IRSs are above ground, channels are more likely to be constant, whereas user-IRS typically have fast varying multipath channels, necessitating fast channel estimation as well as active and passive beamforming optimization. Furthermore, passive double-IRS is incapable of achieving distributed computational performance and disjoint optimization efficiency across the network.

We investigate a multiple-input, single-output (MISO) communication system as depicted in Fig. 1, where we seek to attain the achievable bit rate at the user end by employing double IRS in a hybrid network model. Using the advantages of an active IRS (IRS-2), we propose a decoupled passive beamforming optimization technique which is implemented in three phases and we optimize the joint active and passive beamforming vectors for BS and passive IRS (IRS-1) in phase I. The alternate optimization technique is used to optimize one set of variables iteratively while keeping the other set constant. In phase II, we estimate the fast fading multipath channel between user and IRS-2 with the help of an L-shaped sparse active elements on the IRS-2. Based on the channel knowledge and optimized beamforming vectors of the BS and IRS-1, we compute the ideal passive beamforming vector for IRS-2 and determine the highest bit rate for the user served by the multi-antenna base station (BS) over the double reflection link in phase III.

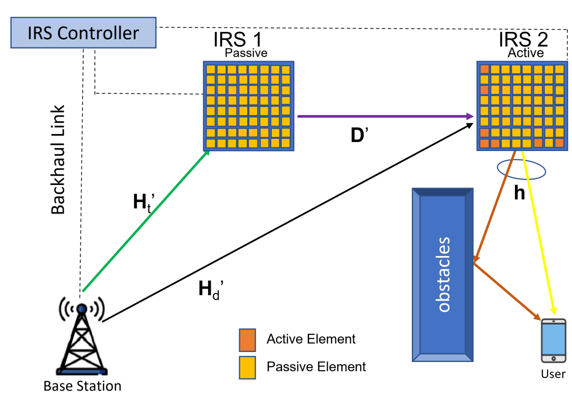

Fig. 1: Double IRS-assisted channel model.

II. System Model

Consider an IRS-enhanced MISO downlink communication system that consists of two distributed IRSs (referred to as IRS-1 and IRS-2) deployed to assist the communication between a single antenna user from an N-antenna BS. We assume that the two IRSs are positioned near the BS and the user respectively, in a practical deployment scenario where direct links between user and the BS are significantly obstructed. We consider the IRS-1 is a passive reflector with M_1 passive reflecting elements. But the IRS-2 has M_2 reflecting elements among which $\bar{M}_2$ active elements which has both sensing and reflecting properties, are placed in sparse L-shaped structure as shown in Fig. 1. Furthermore $\bar{M}_x$ and $\bar{M}_z$ are the active elements in x and z-axis subarrays, where $\bar{M}_2 = \bar{M}_x + \bar{M}_z - 1$. Let $\mathbf{h}_{r,1} \in \mathbb{C}^{M_2 \times 1}$ and $\mathbf{h}_{r,2} \in \mathbb{C}^{M_2 \times 1}$ are two LOS paths between the IRS 2 and user, where $\mathbf{h} \in \mathbb{C}^{M_2 \times 1}$ is the summed channel. In addition, each IRS is connected to a smart controller that controls the phase shifts and communicates with the BS via a separate, reliable backhaul link. Let, the signal transmitted by the BS is given by

$$\mathbf{x} = \mathbf{f}s, \tag{1}$$

where s denotes the transmitted data symbol and follows $\mathbb{E}[|s|^2] = 1$, and $\mathbf{f} \in \mathbb{C}^{N \times 1}$ is the corresponding beamforming vector. The channels between the BS and RIS-1, the BS and IRS-2, and the RIS-1 and IRS-2 are denoted by $\mathbf{H}_t \in \mathbb{C}^{M_1 \times N}$, $\mathbf{H}_d \in \mathbb{C}^{M_2 \times N}$ and $\mathbf{D} \in \mathbb{C}^{M_2 \times M_1}$. Thus the downlink received signal at the user is formulated as

$$\mathbf{y}_T = (\mathbf{h}_k)\mathbf{x} + \mathbf{n}_T, \tag{2}$$

where, $\mathbf{y}_T$ and $\mathbf{h_k} \in \mathbb{C}^{1 \times N}$ are the received signal and the superimposed channel from BS to user, respectively. $\mathbf{n}_T \sim \mathcal{CN}(0, \sigma_T^2)$ represents the additive white Gaussian noise (AWGN) at the user with noise variance σ_T^2. From fig.1 we can write the superimposed channel as

$$\begin{aligned} \mathbf{h}_k &= \mathbf{h}^{\mathrm{H}}\Phi\mathbf{H}_d + \mathbf{h}^{\mathrm{H}}\Phi\mathbf{D}\Theta\mathbf{H}_t \\ &= \boldsymbol{\phi}\,\mathrm{diag}(\mathbf{h}^{\mathrm{H}})\mathbf{H}_d + \mathbf{h}^{\mathrm{H}}\Phi\{\bar{\boldsymbol{\theta}}\,\mathrm{diag}(\mathbf{d}_1), \bar{\boldsymbol{\theta}}\,\mathrm{diag}(\mathbf{d}_2), \\ &\qquad \cdots, \bar{\boldsymbol{\theta}}\,\mathrm{diag}(\mathbf{d}_{M_1})\}\mathbf{H}_t, \end{aligned} \tag{3}$$

here, $\Theta = \mathrm{diag}(\bar{\boldsymbol{\theta}})$ and $\Phi = \mathrm{diag}(\boldsymbol{\phi})$ represents the diagonal reflection matrix of IRS-1 and IRS-2, respectively [8]. $\mathbf{d}_m \in \mathbb{C}^{1 \times M_1}$ is the channel between IRS-1 and m-th patch of IRS-2. For simplicity, we assume that the BS perfectly knows all the channel state information (CSI) between BS $\rightarrow$ IRS-1, inter IRS channel (IRS-1 $\rightarrow$ IRS-2) and BS$\rightarrow$ IRS-2 channel. But the multipath channel between IRS-2 and user is unknown. To make the system more robust we have included the distortion noise in the BS. In this work, our goal is to optimize passive beamforming vectors ($\bar{\boldsymbol{\theta}}$ and $\boldsymbol{\phi}$) in a decoupled fashion to achieve the performance gains over multipath channel between IRS-2 and user, brought by the passive and active IRS system. So, the problem can be expressed as,

$$\begin{aligned} \underset{\mathbf{f}, \bar{\boldsymbol{\theta}}, \boldsymbol{\phi}}{\text{maximize}} \quad & [R]^+ && \text{(4a)} \\ \text{subject to} \quad & \|\mathbf{f}\|_2^2 \leq P_{max}, && \text{(4b)} \\ & |\theta_m| = 1, \ |\phi_m| = 1, \quad \forall m = 1, 2, \cdots, M, && \text{(4c)} \end{aligned}$$

here, P_{max} is the base station power budget. The passive beamforming vectors are subject to the unit modulus constraints. The bit rate R can be formulated as,

$$R = \log_2(1 + \frac{|\mathbf{h}_k \mathbf{f}^{\mathrm{H}}|^2}{\Xi_T}), \tag{5}$$

where Ξ_T is the sum power of distortion noise and thermal noise of the system. Due to the non-concave objective function with respect to $\mathbf{f}, \bar{\boldsymbol{\theta}}, \boldsymbol{\phi}$ in (4a) and the non-convex unit-modulus constraints in (4c) the optimization problem which becomes NP hard and is not easy to solve. Moreover we also need the estimated channel between IRS-2 and user. So, the total system is solved in three phase which is described in next section.

III. Proposed Channel Decouple Method

A. Phase I

Assume m-th element of the IRS-2 is an active patch. From (3), the equivalent channel between BS$\rightarrow$ IRS-1$\rightarrow$ m-th element of IRS-2 ($\mathrm{diag}(\mathbf{d}_m)\mathbf{H}_t \in \mathbb{C}^{M_1 \times N}$) and BS$\rightarrow$ m-th element of IRS-2 ($\mathbf{h}_{d,m} \in \mathbb{C}^{1 \times N}$) is, $\mathbf{G}_1 = \begin{bmatrix} \mathrm{diag}(\mathbf{d}_m)\mathbf{H}_t \\ \mathbf{h}_{d,m} \end{bmatrix} \in \mathbb{C}^{(M_1+1) \times N}$. And $\boldsymbol{\theta} = [\theta_1, \theta_2, \cdots, \theta_{M_1}, 1]^{\mathrm{T}} \in \mathbb{C}^{(M_1+1) \times 1}$ is the equivalent reflection coefficient vector. So in the Phase I we determine the bit rate, R_1 over the equivalent channel and can be expressed as,

$$R_1 = \log_2(1 + \frac{1}{\Xi_1(\mathbf{f}, \boldsymbol{\theta})} \mathbf{f}^{\mathrm{H}} \mathbf{G}_1^{\mathrm{H}} \boldsymbol{\theta}\boldsymbol{\theta}^{\mathrm{H}} \mathbf{G}_1 \mathbf{f}), \tag{6}$$

here, $\Xi_1(\mathbf{f}, \boldsymbol{\theta}) = \mu_t \mathbf{f}^{\mathrm{H}} \widetilde{\mathrm{diag}} \mathbf{G}_U^{\mathrm{H}} \boldsymbol{\theta}\boldsymbol{\theta}^{\mathrm{H}} \mathbf{G}_U \mathbf{f} + \sigma_1^2$ is the sum power of distortion noise and thermal noise of the system [9].

Phase I focuses on optimizing the received bit rate, R_1 at the m-th element of IRS-2 by combining the transmit beamforming vector $\mathbf{f}$ and the passive beamforming $\boldsymbol{\theta}$, while keeping the maximum transmit power constraint at the BS in mind. The associated optimization issue can be formulated as,

$$\begin{aligned} \underset{\mathbf{f}, \boldsymbol{\theta}}{\text{maximize}} \quad & [R_1]^+ && \text{(7a)} \\ \text{subject to} \quad & \|\mathbf{f}\|_2^2 \leq P_{max}, && \text{(7b)} \\ & \boldsymbol{\theta} \in \mathcal{S}, && \text{(7c)} \end{aligned}$$

$\mathcal{S}$ imposes a unit modulus on each entry in $\boldsymbol{\theta}$ with the set $\mathcal{S} = \{\boldsymbol{\theta} | \ |\theta_m|^2 = 1, 1 \leq m \leq M_1, \theta_{M_1+1} = 1\}$. The optimal value of Problem (7a) is always non-negative. If $R_1 \leq 0$, the objective function's value can be set to zero by setting $\|\mathbf{f}\|_2^2 = 0$. The non-concave objective function with respect to $\mathbf{f}$ and $\boldsymbol{\theta}$, as well as the non-convex constraint (7c), make problem (7) difficult to solve.

1) Algorithm Design for Phase I: We adopt the alternating optimization (AO) method to address the coupling of the beamforming vector at the BS and the reflection beamforming at the RIS in Problem (7). Specifically, we alternately update $\mathbf{f}$ and $\boldsymbol{\theta}$ while fixing the other variable. Due to the non concave function in (6) and (7), we introduce auxiliary variables $\mathbf{p}=[p_1,p_2]^{\mathrm{T}}$, which satisfy

$$\begin{cases} \log_2(\Xi_1(\mathbf{f},\boldsymbol{\theta})+\mathbf{f}^{\mathrm{H}}\mathbf{G}_1^{\mathrm{H}}\boldsymbol{\theta}\boldsymbol{\theta}^{\mathrm{H}}\mathbf{G}_1\mathbf{f})\geq p_1, & \text{(8a)}\\ \log_2(\Xi_1(\mathbf{f},\boldsymbol{\theta}))\leq p_2, & \text{(8b)} \end{cases}$$

such that, $R_1\geq p_1-p_2$. With (8), the problem (7) can be reformulated as,

$$\begin{aligned} &\underset{\mathbf{f},\boldsymbol{\theta},\mathbf{p}}{\text{maximize}} \quad p_1-p_2 && \text{(9a)}\\ &\text{subject to} \quad (7b),(7c),(8). && \text{(9b)} \end{aligned}$$

2) Optimize $\mathbf{f}$ *with respect to* $\boldsymbol{\theta}$*:* When $\boldsymbol{\theta}$ is given, we further introduce auxiliary variables $\mathbf{r}=[r_{f,1},r_{f,2}]$ such that non-convex constraints at (8) are respectively equivalent to,

$$(8)\Rightarrow\begin{cases} \log_2(r_{f,1})\geq p_1, & \text{(10a)}\\ \log_2(r_{f,2})\leq p_2, & \text{(10b)}\\ (\Xi_1(\mathbf{f})+\mathbf{f}^{\mathrm{H}}\mathbf{G}_1^{\mathrm{H}}\boldsymbol{\theta}\boldsymbol{\theta}^{\mathrm{H}}\mathbf{G}_1\mathbf{f})\geq r_{f,1}, & \text{(10c)}\\ (\Xi_1(\mathbf{f}))\leq r_{f,2}. & \text{(10d)} \end{cases}$$

It is observed from (10) that constraints (10a), (10d) are convex, while constraints (10b), (10c) are concave. According to [10], the successive convex approximation (SCA) can be used to address the concave constraints. First-order Taylor approximation can provide the following equivalent constraints,

$$(10b)\Rightarrow\log_2(r_{f,2}^n)+\frac{r_{f,2}-r_{f,2}^n}{r_{f,2}^n\ln(2)}\leq p_2, \tag{11}$$

$$(10c)\Rightarrow 2\mathrm{Re}\{\mathbf{f}^{n,\mathrm{H}}\mathbf{A}_1\mathbf{f}\}-\mathbf{f}^{n,\mathrm{H}}\mathbf{A}_1\mathbf{f}^n+{\sigma_1}^2\geq r_{f,1} \tag{12}$$

where, $\mathbf{A}_1=\mathbf{G}_1^{\mathrm{H}}\boldsymbol{\theta}\boldsymbol{\theta}^{\mathrm{H}}\mathbf{G}_1$, and $r_{f,2}^n$ and $\mathbf{f}$ are the solutions obtained at the n-th iteration. Finally, the suboptimal problem for solving $\mathbf{f}$ is formulated as,

$$\begin{aligned} &\underset{\mathbf{f},\mathbf{p},\mathbf{r}_f}{\text{maximize}} \quad p_1-p_2 && \text{(13a)}\\ &\text{subject to} \quad (7b),(10a),(10d),(11),(12). && \text{(13b)} \end{aligned}$$

Problem (13) is a second order cone program (SOCP) and can be solved by CVX tool.

3) Optimize $\boldsymbol{\theta}$ *with respect to* $\mathbf{f}$*:* In order to tackle the non-convex unit-modulus constraint $\boldsymbol{\theta}\in\mathcal{S}$, we adopt the semidefinite relaxation (SDR) technique to update $\boldsymbol{\theta}$. In particular, by defining a new variable $\tilde{\mathbf{E}}=\underline{\boldsymbol{\theta}\boldsymbol{\theta}}^{\mathrm{H}}$, constraint $\boldsymbol{\theta}\in\mathcal{S}$ is replaced by $\{\tilde{\mathbf{E}}\succeq 0,\mathrm{rank}(\tilde{\mathbf{E}})=1,\widetilde{\mathrm{diag}}(\tilde{\mathbf{E}})=\mathbf{I}_{M_1+1}\}$, Furthermore, with fixed $\mathbf{f}$ and new auxiliary variables $\mathbf{r}_\theta=[r_{\theta,1},r_{\theta,2}]^T$, non-convex constraints (8a) and (8b) are equivalent to,

$$(8a),(8b)\Rightarrow\begin{cases} \log_2(r_{\theta,1})\geq p_1, & \text{(14a)}\\ \log_2(r_{\theta,2})\leq p_2, & \text{(14b)}\\ \mathrm{Tr}\{\mathbf{B}_1\mathbf{E}\}+\sigma_1^2\geq r_{\theta,1}, & \text{(14c)}\\ \mathrm{Tr}\{\mathbf{B}_2\mathbf{E}\}+\sigma_1^2\leq r_{\theta,2}, & \text{(14d)} \end{cases}$$

where, $\mathbf{B}_1=\mathbf{B}_2+\mathbf{G}_1\mathbf{f}\mathbf{f}^{\mathrm{H}}\mathbf{G}_1^{\mathrm{H}}$ and $\mathbf{B}_2=\mu_t\mathbf{G}_1\widetilde{\mathrm{diag}}(\mathbf{f}\mathbf{f})^{\mathrm{H}}\mathbf{G}_1^{\mathrm{H}}$ and $\mathrm{Tr}(.)$ denotes the trace operation. In (14), the only non-convex constraints (14b), which can be approximated by using the first-order Taylor approximation as

$$\log_2(r_{\theta,2}^n)+\frac{r_{\theta,2}-r_{\theta,2}^n}{r_{\theta,2}^n\ln(2)}\leq p_2, \tag{15}$$

$r_{\theta,2}^n$ is the solutions obtained at the n-th iteration. Finally, the relaxed subproblem of Problem (9) can be formulated as,

$$\begin{aligned} &\underset{\tilde{\mathbf{E}},\mathbf{p},\mathbf{r}_\theta}{\text{maximize}} \quad p_1-p_2 && \text{(16a)}\\ &\text{subject to} \quad \tilde{\mathbf{E}}\succeq 0,\mathrm{rank}(\tilde{\mathbf{E}})=1,\widetilde{\mathrm{diag}}(\tilde{\mathbf{E}})=\mathbf{I}_{M+1}, && \text{(16b)}\\ &\qquad\qquad\quad (14a),(14c),(14d),(15). && \text{(16c)} \end{aligned}$$

In order to ensure the non-decreasing objective value sequence generated in each iteration, we update $\boldsymbol{\theta}$ as [9]. As the IRS-1 and IRS-2 are in LOS, and the angle of arrival (AOA) from the reflected IRS-1 signal for all the active and passive elements of IRS-2 are same, thus the achievable bit rate for each element at IRS-2 is also same. At phase I, by the above joint active and passive beamforming we can determine the optimal active beamforming vector $\mathbf{f}$ for the BS and passive beamfoming vector $\bar{\boldsymbol{\theta}}$ of IRS-1. The total complexity of phase I is $\mathcal{O}(N^3)+\mathcal{O}(M_1^{3.5})$. As the computational complexity is proportional to the channel dimension, we have reduced the computational time by optimizing the cascade channel over a single element of IRS-2.

B. Phase II

At phase II, we will estimate the multipath channel between user and IRS-2. It is assumed that uncorrelated far-field narrowband signals impinge to the IRS-2 described as L paths between the user and the IRS, and the AOAs corresponding to the L-shaped sparse array are $\{\varphi_l,\vartheta_l\}$ for $l=1,\cdots,L$.

Considering the IRS-2 as an uniform planar array, the 2-D steering vector $\mathbf{a}_{\mathrm{IRS}}(\varphi,\vartheta)$ for can be written as [11]:

$$\mathbf{a}_{\mathrm{IRS}}(\varphi,\vartheta)=\mathbf{a}_z(\varphi)\otimes\mathbf{a}_x(\vartheta), \tag{17}$$

where,

$$\mathbf{a}_x(\varphi)=[1,e^{-j\frac{2\pi}{\lambda}d\sin(\varphi)},\ldots,e^{-j\frac{2\pi}{\lambda}d(M_x-1)\sin(\varphi)}]^{\mathrm{T}}, \tag{18}$$

$$\mathbf{a}_z(\vartheta)=[1,e^{-j\frac{2\pi}{\lambda}d\sin(\vartheta)},\ldots,e^{-j\frac{2\pi}{\lambda}d(M_z-1)\sin(\vartheta)}]^{\mathrm{T}}, \tag{19}$$

and M_x denotes the elements in the x-axis and M_z denotes the elements in the z-axis on the IRS-2 surface. On the other hand, since user contains a single antenna, the channel between the IRS and user is denoted by,

$$\mathbf{h}=\mathbf{h}_{r,1}+\mathbf{h}_{r,2}=\sum_{l=1}^{L}\beta_l\mathbf{a}_{\mathrm{IRS}}^{\mathrm{H}}(\vartheta_l^{\mathrm{IRS}},\varphi_l^{\mathrm{IRS}}), \tag{20}$$

where β_l is the path gain, $\vartheta_l^{\mathrm{IRS}}\in[-\pi/2,\pi/2]$ and $\varphi_l^{\mathrm{IRS}}\in[-\pi/2,\pi/2]$ are respectively the elevation and the azimuth angles from the received signal from the user at IRS-2.

1) 2-D AOA and Path Gain Estimation: During sensing mode, the IRS 2 receives signals from the user through its active elements. The received signal at the IRS-2 corresponding to the x-axis and the z-axis are respectively given as [2, 11]:

$$\mathbf{x}(t)=\sum_{l=1}^{L}\beta_l\mathbf{a}_X(\varphi_l)\mathbf{s}_u(t)+\mathbf{n}_X(t), \tag{21}$$

$$\mathbf{z}(t)=\sum_{l=1}^{L}\beta_l\mathbf{a}_Z(\vartheta_l)\mathbf{s}_u(t)+\mathbf{n}_Z(t), \tag{22}$$

where $\mathbf{s}_u(t)$ denotes the source signal vector transmitted by the user, and $\mathbf{n}_X(t)$ and $\mathbf{n}_Z(t)$ are the AWGN vectors.

We consider separately spaced active elements with the ONRA structure [12], and the positions of the active elements along the x- and the z-axes are represented by $\mathbb{X}=\{p_0,p_1,\cdots,p_{\bar{M}_x-1}\}\lambda/2$ and $\mathbb{Z}=\{q_0,q_1,\cdots,q_{\bar{M}_z-1}\}\lambda/2$, respectively, where p_i and q_i are integers for all i, and $p_0=q_0=0$ is assumed. We also denote $W_x=p_{\bar{M}_x-1}+1$ and $W_z=p_{\bar{M}_z-1}+1$ as the length of active and passive elements included within the respective apertures of the x- and z-axis subarrays. $\mathbf{a}_X(\varphi_l)\in\mathbb{C}^{W_x\times 1}$ and $\mathbf{a}_Z(\vartheta_l)\in\mathbb{C}^{W_z\times 1}$ denote the steering vectors corresponding to the received AOAs along the x axis and z axis, respectively.

Assuming the noise is uncorrelated to the signals, the covariance matrices of $\mathbf{x}(t)$ and $\mathbf{z}(t)$ can be respectively expressed as:

$$\mathbf{R}_{X_{\mathrm{IRS}}}=\mathbf{E}[\mathbf{x}(t)\mathbf{x}^{\mathrm{H}}(t)]=\mathbf{A}_X\mathbf{R}_s\mathbf{A}_X^{\mathrm{H}}+\sigma_n^2\mathbf{I}_{W_x}, \tag{23}$$

$$\mathbf{R}_{Z_{\mathrm{IRS}}}=\mathbf{E}[\mathbf{z}(t)\mathbf{z}^{\mathrm{H}}(t)]=\mathbf{A}_Z\mathbf{R}_s\mathbf{A}_Z^{\mathrm{H}}+\sigma_n^2\mathbf{I}_{W_z}, \tag{24}$$

where $\mathbf{R}_s=\mathrm{diag}(\sigma_1^2,\sigma_2^2,\cdots,\sigma_L^2)$, and σ_l^2 represents the power of the l-th path signal and σ_n^2 denote the noise power. Because of the sparse spacing between the elements, the covariance matrices $\mathbf{R}_{X_{\mathrm{IRS}}}$ and $\mathbf{R}_{Z_{\mathrm{IRS}}}$ become sparse with missing holes. So we consider the matrix interpolation of $\mathbf{R}_{X_{\mathrm{IRS}}}$ to obtain a full rank covariance matrix along the x-axis as the following nuclear norm minimization problem [13, 14]:

$$\begin{aligned}&\underset{\boldsymbol{w}}{\text{minimize}} && \|\mathcal{T}(\boldsymbol{w})\mathbf{V}-\mathbf{R}_{X_{\mathrm{IRS}}}\|_F^2+\zeta\|\mathcal{T}(\boldsymbol{w})\|_*\\ &\text{subject to} && \mathcal{T}(\boldsymbol{w})\succcurlyeq 0,\end{aligned} \tag{25}$$

where $\|\mathcal{T}(\boldsymbol{w})\|_*=\mathbf{Tr}(\sqrt{\mathcal{T}^{\mathrm{H}}(\boldsymbol{w})\mathcal{T}(\boldsymbol{w})})$ is the nuclear norm of $\mathcal{T}(\boldsymbol{w})$, $\mathcal{T}(\boldsymbol{w})$ denotes the Hermitian-Toeplitz matrix with $\boldsymbol{w}$ as its first column, ζ is a tunable regularization parameter. $\mathbf{V}=\mathbf{v}_p\mathbf{v}_p^{\mathrm{T}}$ is the binary mask of the sparse covariance matrix,

$$\langle\mathbf{v}_p\rangle_g=\begin{cases}1, & gd\in\mathbb{X},\\ 0, & \text{otherwise},\end{cases} \tag{26}$$

where g is the index of the sensor location with $g\in\{p_0,p_1,\cdots,p_{N_x-1}\}$ and $\langle\cdot\rangle_g$ denotes the element corresponding to sensor positions at gd. The interpolated covariance matrices as $\hat{\mathbf{R}}_{X_{\mathrm{IRS}}}\in\mathbb{C}^{W_x\times W_x}$ and $\hat{\mathbf{R}}_{Z_{\mathrm{IRS}}}\in\mathbb{C}^{W_z\times W_z}$ for the x and the z axes, respectively. Subspace-based methods, such as MUSIC, can be applied to $\hat{\mathbf{R}}_{X_{\mathrm{IRS}}}$ such that the azimuth and elevation AOAs at the IRS can be solved for user-IRS 2 multipath signals.

2) Pair-Matching for 2-D AOA Estimation: Generally, from the user there are multiple incident signals at the same time on the IRS. Therefore, it is important to determine the pairing between the determined azimuth and the corresponding elevation angle. The array steering matrix can be constructed according to the estimated azimuth angles as,

$$\hat{\mathbf{A}}_X=[\mathbf{a}_X(\hat{\varphi}_1),\mathbf{a}_X(\hat{\varphi}_2),\cdots,\mathbf{a}_X(\hat{\varphi}_L)]\in\mathbb{C}^{W_z\times L}. \tag{27}$$

The cross-covariance matrix for $\mathbf{x}(t)$ and $\mathbf{z}(t)$ is given as:

$$\mathbf{R}_{XZ}=\mathbf{E}[\mathbf{x}(t)\mathbf{z}^{\mathrm{H}}(t)]=\mathbf{A}_X\mathbf{R}_S\mathbf{A}_Z^{\mathrm{H}}. \tag{28}$$

The steering matrix $\hat{\mathbf{A}}_Z$ can be obtained as [11],

$$\hat{\mathbf{A}}_Z=(\mathbf{R}_S^{-1}\hat{\mathbf{A}}_X^{\dagger}\mathbf{R}_{XZ})^{\mathrm{H}}, \tag{29}$$

where, $\hat{\mathbf{A}}_Z\in\mathbb{C}^{W_z\times L}$. According to [11], we have reapply the MUSIC algorithm on each l-th path to determine the azimuth angle sequence to reconstruct the steering matrix of the IRS for the user-IRS 2 channel as $\hat{\mathbf{A}}_{\mathrm{IRS}}=[\hat{\mathbf{a}}_{\mathrm{IRS}}(\varphi_1^{\mathrm{IRS}},\vartheta_1^{\mathrm{IRS}}),\cdots,\hat{\mathbf{a}}_{\mathrm{IRS}}(_L^{\mathrm{IRS}},\vartheta_L^{\mathrm{IRS}})]\in\mathbb{C}^{M_2\times L}$.

3) Path Gain Estimation: The path gains are identical for the x- and z-axis subarrays. Therefore, to estimate the path gain of the user-IRS 2 channel, computation in one of these two subarrays will suffice. The received signal at the z-direction subarray can be rearranged as,

$$\mathbf{y}_z(t)=\mathbf{A}_Z\mathbf{g}s_u(t)+\mathbf{n}_Z(t), \tag{30}$$

where $\mathbf{g}=[\beta_1,\beta_2,\cdots,\beta_L]^{\mathrm{T}}$ represents the path gains and can be estimated from

$$\hat{\mathbf{g}}=\frac{1}{\sigma_s^2}(\mathbf{A}_Z^{\mathrm{H}}\mathbf{A}_Z)^{-1}\mathbf{A}_Z^{\mathrm{H}}\bar{\mathbf{y}}_z, \tag{31}$$

where $\bar{\mathbf{y}}_z=\mathbb{E}\{\mathbf{y}_z(t)s_u^*(t)\}$. From the above AoAs and path gain estimation, we can reconstruct the USER-IRS 2 multipath channel $\mathbf{h}_r$.

C. Phase III

Here our goal is to optimize $\boldsymbol{\phi}$ based on the jointly optimized $\boldsymbol{\theta}$, and $\mathbf{f}$ and the estimated $\mathbf{h}$ over the known channels between BS→IRS-1, IRS-1→ IRS-2. From the double IRS channel (BS→IRS-1→ IRS-2→ user), the rate R_U at user,

$$R_U=\log_2(1+\frac{1}{\Xi_U(\mathbf{f},\boldsymbol{\phi})}\mathbf{f}^{\mathrm{H}}\mathbf{G}_U^{\mathrm{H}}\boldsymbol{\phi}\boldsymbol{\phi}^{\mathrm{H}}\mathbf{G}_U\mathbf{f}), \tag{32}$$

where the cascade channel is $\mathbf{G}_U=\mathrm{diag}(\mathbf{h}_r^{\mathrm{H}})\mathbf{D}_r\Theta\mathbf{G}$. At this point we will again adopt SDR to update $\boldsymbol{\phi}$.

IV. Simulation Results

We consider a BS with N = 4 antennas, IRS-1 has $M_1=64$ passive elements where in active IRS-2 has total $M_2=64$ elements where $\bar{M}_2$ = 7 of them are active. The x- and z direction linear subarrays in the L-shape sparse active array each consists of 4 sensors since the 0-th reference sensor is shared by both the subarrays. The position of the active elements along the x- and the z axes are $\mathbb{X}=\mathbb{Z}=\{0,1,5,7\}\lambda/2$.

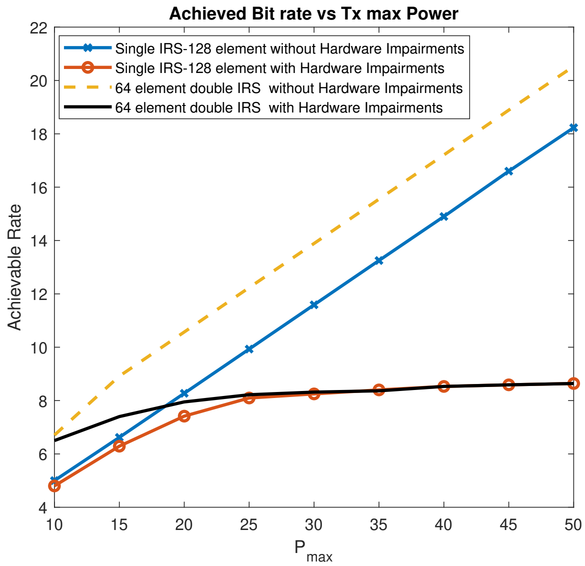

Fig. 2: Achievable bit rate for single and double IRS.

The BS, RIS-1, IRS-2 and user are located at (0 m, 0 m, 10m) and (30 m, 4 m, 10m), (70 m, 4 m, 10m) and (75 m, 5 m, 0m) respectively. For evaluating the decoupled double IRS performance, we have considered a $M_3 = 128$ passive element single IRS-3 at the position of IRS 2 and IRS 1 is also absent. $\sigma_T^2 = -80$dBm, The large-scale path loss is PL$= -30 - 10\alpha \log_{10} b$, where α is the pathloss exponent, b is the link length in meters. The path loss exponent of the BS→RIS-1, IRS-1→IRS-2 and BS→RIS-2 channels are set to 2.4, 2.6, and 3.6, respectively. And user→IRS-2 path loss exponent were set as 2.2 and 2.4.

Fig. 2 investigates the achievable rate of different schemes as a function of the maximum transmit power at the BS through single and double IRS system. Without considering hardware impairment the double IRS performs much better than the single IRS. Considering the hardware impairment scenario, when the transmit power is less than 20dBm, double IRS still achieves bit rate which is much higher than the 128 element single IRS system. But the rate tends to remain steady as P_{max} increases beyond 25 dBm. Because the distortion noise induced by hardware flaws is proportional to the transceiver signal strength, the rate as a function of the maximum transmit power has an upper limit in terms of bit rate it can achieve.

V. Conclusion

In this paper, we have shown that using double IRS setup improves the channel efficiency compared to a single IRS system with the same number of active elements. Furthermore, the computational complexity also reduces because of the distributed computation of passive beamforming vectors. The aperture is increased while the computational overhead is reduced by applying structured matrix completion and pair-matching methods for multipath channel detection. Simulation results confirm the effectiveness and performance of the proposed structured compared to the single IRS setup.

VI. References

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