Dual-Function Radar-Communications Using QAM-based Sidelobe Modulation

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Abstract

Spectrum sharing using a joint platform for radar and communication systems has attracted significant attention in recent years. In this paper, we propose a novel dual-function radar-communications (DFRC) strategy to embed quadrature amplitude modulation (QAM) based communication information in the radar waveforms by exploiting sidelobe control and waveform diversity. The proposed information embedding technique can support multiple communication receivers located in the sidelobe region. In addition to the information broadcasting, the developed approach enables multi-user access by allowing simultaneous transmission of distinct information streams to the communication receivers located in different directions. We prove that the proposed technique ensures a significant data rate enhancement compared to the existing techniques. Moreover, the developed DFRC strategy generalizes the mathematical framework of the existing sidelobe control-based information embedding techniques.

Keywords: Dual-function radar-communications, information embedding, multi-user access, sidelobe control, waveform diversity.

1. Introduction

Spectrum sharing has recently gained a significant attention due to the ongoing congestion of wireless spectrum caused by the broadband wireless communications and ever-increasing deployment of new applications aiming to consume the same spectral resources [1–4]. Modern wireless communication systems require immense expansion of existing spectral allocations in order to achieve high data rates to ensure the success of future generations of wireless systems. Moreover, new technical advancements and emerging applications which bring various advantages to the end users require new allocations of frequency resources [5]. In this context, great efforts have been invested in the field of cognitive radios to improve the spectral efficiency so as to effectively manage the existing usage of electromagnetic spectrum [6]. Recently, the problem of spectral competition between radar and communication systems has been addressed by the coexistence or joint transmission. The coexistence of radar and communication platforms in the same frequency bands requires both systems to work collaboratively in order to mitigate the mutual interference [7–15]. On the other hand, the strategies enabling joint transmission, like dual-function radar-communications (DFRC), perform the secondary communication operation in addition to the primary radar function while utilizing the same spectral resources [16–31].

In a DFRC platform, the waveforms responsible for both radar and communication operations are transmitted from the same physical antenna array. In this case, communication is considered to be the secondary objective of the DFRC system and is enabled by embedding information in the radar waveforms such that the primary radar operation is not compromised. Fig. 1 shows the basic principle of DFRC strategies. The notable DFRC techniques developed so far include waveform diversity-based method [18], sidelobe amplitude modulation (AM) method [20], multi-waveform amplitude shift keying (ASK) method [21–23], and phase shift keying (PSK) method [24–26]. The waveform diversity-based method [18] ensures the secondary objective of communication by exploiting waveform diversity such that the acceptable performance of radar is maintained. The sidelobe AM method [20] exploits multiple beamforming weight vectors corresponding to different sidelobe levels at the communication receivers located in the sidelobe region of radar. Each sidelobe level is mapped to a unique communication symbol. In the multi-waveform ASK-based method [21], multiple orthogonal radar waveforms are employed such that each waveform is transmitted by one of the two beamformers implementing two different sidelobe levels. The communication receiver decodes the transmitted information by matched filtering the received waveform and extracting the sidelobe level information. Although ASK-based method exploits waveform diversity, only two sidelobe levels are utilized. The method in [23] increases the effective signal-to-noise ratio (SNR) at the communication receivers by using only one beamforming weight vector corresponding to the maximum allowable sidelobe level. Unfortunately, all the above ASK-based methods can only broadcast the same communication information to all the communication users. On the other hand, multi-user ASK-based strategy [31] enables the transmission of distinct communication streams towards different communication users located in the sidelobe region of the radar. This objective is achieved by simultaneously transmitting different sidelobe levels towards the communication receivers located in different directions. In PSK-based DFRC schemes [24–26], information embedding is achieved by a dictionary of beamforming weight vectors having the same beampattern but different phase response towards the communication receivers. The communication receivers detect the corresponding phase symbols in the radar waveforms to determine the transmitted information either by coherent demodulation or with the help of a reference radar waveform transmitted through a reference beamfoming weight vector. The PSK-based method cannot exploit the flexibility of having different sidelobe levels. Moreover, the existing PSK-based formulations are only restricted to the case of uniform linear arrays (ULAs).

In this paper, we propose a novel quadrature amplitude modulation (QAM) based DFRC strategy which exploits sidelobe control and waveform diversity to transmit communication information. The proposed approach enables multi-user access, i.e., we can send distinct communication streams in different directions while utilizing the same hardware resources as employed by the existing ASK and PSK-based techniques. In this context, the proposed technique can provide a higher throughput compared to the conventional approaches where multiplexing of communication information will be required to transmit different information to different receivers. Moreover, the proposed technique serves as the generalized mathematical model for the existing sidelobe ASK-based DFRC schemes. Simulation results demonstrate the effectiveness of the proposed strategy.

The rest of the paper is organized as follows. Section II presents the summary of existing sidelobe control-based DFRC methods. In Section III, a new QAM-based information embedding scheme is proposed and its performance is analytically evaluated. Simulation results are compared in Section V, and Section VI concludes the paper.

2. Existing Sidelobe-based DFRC Schemes

Consider a DFRC system equipped with an M-element transmit linear antenna array of an arbitrary configuration. Let P denotes the total power transmitted by the antenna array during each radar pulse, and $\{\psi_1(t), \psi_2(t), \ldots, \psi_{\hat{K}}(t)\}$ be the $\hat{K}$ possible radar waveforms orthogonal to each other such that:

$$\frac{1}{T}\int_0^T \psi_{k_1}(t)\,\psi_{k_2}(t)dt = \delta\left(k_1 - k_2\right), \quad 1 \leq k_1, k_2 \leq \hat{K}, \tag{1}$$

where t is the fast time, T is the time duration of each radar pulse, k_1 and k_2 are positive integers, and $\delta(\cdot)$ is the Kronecker delta function.

The objective of existing sidelobe control-based DFRC schemes is to send information symbols to the communication receivers located in the sidelobe region without introducing perturbation to the primary radar operation [19–28]. This implies that the magnitude of the radar waveform must not vary during each transmitted pulse. In order to realize this objective, ASK-based schemes exploit different beamforming vectors to transmit different sidelobe levels in the directions of communication receivers while keeping the radar's main beam at a constant amplitude. On the other hand, PSK-based schemes rely on the transmission of different phases towards the communication receivers during each radar pulse such that the amplitude levels towards the communication receivers and the radar's main beam are not perturbed.

2.1. Beamforming Weight Vector Design

The following optimization can be used to synthesize the beamforming weight vectors for DFRC schemes [19, 22–26]:

$$\begin{aligned} &\min_{\mathbf{w}_l} \max_{\theta_m} \left| e^{j\varphi(\theta_m)} - \mathbf{w}_l^{\mathrm{H}} \mathbf{a}(\theta_m) \right|, \quad \theta_m \in \Theta, \\ &\text{subject to } \left| \mathbf{w}_l^{\mathrm{H}} \mathbf{a}(\theta_p) \right| \leq \varepsilon, \quad \theta_p \in \bar{\Theta}, \\ &\qquad \mathbf{w}_l^{\mathrm{H}} \mathbf{a}(\theta_r) = \Delta_l, \quad 1 \leq l \leq L, 1 \leq r \leq R. \end{aligned} \tag{2}$$

Here, Θ is the set of angles at which the radar main beam (main lobe) operates, $\bar{\Theta}$ is the complement set of Θ representing the sidelobe region, $\mathbf{a}(\theta)$ is the response vector of the transmitting antenna array at the angle θ, $\varphi(\theta)$ is the phase profile of user's choice, $\mathbf{w}_l$ is the desired beamforming vector which achieves the sidelobe level Δ_l at all the communication receivers located at angles $\theta_r \in \bar{\Theta}$, R is the total number of communication receivers located in the sidelobe region, L denotes the total number of allowable sidelobe levels, and $(\cdot)^{\mathrm{H}}$ represents the Hermitian operator.

In the following, we summarize the sidelobe control-based DFRC schemes [19–28].

2.2. ASK-based schemes

The information embedding in the radar waveform by exploiting ASK-based techniques [19–23] can be realized by projecting varying sidelobe levels towards the directions of communication receivers located in the sidelobe region of the radar. These sidelobe levels change from one pulse to other but remain constant during the course of each radar pulse. We can generate L beamforming weight vectors by using the optimization in Eq. (2) such that each vector results in a unique sidelobe level Δ_l in the directions of communication receivers. If one radar waveform is exploited, the signal transmitted from the DFRC platform during one radar pulse using one of the L available beampatterns can be expressed as follows [19, 20]:

$$\mathbf{s}(t, \tau) = \sqrt{P} \sum_{l=1}^{L} b_l(\tau) \mathbf{w}_l{}^{*} \psi_k(t), \tag{3}$$

where τ is the slow time (i.e., pulse index), $\psi_k(t)$ is the arbitrary waveform selected from $\hat{K}$ possible radar waveforms, $b_l(\tau)$ is the binary selection coefficient such that $\sum_{l=1}^{L} b_l(\tau) = 1$ for each radar pulse, and $(\cdot)^*$ denotes the conjugate operator.

Multiple orthogonal radar waveforms can be exploited to improve the detection performance of radar and increase the information rate of communication [19, 21–23]. In [21, 22], $K(\leq \hat{K})$ orthogonal radar waveforms are utilized and the transmitted signal vector for this scheme is given as:

$$\mathbf{s}(t,\tau) = \sqrt{\frac{P}{K}} \sum_{k=1}^{K} \left(b_k(\tau)\mathbf{w}_{\text{low}}^* + (1 - b_k(\tau))\,\mathbf{w}_{\text{high}}^*\right)\psi_k(t), \tag{4}$$

where only two beamforming weight vectors $\mathbf{w}_{\text{low}}$ and $\mathbf{w}_{\text{high}}$ are exploited which, respectively, result in the sidelobe levels of Δ_{low} and Δ_{high} ($\Delta_{\text{low}} < \Delta_{\text{high}}$) at all the communication receivers. The value of each coefficient $b_k(\tau)$ is either 0 or 1 during each radar pulse. These coefficients select the desired beamforming weight vector for each of the K transmitted waveforms, thereby carrying one bit of information for each of these waveforms. During each radar pulse, each radar waveform is transmitted with an amplitude of either $\Delta_{\text{low}}\sqrt{P/K}$ or $\Delta_{\text{high}}\sqrt{P/K}$ towards each communication receiver for the detection of embedded information. This means that the same communication symbols are broadcast to all the receivers. Obviously, it is not possible to transmit different information streams to different communication users located at different directions.

Another ASK-based DFRC technique employs only one beamforming weight vector, which corresponds to the highest allowable sidelobe level at all the communication receivers resulting in highest possible SNR for the communication receivers. During each radar pulse, $\hat{K} - 1$ bits are transmitted such that the coefficients corresponding to $K(\leq \hat{K} - 1)$ bits are equal to 1 and the remaining $\hat{K} - K - 1$ bits are equal to zero. This is achieved by transmitting $\hat{K}$ distinct orthogonal waveforms. The transmitted signal is given as [23]:

$$\mathbf{s}(t,\tau) = \sqrt{\frac{P}{K}} \sum_{k=1}^{\hat{K}-1} b_k(\tau)\mathbf{w}_{\text{high}}^*\psi_k(t) + \sqrt{P} \prod_{k=1}^{\hat{K}-1} (1 - b_k(\tau))\,\mathbf{w}_{\text{high}}^*\psi_{\hat{K}}(t). \tag{5}$$

Each coefficient $b_k(\tau)$ for $1 \leq k \leq \hat{K} - 1$ is either 0 or 1 such that only K coefficients are equal to 1 and the rest of them are equal to 0. The second term in the above equation expresses the case when all zeros are transmitted (i.e., all coefficients $b_k(\tau)$ for $1 \leq k \leq \hat{K} - 1$ are equal to 0) using the reference orthogonal waveform $\psi_{\hat{K}}(t)$. In this scheme, same information is broadcast to all the communication receivers because the transmission is formulated to achieve same sidelobe level at each communication receiver.

2.3. PSK-based Schemes

The fundamental principle underlying PSK-based DFRC is to embed communication information by controlling the phase of the signals transmitted towards the communication receivers while keeping the amplitude levels constant in the direction of communication [19, 24–26]. This is achieved by exploiting the radiation pattern invariance property of ULAs which states that for each of the possible transmit radiation patterns, there exists a set of beamforming weight vectors $\mathbf{W}_r = [\mathbf{w}_1, \mathbf{w}_2, \cdots, \mathbf{w}_{2^{M-1}}]$ such that each member of $\mathbf{W}_r$ provides exactly the same radiation pattern but exhibits different phase profile. The complete set $\mathbf{W}_r$ can be determined if any of the beamforming vectors presented in $\mathbf{W}_r$ is known [32]. The transmitted signal for PSK-based DFRC can be expressed as [19, 24, 25]:

$$\mathbf{s}(t,\tau) = \sqrt{\frac{P}{K}} \sum_{k=1}^{K} \mathbf{W}_r^* \mathbf{b}_k(\tau) \psi_k(t), \quad K \leq \hat{K}, \tag{6}$$

where $\mathbf{b}_k(\tau)$ is a binary vector of size $(2^{M-1} \times 1)$ such that all of its elements are zero except one element which is equal to 1. The vector $\mathbf{b}_k(\tau)$ is responsible to select the desired beamforming vector $\mathbf{w}_k$ from $\mathbf{W}_r$. Here, $\mathbf{w}_1$ is calculated by solving the optimization problem in Eq. (2) and rest of the beamforming vectors in $\mathbf{W}_r$ are calculated using the method developed in [32]. If coherent communication is considered, each radar pulse consists of K orthogonal waveforms transmitted towards communication receivers with the embedded phase information given as:

$$\phi_k = \angle \left\{ \mathbf{w}_k^{\mathrm{H}} \mathbf{a}(\theta_r) \right\}, \quad 1 \leq k \leq K, \tag{7}$$

where $\angle\{\cdot\}$ denotes the angle of a complex number. For non-coherent communications, we select $\psi_1(t)$ as the reference waveform transmitted using the reference beamforming weight vector $\mathbf{w}_1$ by setting $\mathbf{b}_1(\tau) = [1, \quad \mathbf{0}_{2^{M-1}-1}]^{\mathrm{T}}$ where $(\cdot)^{\mathrm{T}}$ is the transpose operator and $\mathbf{0}_{2^{M-1}-1}$ is a row vector of all zeros having the length of $2^{M-1}-1$. In this case, each radar pulse projects the set of $(K-1)$ phase rotations towards the communication receivers. The phase ϕ_k corresponding to the k-th waveform can be calculated as:

$$\phi_k = \angle\left\{\frac{\mathbf{w}_k^{\mathrm{H}}\mathbf{a}(\theta_r)}{\mathbf{w}_1^{\mathrm{H}}\mathbf{a}(\theta_r)}\right\}, \quad 2 \leq k \leq K. \tag{8}$$

3. Proposed QAM-based Sidelobe Modulation

3.1. Signaling Strategy

We have observed that the existing ASK-based DFRC approaches [19–23] can only broadcast the same information to all the communication receivers by controlling the sidelobe levels during each radar pulse. Since the sidelobe level towards all the communication directions is same, it is impossible to enable multi-user access, i.e., transmission of different information to the receivers located in different directions is not feasible. On the other hand, the PSK-based DFRC strategy can be enabled to transmit distinct communication streams to different users; however, the use of radiation pattern invariance property [32] is restricted to ULAs only. Moreover, PSK-based schemes cannot exploit the diversity achieved by the ASK principle, restricting the amount of data which can be transmitted. Nevertheless, it is possible to increase the maximum possible data rate by varying sidelobe levels for different communication receivers and exploiting an efficient PSK-based DFRC strategy. However, inability of ASK-based schemes to transmit different communication streams in different directions makes the problem much more cumbersome.

In the proposed approach, a DFRC system consisting of an arbitrary linear transmit antenna array can serve different communication receivers located in the sidelobe region with different communication information by exploiting

QAM-based communication, i.e., utilizing both amplitude and phase information simultaneously. The proposed DFRC system is powered with two degrees-of-freedom. First, the information embedding exploits different amplitude levels to feed distinct communication streams to multiple communication users located in different directions. Such distinct information transmission is made possible by exploiting different sidelobe levels in different directions simultaneously during each radar pulse. These sidelobe levels are kept constant during each radar pulse which constitutes the symbol period. Second, the DFRC system can transmit the signals with different phase differences in different directions, thus providing an extra degree-of-freedom in addition to the provision of multiple-level sidelobes. The proposed scheme is analogous to QAM-based communication system as it exploits amplitude as well as the phase shift keying to enable the information embedding using multiple radar waveforms. Moreover, we will show that the proposed scheme serves as the generalized mathematical formulation of the existing DFRC techniques [19–28]. Fig. 2 illustrates the basic principle of the proposed approach.

The beamforming weight vectors for the proposed QAM-based communication scheme can be extracted by solving the following optimization:

$$\begin{aligned} &\min_{\mathbf{w}_n} \max_{\theta_m} \left| e^{j\varphi(\theta_m)} - \mathbf{w}_n^{\mathrm{H}} \mathbf{a}(\theta_m) \right|, \quad \theta_m \in \Theta, \\ &\text{subject to } \left| \mathbf{w}_n^{\mathrm{H}} \mathbf{a}(\theta_p) \right| \le \varepsilon, \quad \theta_p \in \bar{\Theta}, \\ &\qquad \mathbf{w}_n^{\mathrm{H}} \mathbf{a}(\theta_r) = \Delta_n(\theta_r) e^{j\phi_n(\theta_r)}, \quad 1 \le r \le R, 1 \le n \le N. \end{aligned} \tag{9}$$

Here, $\mathbf{w}_n$ is the n-th beamforming weight vector resulting in the sidelobe level $\Delta_n(\theta_r)$ and the phase $e^{j\phi_n(\theta_r)}$ towards the r-th communication receiver located at θ_r. Note that the sidelobe levels $\Delta_n(\cdot)$ and the projected phases $e^{j\phi_n(\cdot)}$ are the function of angle θ_r of the communication receivers. Each sidelobe level term $\Delta_n(\theta_r)$ can take any of the L allowable sidelobe levels and each phase term $e^{j\phi_n(\theta_r)}$ can take any of the Q allowable phases. Note that each beamforming weight vector $\mathbf{w}_n$ in Eq. (9) is constructed using R communication constraints, each constraint corresponds to the desired sidelobe level and phase towards the communication receiver located at different angles θ_r $(1 \le r \le R)$.

Note that the possibility of having L unique sidelobe levels and Q unique phases results in LQ unique possibilities for the term $\Delta_n(\theta_r)e^{j\phi_n(\theta_r)}$ towards θ_r. However, each of the R communication constraints selects only one distinct value of $\Delta_n(\theta_r)e^{j\phi_n(\theta_r)}$ out of the LQ possible values for evaluating the beamforming weight vectors. Note that the values of $\Delta_n(\theta_r)e^{j\phi_n(\theta_r)}$ can be different in different directions θ_r which enables multi-user access, i.e. ensures independent communication streams towards different directions. Since there are LQ possible values of $\Delta_n(\theta_r)e^{j\phi_n(\theta_r)}$ at each communication receiver, there will be a total of $(LQ)^R$ possible communication constraints for R users. Each beamforming weight vector $\mathbf{w}_n$ is designed by selecting R constraints out of the total $(LQ)^R$ constraints. Therefore, we can generate $N = (LQ)^R$ unique beamforming weight vectors using Eq. (9) such that each beamforming weight vector projects a unique set of R QAM symbols projected towards the communication directions. Thus, the optimization problem needs to be solved $(LQ)^R$ times to generate all the possible beamforming weight vectors. The desired beamforming weight vector corresponding to the required amplitude levels and phases towards communication directions can be selected from the set of $(LQ)^R$ beamforming weight vectors for transmitting information. Note that, it is possible to transmit radar waveforms with the same (broadcast case) or different (multi-user access case) information by respectively choosing the same or different sidelobe levels and phases at all the communication receivers simultaneously.

The signal transmitted from the DFRC antenna array is expressed as:

$$\mathbf{s}(t,\tau) = \sqrt{\frac{P}{K}} \sum_{k=1}^{K} \mathbf{W}^* \mathbf{b}_k(\tau) \psi_k(t), \tag{10}$$

where $\mathbf{W} = [\mathbf{w}_1, \mathbf{w}_2, \cdots, \mathbf{w}_N]$ is an $M \times N$ matrix which serves as the dictionary of N beamforming weight vectors optimized using Eq. (9) and $\mathbf{b}_k(\tau) = [b_{1,k}(\tau), b_{2,k}(\tau), \cdots, b_{N,k}(\tau)]^{\mathrm{T}}$ is an $N \times 1$ binary selection vector selecting the desired beamforming weight vector from the dictionary $\mathbf{W}$ for each transmitted waveform $\psi_k(t)$. All the elements in $\mathbf{b}_k(\tau)$ are 0 except only one element which is equal to 1. We utilize $K(\leq \hat{K})$ orthogonal waveforms during each radar pulse and it is possible to use different values of K for each pulse. Note that the individual sidelobe levels and phases towards each communication receiver obtained by the weight vectors present in $\mathbf{W}$ are not the same, thus enabling us to transmit distinct QAM-based communication streams towards each communication receiver. Since the transmission scheme allows L unique sidelobe levels and Q unique phases at each receiver, each transmitted waveform carries $\log_2(LQ)$ bits of distinct information for each receiver. The proposed signaling strategy is outlined in Fig. 3.

The transmitted signal $\mathbf{s}(t,\tau)$ in (10) can be rewritten in a compact form as:

$$\mathbf{s}\left(t,\tau\right)=\sqrt{\frac{P}{K}}\mathbf{W}\mathbf{B}(\tau)\boldsymbol{\psi}\left(t\right), \tag{11}$$

where

$$\begin{aligned}\mathbf{B}(\tau)&=\left[\begin{array}{c}\mathbf{b}_1(\tau),\mathbf{b}_2(\tau),\cdots,\mathbf{b}_K(\tau)\end{array}\right],\\ \boldsymbol{\psi}\left(t\right)&=\left[\begin{array}{c}\psi_1(t),\psi_2(t),\cdots,\psi_K(t)\end{array}\right]^{\mathrm{T}}.\end{aligned} \tag{12}$$

For the case of coherent communications where carrier synchronization is not an issue, the transmitted information in the direction θ_r can be expressed as:

$$G_{\mathrm{T}}\left(\theta_r\right)=\mathbf{w}_n^{\mathrm{H}}\mathbf{a}(\theta_r)=\Delta_n(\theta_r)e^{j\phi_n(\theta_r)},\quad 1\leq n\leq N, \tag{13}$$

where $\Delta_n(\theta_r)$ and $e^{j\phi_n(\theta_r)}$ vary with respect to θ_r.

For the case of non-coherent communications where it is difficult to achieve carrier synchronization, we can exploit a reference waveform $\psi_1(t)$ along with a reference beamforming vector $\mathbf{w}_1$. In this way, the transmitted information in the direction θ_r can be expressed as:

$$G_{\mathrm{T}}\left(\theta_r\right)=\frac{\mathbf{w}_n^{\mathrm{H}}\mathbf{a}(\theta_r)}{\mathbf{w}_1^{\mathrm{H}}\mathbf{a}(\theta_r)},\quad 2\leq n\leq N. \tag{14}$$

Note in Eqs. (13) and (14) that $\mathbf{w}_n$ varies with τ depending on the value of $\mathbf{b}_k(\tau)$ depicted in Eq. (10), and $G_{\mathrm{T}}(\theta_r)$ provides an estimate of the transmitted QAM symbol where $|G_{\mathrm{T}}(\theta_r)|$ represents the amplitude component and $\angle\{G_{\mathrm{T}}(\theta_r)\}$ determines the phase component in the communication direction θ_r.

3.2. Information Decoding at Communication Receivers

The signal received at the r-th communication receiver located in the sidelobe region at angle θ_r can be described as:

$$x_r(t,\tau) = \alpha_r(\tau)\mathbf{a}^{\mathrm{T}}(\theta_r)\mathbf{s}(\mathbf{t},\tau) + n(t), \tag{15}$$

where $\alpha_r(\tau)$ is the complex channel response which is considered constant during each radar pulse, and $n(t)$ is the zero-mean complex white Gaussian noise. Matched filtering the received signal $x_r(t,\tau)$ in (15) to each of the $\hat{K}$ possible waveforms at the communication r-th receiver yields the following scalar output:

$$\begin{aligned} y_{r,k}(\tau) &= \frac{1}{T}\int_0^T x_r\left(t,\tau\right)\psi_k\left(t\right)dt \\ &= \begin{cases} \sqrt{\frac{P}{K}}\alpha_{ch}(\tau)\Delta_n(\theta_r)e^{j\phi_n(\theta_r)} + n_k(\tau), & \text{if } \psi_k(t) \text{ was transmitted,} \\ n_k(\tau), & \text{otherwise.} \end{cases} \end{aligned} \tag{16}$$

Here, $n_k(\tau)$ is the zero mean complex white Gaussian noise at the output of the matched filter. By analyzing $y_{r,k}(\tau)$ at the r-th receiver using all radar waveforms $\psi_k(t)$, $1 \leq k \leq \hat{K}$, it is possible to determine the transmitted sidelobe levels $\Delta_n(\theta_r)$ and phases $e^{j\phi_n(\theta_r)}$ which decodes the embedded communication information.

For the case of coherent communication, the receiver at θ_r determines the transmitted QAM signals during each radar pulse as:

$$G_{\mathrm{R}}(\theta_r) = y_{r,k}(\tau). \tag{17}$$

For the case of non-coherent communication, $y_{r,1}(\tau)$ is considered as the reference, and the receiver determines the transmitted QAM signals during each radar pulse as:

$$G_{\mathrm{R}}(\theta_r) = \frac{y_{r,k}(\tau)}{y_{r,1}(\tau)}. \tag{18}$$

In Eqs. (17) and (18), $G_{\mathrm{R}}(\theta_r)$ denotes the received QAM communication symbol having magnitude $|G_{\mathrm{R}}(\theta_r)|$ and phase $\angle\{G_{\mathrm{R}}(\theta_r)\}$ at the receiver located at angle θ_r.

We can observe that the proposed multi-waveform sidelobe QAM-based signaling strategy treats any of the existing DFRC techniques discussed in [19–28, 31] as special cases. Table I shows the parameters which can be changed in Eq. (10) to yield these existing DFRC signaling methods. This implies that the proposed signaling scheme represents a generalized mathematical framework of existing DFRC schemes [19–28, 31].

The following proposition addresses the number of users which can be supported by the proposed DFRC technique.

Proposition 1: For the DFRC system consisting of an M-element ULA, the number of maximum possible supportable communication users located in unique directions is $M-1$.

Proof: For the case of ULA, we can consider the beamforming weight vector in Eq. (9) as a polynomial of degree $M-1$ which can have a maximum $M-1$ number of unique roots. If all these $M-1$ roots correspond to the equality constraints in Eq. (9), it may still be possible to solve for other constraints without losing any degrees-of-freedom in ideal cases. However, some of the degrees-of-freedom might be utilized to satisfy the inequality constraint and the minimization function in Eq. (9). Thus, the maximum possible number of supported communication users located in distinct directions for ULA is $M-1$.

3.3. Sum Data Rate Analysis

In this section, we evaluate the sum of the number of bits which can be transmitted during one radar pulse using the proposed QAM-based DFRC technique to the sidelobe communication receivers located at distinct angles. We consider R receivers utilizing $L(\geq 1)$ sidelobe levels and $Q(\geq 1)$ phase constellations with $LQ > 1$ such that $K \leq \hat{K}$ fixed number of orthogonal waveforms are used during each radar pulse. The following proposition addresses the achievable data rate.

Proposition 2: The maximum data rate achieved from the proposed QAM-based DFRC strategy is $RK\log_2(LQ)$ for coherent communication.

Proof: According to Eq. (10), for each radar waveform $\psi_k(t)$, we can transmit L possible sidelobe levels and Q distinct phases to each communication receiver. Thus, each communication receiver deciphers $\log_2(LQ)$ bits of distinct information during each radar pulse for each of the K transmitted radar waveforms. Therefore, the sum data rate for all the R communication receivers becomes $RK\log_2(LQ)$ for the case of coherent communications. Similarly, the sum data rate offered by the proposed strategy for non-coherent communication is $R(K-1)\log_2(LQ)$ if one waveform is used as the reference waveform.

The achievable data rate offered by the proposed QAM-based information embedding strategy is compared with existing DFRC methods [19–28, 31] in Table I. It is evident that the proposed technique outperforms existing approaches in terms of overall throughput or sum data rate because it is benefited from multiple sidelobe levels and phase possibilities simultaneously at different communication receivers. Moreover, the ability to transmit different information streams (sidelobe levels and phases) to different users further enhances the maximum achievable data rate.

4. Simulation Results

In this section, we present simulation results to illustrate the performance of the proposed QAM-based DFRC strategy. In all simulations, we consider a ULA of 10 transmit antennas to serve two communication users located in the sidelobe region of the radar.

4.1. Example 1: Beampattern synthesis and data rate analysis for equal number of sidelobe levels and phase constellations ($R = L = Q = 2$)

We consider that the DFRC system is capable of projecting two sidelobe levels and can transmit the waveforms with two different phases towards the communication receivers located in the sidelobe region at angles 35^{o} and 40^{o}, respectively. Here, the primary function of the radar is to project the main beam at 0^o. For the communication purpose, ASK and PSK-based schemes will have the ability to exploit only the magnitude or phase variation, respectively. In contrast, the QAM-based scheme can utilize the variation in both magnitude as well as the phase of the transmitted waveform.

Fig. 4(a) shows the transmit power pattern corresponding to two beamforming weight vectors for multi-waveform ASK [21, 22]. These beamforming vectors respectively broadcast the amplitude of either -30 dB or -40 dB in the directions of communication receivers. The sidelobe level at both communication receivers also remains identical for the existing ASK-based schemes [21, 22] during each radar pulse. For PSK-based method [24–26], two beamforming vectors are generated to have the same magnitude response but different phase response in the directions of communication receivers. Fig. 4(b) shows the power pattern of these beamforming weight vectors projecting a sidelobe level of -32.61 dB at both communication receivers. Each of the two beamforming vectors for the PSK-based method broadcast a unique phase towards all the communication receivers during each radar pulse. Just like ASK-based methods, the transmitted information is broadcast to all the communication users.

Unlike the ASK-based technique, the QAM-based method can assign different sidelobe levels at the two communication receivers. Moreover, in contrast to the existing PSK-based strategy, the proposed QAM-based technique can transmit different phases to different receivers at the same time. Thus, the QAM-based strategy can independently project two different levels of amplitudes as well as phases at the two receivers so as to transmit distinct information to each user. We can generate 16 beamforming weight vectors for $R = L = Q = 2$ using Eq. (9). Fig. 4(c) shows the four possible power patterns for the proposed QAM-based scheme generated using Eq. (9). Since the number of possible transmitted phases towards each receiver are 2, each beampattern for the QAM-based scheme corresponds to four distinct beamforming weight vectors projecting the same magnitude but different phase responses towards the two communication receivers.

Let us examine the achievable throughput (sum data rate) for this experiment. Note that the average power transmitted to each communication receiver is kept constant for all the three schemes. For a single radar waveform, the ASK-based method exploiting two sidelobe levels will be able to transmit (broadcast) $\log_2 L = 1$ bit per waveform for each radar pulse. Coherent PSK-based method utilizing two phase options will also be able to broadcast $\log_2 Q = 1$ bit per radar waveform for each radar pulse. On the other hand, the coherent QAM-based information embedding strategy is able to transmit 2 bits per user resulting in a total throughput of $R \log_2(LQ) = 4$ bits transmitted using each radar waveform during each radar pulse. For two waveforms, this data rate for all the coherent schemes will double. The non-coherent schemes will utilize one waveform as the reference and one waveform will be used for communication purpose; therefore, the data rate of coherent scheme will be half compared to the coherent counterpart if two orthogonal waveforms are utilized. Fig. 5 shows the maximum throughput with respect to the number of available waveforms for this simulation parameters ($R = L = Q = 2$). The simulation results clearly illustrate the superiority of our proposed technique in terms of sum data rate.

4.2. Example 2: Bit error rate comparison for same overall throughput (sum data rate)

In the second example, we investigate the possibility of synthesizing beamforming weight vectors for the case of extended radar's main beam and compare the bit error rate (BER) for the DFRC strategies by keeping the sum data rate and transmitted power constant. Two communication users ($R = 2$) are considered at 35^o and 42^o with respect to the DFRC transmit antenna array. The desired main lobe region is from -10^o to 10^o and two orthogonal waveforms ($K = 2$) are available to the DFRC system. The objective is to transmit 4 bits of information to each communication user (a total throughput of 8 bits). For the case of multi-waveform ASK [21, 22], we have to design 16 beamforming weight vectors having sidelobe levels uniformly distributed from 0 to 0.1 towards the intended communication directions ($L = 16$, $K \log_2 L = 8$ bits throughput).

For the PSK-based technique [25, 26], 16 beamforming vectors having sidelobe level of 0.2347 towards the communication users are synthesized such that the respective phases of these beamforming vectors at the receivers is uniformly distributed from 0^o to 360^o ($Q = 16, K \log_2 Q = 8$ bits throughput). Since there are only two available waveforms, the non-coherent PSK will have 32 beamforming weight vectors to match the data rate because one waveform will be used as a reference to realize non-coherent communications ($Q = 32, (K-1)\log_2 Q = 8$). For the case of the proposed coherent QAM-based sidelobe modulation , we have generated 16 beamforming vectors using Eq. (9) such that there are a total number of four groups of QAM-based beamforming weight vectors corresponding to the same amplitude response. Each group comprises four beamforming weight vectors corresponding to unique phase combinations at the both communication receivers. We have the ability to transmit waveforms towards the communication receivers with two distinct power levels and four phase possibilities. In a similar manner, the non-coherent QAM-based strategy will result in the generation of 32 beampatterns such that we have the ability to project the radar waveforms at two power levels and four unique phase constellations at each communication receiver. The beampatterns for the designed beamforming vectors for the case of ASK, PSK and QAM-based information embedding is shown in Fig. 6. We observe that the multi-waveform ASK and all the coherent communication based algorithms result in an equal number of beamforming weight vectors. Moreover, non-coherent based DFRC techniques also have the same number of beamforming vectors.

Fig. 7 compares the BER performance of the proposed technique with existing techniques. For error reduction, all the symbols for the DFRC transmission schemes are grey coded before transmission. We observe that the proposed QAM-based method is more capable to combat noise compared to existing DFRC techniques. This is because the proposed QAM-based method is designed to offer a higher throughput with the same resources and, therefore, results in increased distance between the signals in the symbol space. The QAM symbol space in this simulation contains 8 symbols compared to 16 symbols each for

PSK and ASK, respectively. In addition to this, 8-QAM symbols are distributed as two levels of amplitudes and 4 levels of phases which further increases the effective distance between the transmitted symbol constellations. This increased distance between the symbol constellations along with the flexibility to transmit different symbols to different users results in reduced BER for the proposed QAM-based technique. Moreover, the coherent communication based methods have better performance compared to the non-coherent counterparts because the number of symbols in the symbol space is increased for the latter, resulting in higher error rates. However, the proposed non-coherent QAM-based information embedding strategy still outperforms the existing methods in terms of the BER.

In short, the simulation results illustrate the effectiveness of the proposed QAM-based information embedding in comparison to the existing schemes in terms of BER and the overall throughput of the communication system.

5. Conclusion

In this paper, a novel multi-waveform QAM-based scheme exploiting sidelobe control and waveform diversity was proposed, which enables the transmission of distinct communication information to the communication receivers located in distinct directions. The information embedding has been discussed for coherent as well as non-coherent communication cases. Compared to the existing ASK and PSK-based signaling strategies, the proposed method achieves higher transmission capacity by simultaneously enabling both amplitude and phase shift keying in the radar sidelobe region. Moreover, the ability to transmit distinct information in different directions enhances the sum data rate by a factor of number of communication users. As such, the proposed signaling strategy also serves as a generalized mathematical framework of existing DFRC strategies. Simulation results evidently verify the effectiveness of the proposed scheme.

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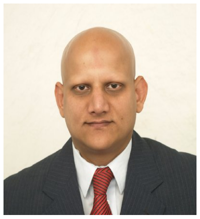

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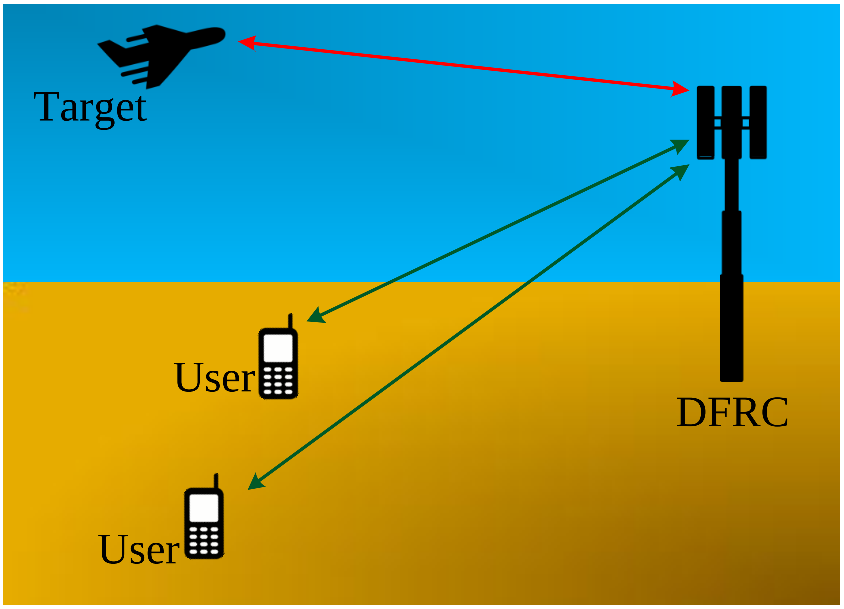


Figure 1: The principle of dual-function radar-communications.

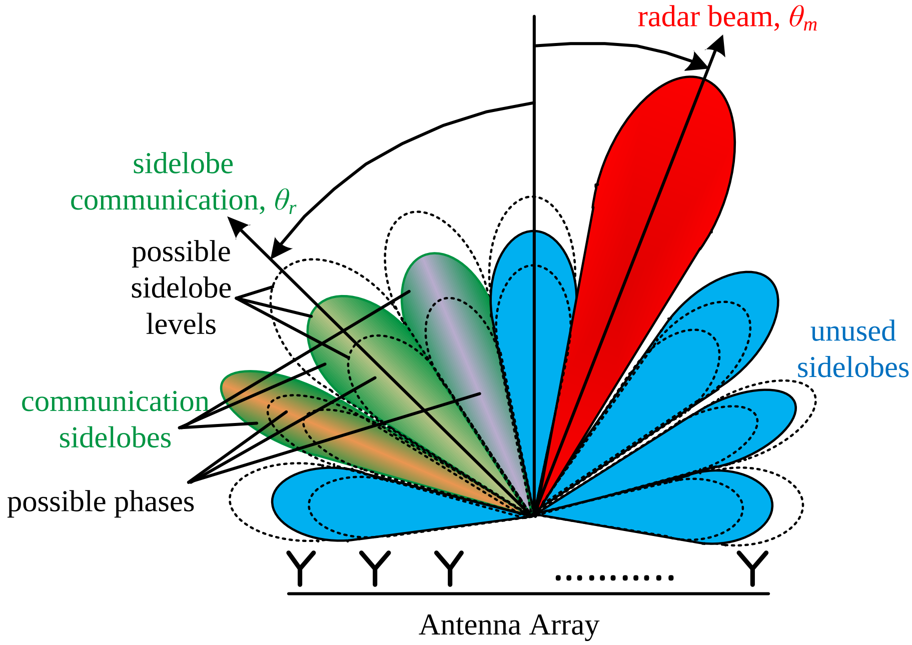


Figure 2: The proposed DFRC strategy using QAM-based sidelobe modulation.

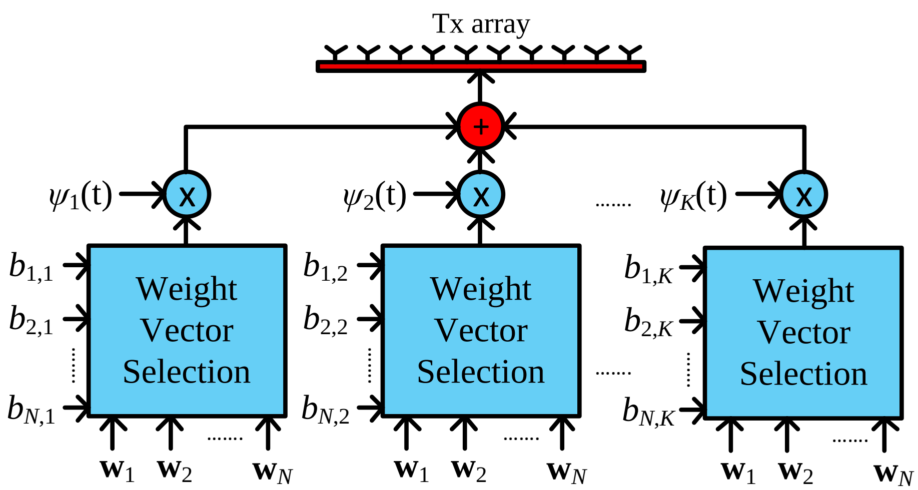
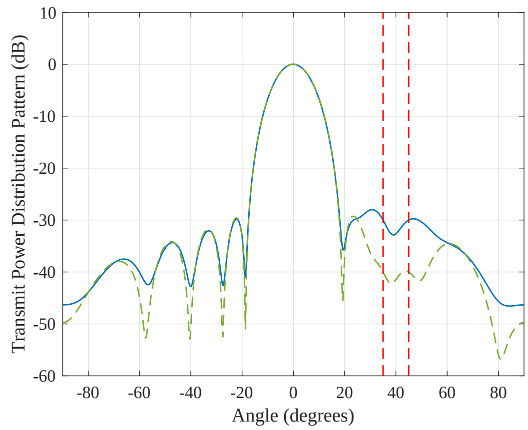


Figure 3: The proposed QAM-based DFRC approach using QAM-based sidelobe modulation (coefficients $b_{n,k}$ are the function of τ).

Table 1: Generalization of existing DFRC strategies by the proposed technique

Signaling Strategy	**Parameters for Eq. (9)-(10)**	**Maximum data rate (bits/pulse)**
Sidelobe AM [20]	$K = 1, N = L \geq 2,$ $Q = 1, R = 1.$	$\log_2 L$
Multiwaveform ASK [21, 22]	$N = L = 2, Q = 1,$ fixed $K(\leq \hat{K})$, varying R, $\mathbf{u}_1 = \mathbf{u}_{\text{low}}, \mathbf{u}_2 = \mathbf{u}_{\text{high}},$ $b_{1,k}(\tau) = 1 - b_{2,k}(\tau)$	$K \log_2 L$
Multi-waveform single-level ASK[23]	$N = L = 1, Q = 1,$ $\mathbf{u}_1 = \mathbf{u}_{\text{high}},$ varying K and R, For all zeros: $K = 1, b_{1,K} = 1$ else: $b_{1,1} = 0$, $b_{1,k} = 0$ or 1, $\sum_{k=2}^{\hat{K}} b_{1,k} = K.$	$K \leq \hat{K}$
Multi-user ASK [31]	varying $N, L, R, K,$ $Q = 1$, $1 < K \leq \hat{K}$	$RK \log_2 L$
Multi-waveform coherent PSK [25, 26]	$N = L = 1, \mathbf{u}_1 = \mathbf{u}_{\text{high}},$ fixed $K(\leq \hat{K})$, varying R and $Q(\geq 2)$.	$K \log_2 Q$
Multi-waveform non-coherent PSK [25, 26]	$N = L = 1, \mathbf{u}_1 = \mathbf{u}_{\text{high}},$ fixed $K(2 \leq K \leq \hat{K})$, varying R and $Q(\geq 2)$	$(K - 1) \log_2 Q$
Coherent Multi-user QAM (proposed)	varying N, L, R, Q, K	$RK \log_2 LQ$
Non-coherent Multi-user QAM (proposed)	varying $N, L, R, Q, K,$ $1 < K \leq \hat{K}$	$R(K - 1) \log_2 LQ$



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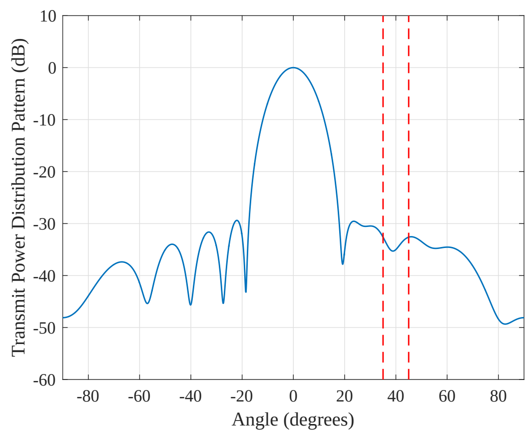

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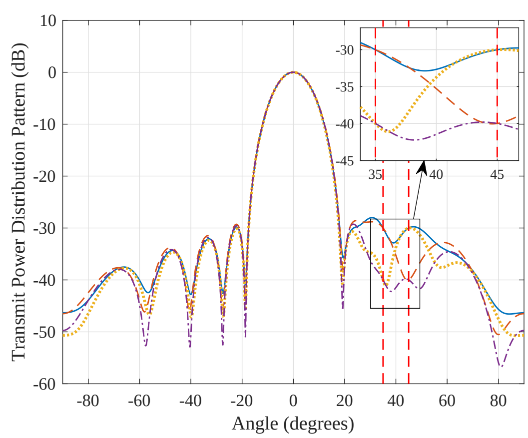


(c)

Figure 4: Beampatterns for DFRC strategies ($R = Q = L = 2$): (a) Multi-waveform ASK-based method [21, 22], (b) Multi-waveform PSK-based method [24–26], (c) Proposed QAM-based method.

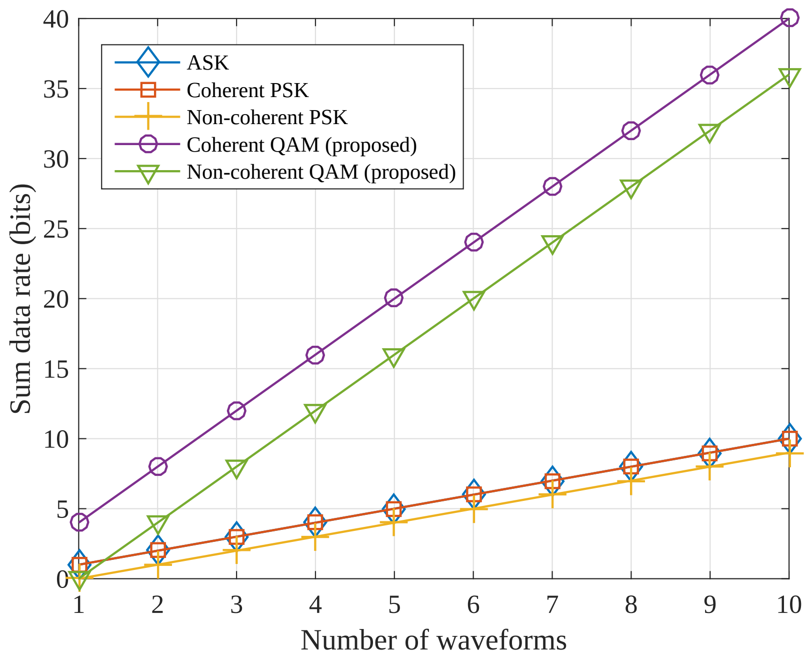
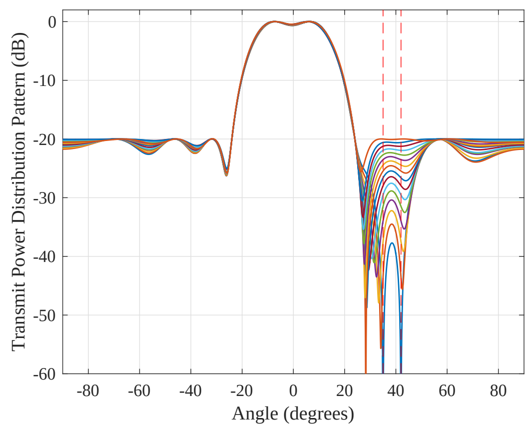
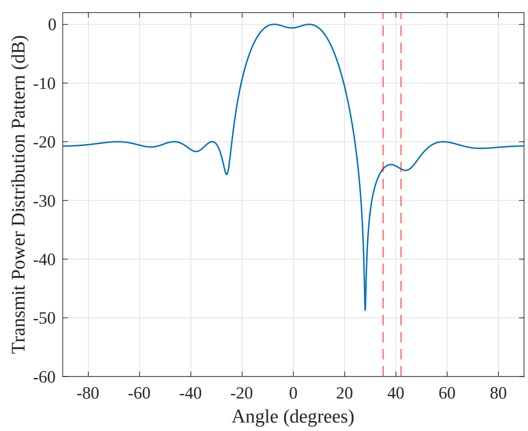


Figure 5: The comparison of throughput for the case of $R = L = Q = 2$ and varying number of available waveforms.



(a)



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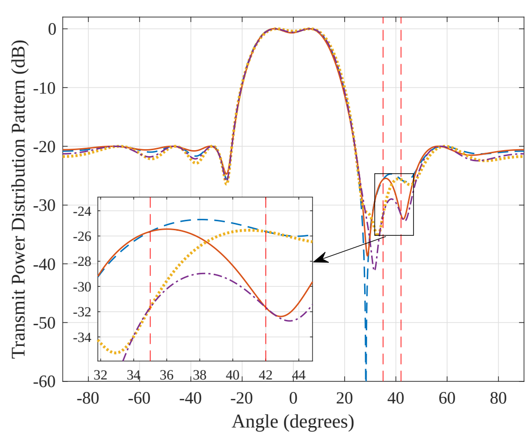

(c)

Figure 6: Beampatterns for DFRC strategies. (a) Multi-waveform ASK-based method [21, 22], (b) Multi-waveform PSK-based method [24–26], (c) Proposed QAM-based method

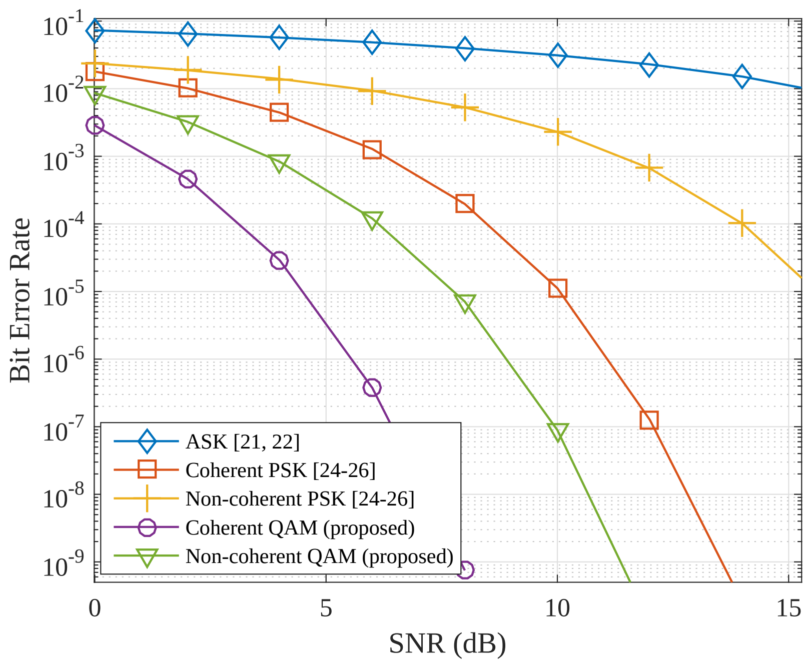

Figure 7: Bit error rate comparison for the multi-waveform ASK-based method [21, 22], multi-waveform PSK-based method (coherent and non-coherent case) [24–26] and the proposed QAM-based method (coherent and non-coherent case).