

On Unscented Kalman Filter for NeQuick-G Based Ionosphere Estimation

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Abstract --- Ionospheric delay significantly affects the accuracy of positioning applications, posing a challenge due to the nonhomogeneous electron densities and magnetic fields that characterize the global ionosphere. To address this issue, researchers have recently introduced ionospheric correction models aimed at effectively mitigating ionospheric delay. One such model is NeQuick, which offers a comprehensive 3-D representation of electron density over time, as well as the longitudes, latitudes, and heights of both the satellite transmitter and ground receiver. NeQuick-G relies on three ionospheric coefficients, which are transmitted by Galileo Satellites. These coefficients are optimized for all Galileo sensor stations worldwide, making them less than optimal for local users. Moreover, the coefficient updates are not immediate. To address these limitations, an unscented Kalman filter (UKF) for tracking the three ionospheric coefficients is proposed in this paper. This is achieved by utilizing four local reference emitters and one Low Earth Orbit (LEO) satellite, with the objective of passively geolocating ground-based electromagnetic interference (EMI) sources. The accurate and real-time estimation of the ionosphere provided by the UKF will significantly enhance geolocation accuracy. In the design, the satellite and four ground reference emitters, strategically deployed around the estimated EMI position, are used to measure the ionosphere. The UKF tracks the ionospheric coefficients, updating them in real time and optimizing them specifically for the local region where the EMI is located. These precise values are then employed in the NeQuick-G model to estimate the ionosphere along the path from the EMI source to the individual satellite. Numerical results validate the effectiveness of the proposed approach, combining UKF-enabled NeQuick-G for ionosphere estimation and the subsequent enhancement of single satellite geolocation (SSG) accuracy.

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1. INTRODUCTION

Interference of satellite communications is a frequent and ongoing concern for both DoD and civilian enterprises. Satellite communications are facing increasingly diverse physical and electromagnetic interference (EMI) that transmit radio frequency (RF) signals in X/Ku/K/Ka/Q-bands. Therefore, RF emitter detection and localization is a key enabler for reliable space control, space situational awareness, intelligence surveillance and reconnaissance, as well as satellite communications together with positioning, navigation and timing. Geolocation [1] of the interfering source is an essential step in mitigating or eliminating the interference and restoring operation of the communication services. The DDDAS program (which combines the theoretical simulations with real-time data monitoring) looked into variations of the ionospheric models in support of SSA drag, as highlighted by the DDDAS methods [2]-[5].

Ionospheric delay [6] significantly affects the accuracy of positioning applications, posing a challenge due to the nonhomogeneous electron densities and magnetic fields that characterize the global ionosphere. To address this issue, researchers have recently introduced ionospheric correction models aimed at effectively mitigating ionospheric delay. One such model is NeQuick [7], which offers a comprehensive 3-D representation of electron density over time, as well as the longitudes, latitudes, and heights of both the satellite transmitter and ground receiver. NeQuick finds extensive application in various fields, including Global Navigation Satellite Systems (GNSS) navigation, radio communication, and space weather research. The latest iteration, NeQuick-G [8], is an adaptation that caters to real-time users, utilizing the International Telecommunication Union (ITU)-R NeQuick ionospheric electron density model.

NeQuick-G relies on three ionospheric coefficients, which are transmitted by Galileo Satellites. These coefficients are optimized for all Galileo sensor stations worldwide, making them less than optimal for local users. Additionally, the coefficient updates are not immediate. To address these limitations, we propose the implementation of an unscented Kalman filter (UKF) for tracking the three ionospheric coefficients. This is achieved by utilizing four local reference emitters and one Low Earth Orbit (LEO) satellite, with the objective of passively geolocating ground-based EMI sources. The accurate and real-time estimation of the ionosphere provided by the UKF will significantly enhance geolocation accuracy.

In our design, the satellite and four ground reference emitters, strategically deployed around the estimated EMI position, are used to measure the ionosphere. The UKF tracks the ionospheric coefficients, updating them in real time and optimizing [9]-[12] them specifically for the local region where the EMI is located. These precise values are then employed in the NeQuick-G model to estimate the ionosphere along the path from the EMI source to the individual satellite.

The rest of the paper is organized as follows. Section 2 briefly review the NeQuick-G model. Section 3 presents the proposed constrained unscented Kalman filter framework for the locally optimized NeQuick-G. The simulation results and analysis are detailed in Section 4 and Section 5 draws the conclusions.

2. PROBLEM STATEMENT

3-D Ionospheric Model

Ionospheric delay is one of the dominant sources deteriorating the accuracy of positioning applications. Since the ionosphere is characterized by nonhomogeneous electron densities and magnetic fields on the global scale, real-time ionospheric correction models are recently proposed in order to effectively eliminate the ionospheric delay, e.g., [13][14]. In these models, the vertical total electron content (VTEC) is related to the parameters and is updated routinely. The slant total electron content (STEC) and the corresponding range delay are calculated by thin-shell approximation in multiplication with an elevation-dependent mapping function. However, the thin-layer approximation is no longer valid if the electron density profile fluctuates dramatically along the altitude [15]. This can be overcome by the NeQuick model, which can provide a real-time 3-D description of electron density as a function of time and the longitudes, latitudes, and heights of both the satellite transmitter and the ground receiver.

The NeQuick models were initially developed by the Institute of Meteorology and Geophysics of the University of Graz and the Abdus Salam International Centre for Theoretical Physics, Trieste [16][17]. The first version of the model, referred to as NeQuick 1, has been adopted by the ITU Radiocommunication Sector as a procedure for estimating

TEC (Recommendation ITU-R P.531) [18]. Subsequently, NeQuick 1 is refined as NeQuick 2 [19], which is currently recommended by ITU (Recommendation P.531-14; August 2019) [7], and is also included in the Space Environment Information System (SPENVIS) by the European Space Agency (ESA). More recently, the NeQuick model has been adapted to a Galileo specific model, NeQuick-G [8], which is demonstrated to be able to perform ionospheric correction of single-frequency observations from spaceborne applications [15].

The module of NeQuick-G is described in Annex F of [8] in detail. The model is summarized in Figure 1. Unlike the NeQuick1 and 2 models where the solar activity/solar flux is characterized by R12, the 12-month average sunspot number, or F10.7, solar radio flux at 10.7 cm wavelength, the NeQuick-G model uses the effective ionization level, denoted as

$$Az = a_0 + a_1 \mu + a_2 \mu^2 \quad (1)$$

where μ denotes the receiver's modified dip (MODIP) in degree, which is related to the geographic latitude and magnetic field at the receiver. NeQuick-G determines the MODIP value by interpolation of tabulated global grid of longitude/latitude points. Parameters a_0, a_1, a_2 denote the three ionospheric coefficients which are broadcast as part of the navigation message.

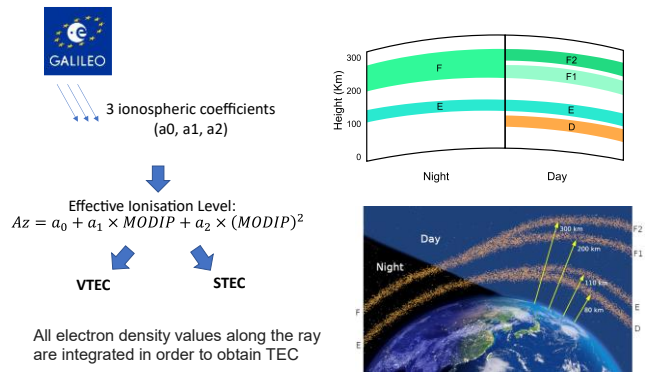


Figure 1. Summary of NeQuick-G

Based on the calculated Az values, NeQuick-G determines the STEC or VTEC values from the numerical integral $\int_{\text{path}} N d\ell$, where N is electron density, along the propagation path l between satellite transmitter and the ground receiver. Generally, the propagation path will go through several ionospheric layers as shown in Figure 1. Different layers have different parameter values to compute the N .

Problem Statement and Proposed Solution

NeQuick-G relies on three ionospheric coefficients, which are transmitted by Galileo Satellites. These coefficients are optimized for all Galileo sensor stations worldwide, making them less than optimal for local users.

Additionally, the coefficient updates are not frequently. The problem and proposed solution are depicted in Figure 2.

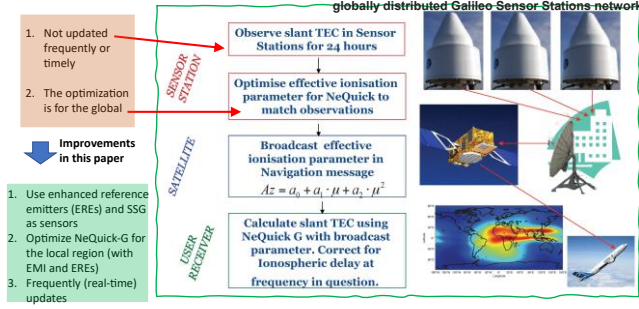


Figure 2. Problem Statement and the Proposed Solution

3. cUKF FOR NEQUICK-G

Nonlinear Filtering Problem

For a general nonlinear filtering problem [20], we have

$$\mathbf{x}_{k+1} = \mathbf{f}_k(\mathbf{x}_k, \mathbf{w}_k) \quad (2)$$

$$\mathbf{z}_k = \mathbf{h}_k(\mathbf{x}_k, \mathbf{v}_k) \quad (3)$$

$$\mathbf{0} = \mathbf{g}_k(\mathbf{x}_k) \quad (4)$$

where \mathbf{x}_k is the state vector ($L \times 1$) at time instant k , and \mathbf{z}_k is the measurement vector ($N \times 1$), \mathbf{f}_k and \mathbf{h}_k are the nonlinear functions, and \mathbf{w}_k and \mathbf{v}_k are independent white noise processes of the state and measurement equations, with zero mean and covariances \mathbf{Q}_k and \mathbf{V}_k , respectively. \mathbf{g}_k are the constraints of the system.

For the traditional Kalman filter, we assume the f (process) and h (measurement) are linear. The state variables are Gaussian random variables (GRVs). We know that a GRV put through a linear system is still a GRV. So, the Kalman filter is optimal for linear systems. For a nonlinear system, characterizing the resulting distribution of the propagated GRVs is non-trivial. For UKF, the resulting distribution is represented by a set of $2L+1$ deterministic sample points called **sigma** points.

In our NeQuick-G tracking problem, the state x_k is the ionospheric coefficients (a_0, a_1, a_2). For the feasibility research, we assume the ionospheric coefficients are constants during the tracking period (~500 seconds), $\mathbf{x}_{k+1} = \mathbf{x}_k$. \mathbf{h}_k is the NeQuick-G model of the STEC. \mathbf{g}_k is the condition such that \mathbf{x}_k should satisfy $A_z \leq 400$ as specified in [8].

We preferred a constrained UKF (cUKF) design for our problem based on the following observations:

- 1) cUKF is ideally suited for dealing with the nonlinear in the NeQuick-G measurement model;
- 2) The UKF provides increased modeling capabilities and robustness compared to the Nonlinear Least Squares (NLLS) and Extended Kalman Filter (EKF) approaches;

- 3) cUKF maintains fast computation capabilities and does not need the large number of samples that are required for the Particle Filter (PF) in order to map the nonlinear measurements;
- 4) Adding the $A_z \leq 400$ bound constraint provides faster convergence and greatly increases the search area and accuracy with a straightforward implementation within the UKF framework.

Measurement Model

Given locations of the ground transmitters (enhanced reference emitters, EREs) and a single satellite for EMI geolocation (SSG), Ne-Quick G model maps the states, ionospheric coefficients (a_0, a_1, a_2) to the ionospheric density values in STEC or VTEC.

The ionospheric density can be measured in real-time using ERE and SSG via the ionospheric delay I_f (at the frequency f), denoted by

$$I_f = \alpha_f \times STEC \quad (5)$$

where the values of STEC are in the unit of total electron content unit (TECU), $1 \text{ TECU} = 10^{16} \text{ electrons/m}^2$.

$$\alpha_f = \frac{40.3 \times 10^{16}}{f^2} \quad (6)$$

Sigma Points

The sigma points in the cUKF are calculated as

$$\chi_0 = \bar{\mathbf{x}}_{k-1} \quad (7)$$

$$\chi_i = \bar{\mathbf{x}}_{k-1} + \zeta(\sqrt{P_{x_{k-1}}})_i, \quad \text{for } i = 1, \dots, L \quad (8)$$

$$\chi_i = \bar{\mathbf{x}}_{k-1} - \zeta(\sqrt{P_{x_{k-1}}})_i, \quad \text{for } i = L + 1, \dots, 2L \quad (9)$$

where ζ is a scaling factor that determines the spread of the sigma points about the mean. These sigma points are then fed through the state and measurement equations, and the resulting distributions are approximated with weighted sample means and weighted sample covariances.

Time Updates

The time update equations are

$$\chi_{k/t}^x = f(\chi_t^x, \chi_t^w) \quad (10)$$

$$\hat{\mathbf{x}}_k^- = \sum_{i=0}^{2L} w_i^m \chi_{k/ti}^x \quad (11)$$

$$\hat{\mathbf{P}}_{x_k}^- = \sum_{i=0}^{2L} w_i^c (\chi_{k/ti}^x - \hat{\mathbf{x}}_k^-) (\chi_{k/ti}^x - \hat{\mathbf{x}}_k^-)^T + Q_k \quad (12)$$

where Q_k is the covariance of the process noise.

Measurement Updates

The measurement update equations are

$$\chi_{k/t}^z = h(\chi_t^x, \chi_t^v) \quad (13)$$

$$\hat{z}_k^- = \sum_{i=0}^{2L} w_i^m \chi_{k/ti}^z \quad (14)$$

$$\hat{P}_{z_k}^- = \sum_{i=0}^{2L} w_i^c (\chi_{k/ti}^z - \hat{z}_k^-) (\chi_{k/ti}^z - \hat{z}_k^-)^T + R_k \quad (15)$$

$$\hat{P}_{x_k z_k}^- = \sum_{i=0}^{2L} w_i^c (\chi_{k/ti}^x - x_k^-) (\chi_{k/ti}^z - \hat{z}_k^-)^T \quad (16)$$

where the weights can be specified as $w_0^m = \frac{\lambda}{L+\lambda}$, $w_0^c = \frac{\lambda}{L+\lambda} + (1 - \alpha^2 + \beta)$, and $w_i^m = w_i^c = \frac{1}{2(L+\lambda)}$ for $i = 1, 2, \dots, 2L$. The R_k is the covariance of the measurement noises. α , β , and λ are used to taper the spread of the sigma points to the prior mean.

Steps of cUKF for locally optimizing the NeQuick-G

The main steps of performing cUKF for NeQuick-G are listed as:

- 1) Calculate sigma points according to the initial conditions
- 2) Project sigma points that are not in the constrained solution space into the feasible region
- 3) Run projected sigma points through the time update equations
- 4) Project the state estimates outside the constrained solution space into the feasible region using the same algorithm in step 2
- 5) Run projected sigma points through measurement-update equations

4. NUMERICAL SIMULATIONS

CONOPS

The concept of operations (CONOPS) of demonstrating the cUKF for locally optimized NeQuick-G are summarized in Figure 3.

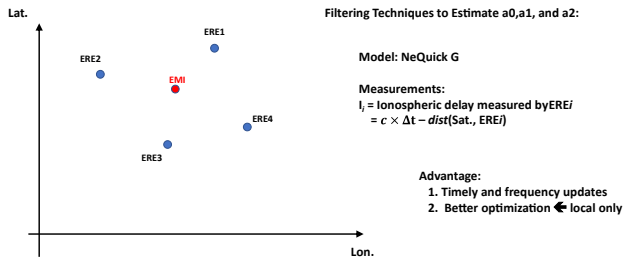


Figure 3. CONOPS for the cUKF for NeQuick-G

Scenario

A scenario we simulated is shown in Figure 4, where the orbit of the LEO satellite is propagated using SGP4 and a two-line-elements (TLE) file from space-track.org.

Satellite TLE:

```
1 04139U 00000 16171.89568986 .00000098 00000-0
25282-4 0 06
2 04139 74.0342 79.1740 0007229 258.8779 101.1575
14.62696590047575
```

EMI location: Latitude, longitude, and altitude (LLA)
Emitter_LLA = [39.18644159, -77.24952161, 10];

Location (LLA) of EREs:

ere1 = EMI + [0.9, 0.1, 0];
ere2 = EMI+ [-0.1, 0.9, 0];
ere3 = EMI+ [-0.9,-0.1, 0];
ere4 = EMI + [0.1,-0.9, 0];

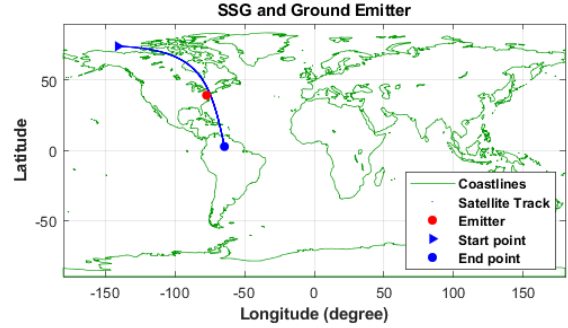


Figure 4. A scenario with a LEO satellite and a EMI emitter and 4 ERE emitters

Simulation Results

The tracked ionospheric coefficients are shown in Figure 5, where the black dashed lines are for the ground truth. The measured TEC and estimated TEC for the 4 EREs are shown in Figure 6.

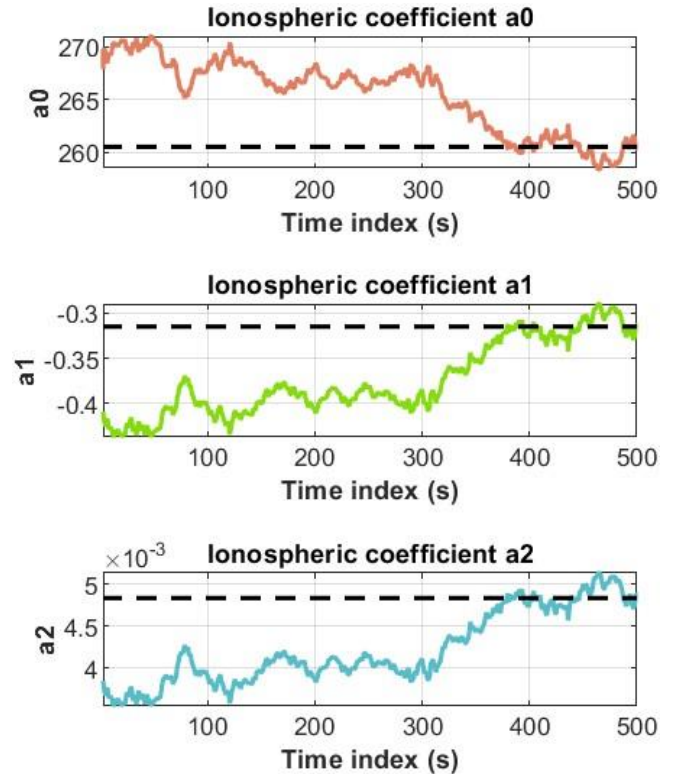


Figure 5. Estimated ionospheric coefficients (a0,a1,a2) from cUKF

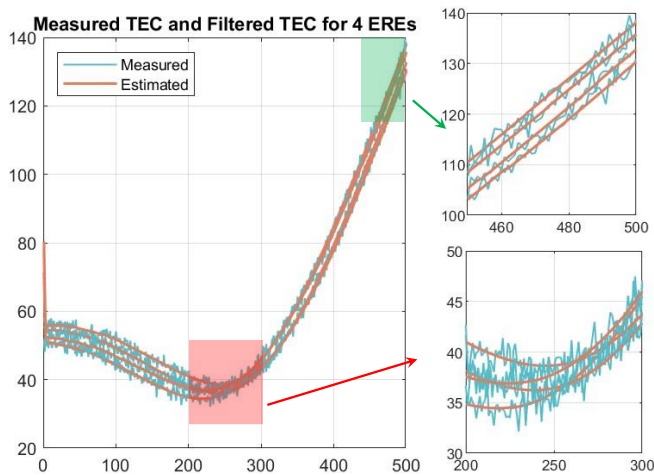


Figure 6. The measured TEC vs Estimated TEC for the four EREs

Using the cUKF and EREs-SSG, we can estimate the locally optimal ionospheric coefficients (a_0 , a_1 , a_2) and then quickly and accurately estimate the ionospheric delays for the potential EMI in the local region.

5. CONCLUSIONS

This paper introduces a locally optimized cUKF for refining the NeQuick-G model used in ionosphere estimation. The methodology employs a single satellite and strategically positioned ground reference emitters, encompassing the estimated EMI location, to measure the ionosphere. In real-time, the UKF continually updates the ionospheric coefficients and fine-tunes them specifically for the local area surrounding the EMI source. These precise coefficients are then integrated into the NeQuick-G model to calculate the ionosphere along the path from the EMI source to the lone satellite. To evaluate the practicality of this approach, a simulation of a real-world scenario is conducted. The numerical results affirm the efficacy of the proposed methodology, showcasing the synergy of cUKF-enabled NeQuick-G for ionosphere estimation and its subsequent impact on improving geolocation accuracy.

ACKNOWLEDGEMENTS

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REFERENCES

[1] Shen, D., Che, G., and Pham, K. "Passive Single Satellite Geolocation of Ground-Based EMI Sources," In *ION GNSS+* 2023.

[2] Morozov, Alexey V., et al. "Data assimilation and driver estimation for the Global Ionosphere-Thermosphere Model using the ensemble adjustment Kalman filter." *Journal of Atmospheric and Solar-Terrestrial Physics* 104 (2013): 126-136.

[3] Bernstein, Dennis S., et al. "Transformative advances in DDDAS with application to space weather monitoring." *AFOSR DDDAS Grant FA9550-12-1-0401 report* (2015).

[4] E. Blasch, "DDDAS Advantages from High-Dimensional Simulation," *IEEE Winter Simulation Conference (WSC)*, 2018

[5] Turner, Herbert, et al. "Machine learning algorithms for improved thermospheric density modeling." *Dynamic Data Driven Applications Systems: Third International Conference, DDDAS 2020*, Springer International Publishing, 2020.

[6] Ionospheric Delay, navipedia, accessed on Oct 2023. https://gssc.esa.int/navipedia/index.php/Ionospheric_Delay

[7] Recommendation ITU-R P. 531, "Ionospheric propagation data and prediction methods required for the design of satellite networks and systems", 2019.

[8] European GNSS (Galileo) Open Service, "Ionospheric Correction Algorithm for Galileo Single Frequency Users", *Issue 1.2, European Union* (2016).

[9] D. Shen, and J. B. Cruz. "An improved game theory based approach to one type of H-infinity optimal control problems." *2006 American Control Conference. IEEE*, 2006.

[10] D. Shen, et al. "An adaptive sequential game theoretic approach to coordinated mission planning for aerial platforms." *Advances in Cooperative Control and Optimization: the 7th International Conference on Cooperative Control and Optimization*. 2007.

[11] D. Shen, et al. "Game optimal sensor management strategies for tracking elusive space objects." *2017 IEEE Aerospace Conference. IEEE*, 2017.

[12] D. Shen, et al. "Adaptive Markov inference game optimization (AMIGO) for rapid Discovery of satellite behaviors." *Sensors and Systems for Space Applications XII. Vol. 11017. SPIE*, 2019.

[13] N. Jakowski, M. M. Hoque, and C. Mayer, A new global TEC model for estimating transionospheric radio wave propagation errors. *J Geodesy* 85(12), 965–974, 2011.

[14] Y. Yuan, N. Wang, Z. Li, and X. Huo, The BeiDou global broadcast ionospheric delay correction model (BDGIM) and its preliminary performance evaluation results. *Navigation* 66(1), 55–69, 2019.

[15] O. Montenbruck and B. González Rodríguez, NeQuick-G performance assessment for space applications. *GPS Solut* 24, 13, 2020. <https://doi.org/10.1007/s10291-019-0931-2>.

[16] G. Hochegger, B. Nava, S. Radicella, and R. Leitinger, A family of ionospheric models for different uses, *Phys Chem Earth Part C*, 25(4), 307–310, 2000.

[17] S. M. Radicella and R. Leitinger, The evolution of the DGR approach to model electron density profiles. *Adv Space Res*, 27(1), 35–40, 2001.

- [18] International Telecommunication Union (ITU) Radiocommunication Sector, *Ionospheric Propagation Data and Prediction Methods Required for The Design of Satellite Networks and Systems*, Report ITU-R P.531-6, February 2001. https://www.itu.int/rec/R-REC-P.531/_page.print
- [19] B. Nava, P. Coisson, and S. M. Radicella, A new version of the NeQuick ionosphere electron density model, *J Atmos Solar Terr Phys* 70(15), 1856–1862, 2008.
- [20] Simon, Dan, *Optimal state estimation: Kalman, H infinity, and nonlinear approaches*. John Wiley & Sons, 2006.



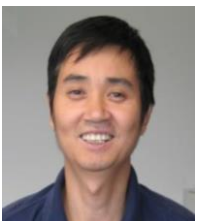
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