Spatial Polarimetric Time-Frequency Distributions for Direction-of-Arrival Estimations

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Abstract

Time-frequency distributions (TFDs) are traditionally applied to a single antenna receiver with a single polarization. Recently, spatial time-frequency distributions (STFDs) have been developed for receivers with multiple single-polarized antennas, and successfully applied for direction-of-arrival (DOA) estimation of nonstationary signals. In this paper, we consider dual-polarized antenna arrays and extend the SPTFD to utilize the source polarization properties. The spatial polarimetric time-frequency distributions (SPTFDs) are introduced as a platform for processing polarized nonstationary signals incident on multi-antenna receivers. The SPTFD platform is applied to dual-polarized double-feed antenna arrays. The source signals are decomposed into two orthogonal polarization components, such as vertical and horizontal. The ability to deal with source signal polarization empowers the STFDs with an additional degree of freedom, leading to improved signal and noise subspace estimates for DOA estimations. The polarimetric time-frequency MUSIC (PTF-MUSIC) method for DOA estimation based on the SPTFD platform is developed and shown to outperform the time-frequency, polarimetric, and conventional MUSIC techniques, when applied separately.

Keywords

Direction-of-arrival estimation, time-frequency distributions, array signal processing, polarization, smart antennas, MUSIC.

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I. Introduction

Time-frequency distributions (TFDs) have been used for nonstationary signal analysis and synthesis in various areas, including speech, biomedicine, automotive industry, and machine monitoring [1], [2]. Over the past few years, the spatial dimension has been incorporated, along with the time and frequency variables, into quadratic and higher-order TFDs, and led to the introduction of spatial time-frequency distributions (STFDs) for non-stationary array signal processing [3], [4]. The relationship between the TFDs of the sensor data to the TFDs of the individual source waveforms is defined by the steering, or the mixing, matrix, and was found to be similar to that encountered in the traditional data covariance matrix approach to array processing. This similarity has allowed subspace-based estimation methods to utilize the source instantaneous frequency for direction-finding. It has been shown that the MUSIC [5] and ESPRIT [6] techniques based on STFDs outperform their counterparts based on covariance matrices, when applied for direction-of-arrival (DOA) estimation of sources of nonstationary temporal characteristics [4], [7], [8], [9].

Polarization and polarization diversities, on the other hand, are commonly used in wireless communications and various types of radar systems [10], [11]. Antenna and target polarization properties are widely employed in remote sensing and synthetic aperture radar (SAR) applications [12], [13], [14]. Airborne and spaceborne platforms as well as meteorological radars include polarization information [15], [16]. Additionally, polarization plays an effective role for target identification in the presence of clutter [17], [18], and has also been incorporated in antenna arrays to improve signal parameter estimation, including DOA and time-of-arrival (TOA) [19], [20].

The two important areas of time-frequency (t-f) signal representations and polarimetric signal processing have not been integrated or considered within the same platform, despite the extensive research work separately performed in each area. In this paper, we introduce the spatial polarimetric time-frequency distributions (SPTFDs) using double-feed dual-polarized arrays, and utilizing both the source time-frequency and polarization signatures. The signal polarization information empowers the STFDs with an additional degree of freedom, leading to improved spatial resolution and source discrimination.

The SPTFD is used to define the polarimetric time-frequency MUSIC (PTF-MUSIC) algorithm, which is formulated based on the source combined t-f and polarization properties for DOA estimation of polarized nonstationary signals. The PTF-MUSIC technique is shown to outperform the MUSIC techniques that only incorporate either the t-f or the polarimetric source characteristics. The application to an ESPRIT-like method is introduced.
separately in [21].

This paper is organized as follows. Section II discusses the signal model and briefly reviews TFDs and STFDs. Section III considers dual-polarized antenna arrays and the concept of spatial polarimetric time-frequency distributions (SPTFDs) is introduced. The PTF-MUSIC algorithm is proposed in Section IV. Sections V and VI, respectively, consider the issues of spatio-polarimetric correlations and DOA estimations of signals with time-varying polarization characteristics. Spatial and polarization averaging methods for coherent signal decorrelation are investigated in Section VII. Computer simulations, demonstrating the effectiveness of the proposed methods, are provided in Section VIII.

Notations

\[ a \] : vector
\[ A \] : matrix
\[ (\cdot)^{[i]} \] : polarization index
\[ (\cdot)^{(i)} \] : subarray index
\[ (\cdot)^* \] : complex conjugate
\[ (\cdot)^T \] : transpose
\[ (\cdot)^H \] : Hermitian
\[ \| \cdot \| \] : vector norm
\[ \odot \] : Hadamard product

II. Signal Model

A. Time-Frequency Distributions

The Cohen’s class of TFDs of a signal \( x(t) \) is defined as \[1\]

\[
D_{xx}(t, f) = \int \int \phi(t - u, \tau)x(u + \frac{\tau}{2})x^*(u - \frac{\tau}{2})e^{-j2\pi f\tau} du d\tau, \tag{1}
\]

where \( t \) and \( f \) represent the time and frequency indexes, respectively. The kernel \( \phi(t, \tau) \) uniquely defines the TFD and is a function of the time and lag variables. In this paper, all the integrals are from \(-\infty\) to \(\infty\).

The cross-term TFD of two signal \( x_i(t) \) and \( x_k(t) \) is defined by

\[
D_{xix_k}(t, f) = \int \int \phi(t - u, \tau)x_i(u + \frac{\tau}{2})x_k^*(u - \frac{\tau}{2})e^{-j2\pi f\tau} du d\tau. \tag{2}
\]
B. Spatial Time-Frequency Distributions

The STFDs have been developed for single-polarized antenna arrays [4], [7]. Consider a narrowband direction-finding problem where the signal bandwidth is small relative to its carrier frequency. We note that the wideband array processing for nonstationary signals, which has been examined in [22] and [23], is outside the scope of the proposed approach. The following linear data model is assumed,

\[ x(t) = y(t) + n(t) = As(t) + n(t), \]

(3)

where the \( m \times n \) matrix \( A = [a_1, a_2, \ldots, a_n] \) is the mixing matrix that holds the spatial information. The number of array elements is \( m \), whereas \( n \) represents the number of signals incident on the array. In the above equation, \( A = A(\Phi) = [a(\phi_1), a(\phi_2), \ldots, a(\phi_n)] \), where \( \Phi = [\phi_1, \phi_2, \ldots, \phi_n] \) and \( a(\phi_i) \) is the spatial signature for source \( i \). Each element of the \( n \times 1 \) vector \( s(t) = [s_1(t) \quad s_2(t) \quad \ldots \quad s_n(t)]^T \) is a mono-component signal. Due to the mixing at each sensor, the elements of the \( m \times 1 \) sensor data vector \( x(t) \) are multi-component signals. \( n(t) \) is an \( m \times 1 \) additive noise vector, which consists of independent zero-mean, white and Gaussian distributed processes.

The STFD of a data vector \( x(t) \) is expressed as [3]

\[ D_{xx}(t, f) = \int\int \phi(t-u, \tau)x(u+\frac{\tau}{2})x^H(u-\frac{\tau}{2})e^{-j2\pi f\tau}dud\tau, \]

(4)

where the \((i,k)\)th element of \( D_{xx}(t, f) \) is given by Eq. (2) for \( i, k = 1, 2, \ldots, m \). The noise-free STFD is obtained by substituting Eq. (3) in Eq. (4),

\[ D_{xx}(t, f) = A(\Phi)D_{ss}(t, f)A^H(\Phi), \]

(5)

where \( D_{ss}(t, f) \) is the TFD matrix of \( s(t) \) which consists of auto- and cross-source TFDs. With the presence of the noise, which is uncorrelated with the signals, the expected value \( D_{xx}(t, f) \) yields

\[ E[D_{xx}(t, f)] = A(\Phi)E[D_{ss}(t, f)]A^H(\Phi) + \sigma^2 I. \]

(6)

In the above equation, \( \sigma^2 \) is the noise power, \( I \) is the identity matrix, and \( E[\cdot] \) denotes the statistical expectation operator.

Equation (6) is similar to the formula that is commonly used in narrowband array processing problems, relating the source correlation matrix to the sensor spatial correlation matrix. Here, the correlation matrices are replaced by the source and sensor TFD matrices. The two subspaces spanned by the principle eigenvectors of \( D_{xx}(t, f) \) and the columns of...
A(Φ) are, therefore, identical. It is shown in [4], [8], [9] that, by constructing the STFD matrix from the t-f points with highly localized signal energy, the corresponding signal and noise subspace estimations are more robust to noise than their counterparts obtained using the data covariance matrix, \( R_{xx} = E[x(t)x^H(t)] \). Further, the source discriminations, rendered through the selection of specific t-f regions, also permit DOA estimations to be performed for only individual or subgroup of sources. In this respect, the number of impinging sources may exceed the number of array sensors. The above attractive properties allow key problems in various array processing applications to be addressed and solved using a new formulation, Eq. (6), that is more tuned to nonstationary signal environments.

III. Spatial Polarimetric Time-Frequency Distributions

A. Polarimetric Modeling

For a transverse electromagnetic (TEM) wave incident on the array, shown in Fig. 1, the electric field can be described as

\[
\mathbf{E} = E_\theta \hat{\theta} + E_\phi \hat{\phi}
\]

\[
= [E_\theta \cos(\theta) \cos(\phi) - E_\phi \sin(\phi)] \hat{x} + [E_\theta \cos(\theta) \sin(\phi) + E_\phi \cos(\phi)] \hat{y} + E_\theta \sin(\theta) \hat{z},
\]

where \( \hat{\phi} \) and \( \hat{\theta} \) are, respectively, the spherical unit vectors along the azimuth and elevation angles \( \phi \) and \( \theta \), viewed from the source, whereas \( \hat{x} \), \( \hat{y} \) and \( \hat{z} \) are the unit vectors along the \( x \), \( y \), and \( z \) directions, respectively. For simplicity and without loss of generality, it is assumed that the source signal is in the \( x-y \) plane whereas the array is located in the \( y-z \) plane. Accordingly, \( \theta = 90 \) degrees, \( \hat{\theta} = -\hat{z} \), and

\[
\mathbf{E} = s[-\cos(\gamma) \sin(\phi) \hat{x} + \cos(\phi) \sin(\gamma) e^{j\eta} \hat{y} + \cos(\gamma) \hat{z}].
\]

For a signal with magnitude \( s \), polarization angle \( \gamma \in [0, \frac{\pi}{2}] \), and polarization phase difference \( \eta \in (-\pi, \pi] \), the two polarization components can be described, respectively, as

\[ s^{[v]} = s \cos(\gamma), \]

which is in the \( z \) direction and induces \( E_\theta \), and

\[ s^{[h]} = s \sin(\gamma) e^{j\eta}, \]

which is in the \( x-y \) plane and determines \( E_\phi \). A signal is said to be linearly polarized if \( \eta = 0 \) or \( \eta = 180 \) degrees.
Now we consider that $n$ signals impinge on the array consisting of $m$ dual-polarized antennas. For the $i$th source whose vertical and horizontal components are expressed as

$$
\begin{align*}
    s_i^{[v]}(t) &= s_i(t) \cos(\gamma) \triangleq c_{i1}s_i(t), \\
    s_i^{[h]}(t) &= s_i(t) \sin(\gamma)e^{j\eta} \triangleq c_{i2}s_i(t),
\end{align*}
$$

the noise-free signal received at the $l$th dual-polarization antenna with vertical and horizontal sensors located in the $\hat{z}$ and $\hat{y}$ directions, respectively, is expressed as

$$
    y_l(t) = [y_l^{[v]}(t), y_l^{[h]}(t)]^T = [a_{il}^{[v]} E \cdot \hat{z}, a_{il}^{[h]} E \cdot \hat{y}]^T = [a_{il}^{[v]} s_i^{[v]}(t), a_{il}^{[h]} s_i^{[h]}(t) \cos(\phi_i)]^T,
$$

where "\cdot" represents the dot product, and $a_{il}^{[v]}$ and $a_{il}^{[h]}$, respectively, are the $l$th elements of the vertically and horizontally polarized array vectors, $a^{[v]}(\phi_i)$ and $a^{[h]}(\phi_i)$. It is assumed that the array has been calibrated and both $a_{il}^{[v]}(\phi)$ and $a_{il}^{[h]}(\phi)$ are known and are normalized such that $||a^{[v]}(\phi)||^2 = ||a^{[h]}(\phi)||^2 = m$. It is noted that the $\cos(\phi_i)$ term in the horizontally polarized array manifold can be absorbed in the array calibration for the region of interest and, therefore, removed from further consideration. Then, the above equation is simplified as

$$
    y_l(t) = [a_{il}^{[v]} s_i^{[v]}(t), a_{il}^{[h]} s_i^{[h]}(t)]^T = s_i(t) \left( [a_{il}^{[v]} a_{il}^{[h]}] \right)^T \circ [c_{i1} c_{i2}]^T \triangleq s_i(t) a_l \odot c_i,
$$

where the vector $c_i = [c_{i1}, c_{i2}]^T$ represents the polarization signature of the $i$th signal with the parameters $c_{i1} = \cos(\gamma_i)$ and $c_{i2} = \sin(\gamma_i)e^{j\eta_i}$ denoting the polarization coefficients corresponding to the vertical and horizontal polarizations, respectively.

### B. Polarimetric Time-Frequency Distributions

For a dual-polarized sensor, $l$, we define the self- and cross-polarized TFDs, respectively, as

$$
    D_{x_l^{[v]}x_l^{[v]}}(t, f) = \int \int \phi(t-u, \tau) x_l^{[v]}(u+\frac{\tau}{2}) (x_l^{[v]}(u-\frac{\tau}{2}))^* e^{-j2\pi ft} dud\tau
$$

and

$$
    D_{x_l^{[v]}x_l^{[h]}}(t, f) = \int \int \phi(t-u, \tau) x_l^{[v]}(u+\frac{\tau}{2}) (x_l^{[h]}(u-\frac{\tau}{2}))^* e^{-j2\pi ft} dud\tau,
$$

where the superscripts $i$ and $k$ denote either $v$ or $h$. The self- and cross-polarized TFDs constitute the $2 \times 2$ polarimetric TFD (PTFD) matrix,

$$
    D_{x_l x_l}(t, f) = \int \int \phi(t-u, \tau) x_l(u+\frac{\tau}{2}) x_l^H(u-\frac{\tau}{2}) e^{-j2\pi ft} dud\tau.
$$

The diagonal entries of $D_{x_l x_l}(t, f)$ are the self-polarized TFDs, $D_{x_l^{[v]}x_l^{[v]}}(t, f)$, whereas the off-diagonal elements are the cross-polarized terms $D_{x_l^{[v]}x_l^{[h]}}(t, f)$, $i \neq k$. 


C. Spatial Polarimetric Time-Frequency Distributions

Equations (12)–(16) correspond to the case of a single dual-polarization sensor. With an \( m \)-sensor array, the data vector, for each polarization \( i, i = v \) or \( h \), is expressed as,

\[
x^{[i]}(t) = [x_1^{[i]}(t), x_2^{[i]}(t), \ldots, x_n^{[i]}(t)]^T = y^{[i]}(t) + n^{[i]}(t) = A^{[i]}(\Phi)s^{[i]}(t) + n^{[i]}(t). \tag{17}
\]

The generalization of polarimetric time-frequency distributions to a multi-sensor receiver is obtained using Eq. (17). Instead of the scalar variable TFD of Eq. (14), we define the self-polarized STFD matrix of vector \( x^{[i]}(t) \) for polarization \( i \) as

\[
D_{x^{[i]}x^{[i]}}(t, f) = \int \int \phi(t - u, \tau)x^{[i]}(u + \frac{\tau}{2})(x^{[i]}(u - \frac{\tau}{2}))^H e^{-j2\pi f \tau} dud\tau, \tag{18}
\]

which, in the noise-free environment, can be expressed as

\[
D_{x^{[i]}x^{[i]}}(t, f) = A^{[i]}(\Phi)D_{s^{[i]}s^{[i]}}(t, f) (A^{[i]}(\Phi))^H . \tag{19}
\]

In a similar manner, the cross-polarization STFD matrix between the data vectors with two different polarizations \( i \) and \( k \) can be expressed as,

\[
D_{x^{[i]}x^{[k]}}(t, f) = \int \int \phi(t - u, \tau)x^{[i]}(u + \frac{\tau}{2})(x^{[k]}(u - \frac{\tau}{2}))^H e^{-j2\pi f \tau} dud\tau, \tag{20}
\]

which becomes

\[
D_{x^{[i]}x^{[k]}}(t, f) = A^{[i]}(\Phi)D_{s^{[i]}s^{[k]}}(t, f) (A^{[k]}(\Phi))^H \tag{21}
\]

when the noise is ignored.

Based on Eq. (17), the following extended data vector can be constructed for both polarizations,

\[
x(t) = \begin{bmatrix} x^{[v]}(t) \\ x^{[h]}(t) \end{bmatrix} = \begin{bmatrix} A^{[v]}(\Phi) & 0 \\ 0 & A^{[h]}(\Phi) \end{bmatrix} \begin{bmatrix} s^{[v]}(t) \\ s^{[h]}(t) \end{bmatrix} + \begin{bmatrix} n^{[v]}(t) \\ n^{[h]}(t) \end{bmatrix}
\]

\[
= \begin{bmatrix} A^{[v]}(\Phi) & 0 \\ 0 & A^{[h]}(\Phi) \end{bmatrix} \begin{bmatrix} Q^{[v]} \\ Q^{[h]} \end{bmatrix} s(t) + \begin{bmatrix} n^{[v]}(t) \\ n^{[h]}(t) \end{bmatrix}
\]

\[
= B(\Phi)Qs(t) + n(t), \tag{22}
\]

where

\[
B(\Phi) = \begin{bmatrix} A^{[v]}(\Phi) & 0 \\ 0 & A^{[h]}(\Phi) \end{bmatrix} \tag{23}
\]
is block-diagonal, and
\[ Q = \begin{bmatrix} Q^v \\ Q^h \end{bmatrix} \] (24)
is the polarization signature vector of the sources, where
\[ q^v = [\cos(\gamma_1), \ldots, \cos(\gamma_n)]^T, \quad Q^v = \text{diag}(q^v), \] (25)
\[ q^h = [\sin(\gamma_1)e^{jn_1}, \ldots, \sin(\gamma_n)e^{jn_n}]^T, \quad Q^h = \text{diag}(q^h). \] (26)
Accordingly,
\[ B(\Phi)Q = \begin{bmatrix} a^v(\phi_1) \cos(\gamma_1) & \cdots & a^v(\phi_n) \cos(\gamma_n) \\ a^h(\phi_1) \sin(\gamma_1)e^{jn_1} & \cdots & a^h(\phi_n) \sin(\gamma_n)e^{jn_n} \end{bmatrix} = \begin{bmatrix} \tilde{a}(\phi_1) & \cdots & \tilde{a}(\phi_n) \end{bmatrix}. \] (27)
The above matrix can be viewed as the extended mixing matrix, with \( a(\tilde{\phi}_k) \) representing the joint spatial-polarimetric signature of signal \( k \). The extended spatial polarization signature vector for the \( k \)th source is
\[ \tilde{a}(\phi_k) = \begin{bmatrix} a^v(\phi_k) \cos(\gamma_k) \\ a^h(\phi_k) \sin(\gamma_k)e^{jn_k} \end{bmatrix}. \] (28)
It is clear that the dual-polarization array, compared to single-polarization case, doubles the vector space dimensionality.

It is now possible to combine the polarimetric, spatial, and t-f properties of the source signals incident on the receiver array. The STFD of the dual-polarization data vector, \( x(t) \), can be written as
\[ D_{xx}(t, f) = \iint \phi(t - u, \tau)x(u + \frac{\tau}{2})x^H(u - \frac{\tau}{2})e^{-j2\pi f\tau} du d\tau = B(\Phi)QD_{ss}(t, f)Q^H B^H(\Phi). \] (29)
\( D_{xx}(t, f) \), formulated in Eq. (29), is referred to as the spatial polarimetric time-frequency distribution (SPTFD) matrix. This distribution, or matrix, serves as a general framework within which typical problems in array processing, including direction-finding, can be addressed as will be shown in the next section.

IV. Polarimetric Time-Frequency MUSIC

Time-frequency MUSIC (TF-MUSIC) has been recently introduced for improved spatial resolution for signals with clear t-f signatures [7]. The proposed PTF-MUSIC is an
important generalization of the TF-MUSIC to deal with diversely polarized signals and polarized arrays. It is based on the orthogonal projection of the searching vector, defined in the joint spatial and polarimetric domains, to the noise subspace obtained from the SPTFD matrix over selected t-f regions.

Consider the following spatial signature matrix

$$F(\phi) = \frac{1}{\sqrt{m}} \begin{bmatrix} a^{[i]}(\phi) & 0 \\ 0 & a^{[h]}(\phi) \end{bmatrix}$$

(30)

corresponding to DOA $\phi$. Because $\|a^{[i]}(\phi)\|^2 = m$, $F^H(\phi)F(\phi)$ is the $2 \times 2$ identity matrix.

To search in the joint spatial and polarimetric domains, we define the following spatio-polarimetric search vector

$$f(\phi, c) = \frac{F(\phi)c}{\|F(\phi)c\|} = F(\phi)c,$$

(31)

where the vector $c = [c_1 \ c_2]^T$ is a unit norm vector with unknown polarization coefficients. In Eq. (31), we have used the fact that $\|F(\phi)c\| = [c^H F^H(\phi) F(\phi)c]^\frac{1}{2} = (c^H c)^\frac{1}{2} = 1$.

The PTF-MUSIC spectrum is given by the following function,

$$P(\phi) = \left[ \min_c c^H F^H(\phi) U_n U_n^H F(\phi) c \right]^{-1} = \left[ \min_c c^H F^H(\phi) U_n U_n^H F(\phi) c \right]^{-1},$$

(32)

where $U_n$ is the noise subspace obtained from the SPTFD matrix using selected t-f points. For t-f based DOA estimation methods, t-f averaging and joint block-diagonalization are two known techniques that can be used to integrate the different STFD or SPTFD matrices constructed at multiple t-f points [4], [7], [26]. The selection of those points from high energy concentration regions pertaining to all or some of the sources enhances the SNR and allows the t-f based MUSIC algorithms to be more robust to noise [4] compared to its conventional MUSIC counterpart.

In (32), the term in brackets is minimized by finding the minimum eigenvalue of the $2 \times 2$ matrix $F^H(\phi) U_n U_n^H F(\phi)$. Thus, a computationally expensive search in the polarization domain is avoided by performing a simple eigen-decomposition on a $2 \times 2$ matrix. As a result, the PTF-MUSIC spectrum can be expressed as

$$P(\phi) = \lambda_{\min}[F^H(\phi) U_n U_n^H F(\phi)],$$

(33)

where $\lambda_{\min}[\cdot]$ denotes the minimum eigenvalue operator. The DOAs of the sources are estimated as the locations of the highest peaks in the PTF-MUSIC spectrum. For each
angle $\phi_k$ corresponding to the $n$ signal arrivals, $k = 1, 2, \ldots, n$, the polarization parameters of the respective source signal can be estimated from

$$
\hat{c}(\phi_k) = v_{\text{min}}[F^H(\phi_k)U_nU_n^H F(\phi_k)],
$$

where $v_{\text{min}}[\cdot]$ is the eigenvector corresponding to the minimum eigenvalue $\lambda_{\text{min}}[\cdot]$.

V. Spatio-Polarimetric Correlations

The spatial resolution capability of an array highly depends on the correlation between the propagation signatures of the source arrivals [4], [27]. This is determined by the normalized inner product of the respective array manifold vectors. In the underlying problem where both the spatial and polarimetric dimensions are involved, the joint spatio-polarimetric correlation coefficient between sources $l$ and $k$ is defined using the extended array manifold $\tilde{a}(\phi)$, i.e.,

$$
\beta_{l,k} = \left( \frac{1}{m} \tilde{a}^H(\phi_k)\tilde{a}(\phi_l) \right) = \frac{1}{m} \left( c_{k1}^*c_{l1}(a^{[v]}(\phi_k))^H a^{[v]}(\phi_l) + c_{k2}^*c_{l2}(a^{[h]}(\phi_k))^H a^{[h]}(\phi_l) \right)
$$

$$
= c_{k1}^*c_{l1}\beta_{l,k}^{[v]} + c_{k2}^*c_{l2}\beta_{l,k}^{[h]},
$$

where $\beta_{l,k}^{[i]} = \frac{1}{m} (a^{[i]}(\phi_k))^H a^{[i]}(\phi_l)$ is the spatial correlation coefficient between sources $l$ and $k$ for polarization $i$, with $i = v$ or $h$.

An interesting case arises when the vertically and horizontally polarized array manifolds are identical, i.e., $a^{[v]}(\phi) = a^{[h]}(\phi)$. In this case, $\beta_{l,k}^{[v]} = \beta_{l,k}^{[h]}$, and the joint spatio-polarimetric correlation coefficient becomes the product of the individual spatial and polarimetric correlations, that is,

$$
\beta_{l,k} = \beta_{l,k}^{[v]}\rho_{l,k}
$$

with

$$
\rho_{l,k} = c_k^H c_l = \cos(\gamma_l)\cos(\gamma_k)e^{j(n_k-n_l)} + \sin(\gamma_l)\sin(\gamma_k)
$$

representing the polarimetric correlation coefficient. In particular, for linear polarizations, $\eta_l = \eta_k = 0$, then Eq. (37) reduces to

$$
\rho_{l,k} = \cos(\gamma_l - \gamma_k).
$$

Since $|\rho_{l,k}| \leq 1$, with the equality holds only when the two sources have identical polarization states, the spatio-polarization correlation coefficient is always smaller than that of the individual spatial correlation coefficient. The reduction in the correlation value due to polarization diversity, through the introduction of $\rho_{l,m}$, translates to improved source distinctions. As such, two sources that could be difficult to resolve using the single-polarized
If the source signal polarizations assume constant values, i.e., source signal polarization vectors are defined, similarly to Eqs. (25) and (26), as

$$R_A = \left( \begin{array}{c} R_{xx} \\ R_{xy} \end{array} \right)$$

additive noise is ignored in the formulation within this section. When active or passive sources move or change orientation [28]. The performance of polarimetric MUSIC and PTF-MUSIC are considered and compared. To simplify the expression, time-varying polarizations are often observed when the source spatial correlation is high, but the respective polarimetric correlation is low.

VI. Sources with time-varying polarizations

In this section, we consider the performance of DOA estimation when the source signals have time-varying polarization signatures. Time-varying polarizations are often observed when active or passive sources move or change orientation [28]. The performance of polarimetric MUSIC and PTF-MUSIC are considered and compared. To simplify the expression, additive noise is ignored in the formulation within this section.

A. Polarimetric MUSIC

Given the time-varying nature of the source signal polarizations, the correlation matrix of the received signal vector is

$$R_{xx} = E[x(t)x^H(t)] = E\left\{ \begin{bmatrix} A_1 & A_2 \\ A_2^T & C \end{bmatrix} \right\}$$

where

$$R_{ss} = E[s(t)s^H(t)]$$

is the source signal correlation matrix, and the time-varying source signal polarization vectors are defined, similarly to Eqs. (25) and (26), as

$$q^{[v]}(t) = [\cos(\gamma_1(t)), \ldots, \cos(\gamma_n(t))]^T,$$

$$q^{[h]}(t) = [\sin(\gamma_1(t))e^{j\phi_1(t)}, \ldots, \sin(\gamma_n(t))e^{j\phi_n(t)}]^T.$$

If the source signal polarizations assume constant values, i.e. $q^{[v]}(t) = q^{[v]}$ and $q^{[h]}(t) = q^{[h]}$, then the noise-free received signal correlation matrix becomes

$$R'_{xx} = B \left( \begin{bmatrix} A_1 & A_2 \\ A_2^T & C \end{bmatrix} \right) B^H.$$
The effect of the signal time-varying polarization on the correlation matrix is evident from Eqs. (39) and (42). That is, in the case of time-varying polarizations, the array considers the time-varying polarization scenario identical to their stationary polarized counterpart that has identical covariance matrix. Consider, for example, two source signals with a linearly time-varying polarization over the observation period from 0 to 90 degrees for the first signal and from 90 to 0 degrees for the second, both sources are considered to be of the same stationary polarization with \( \gamma = 45 \) degrees. As a result, their polarization diversity is consequently not utilized in DOA estimation when polarimetric MUSIC is used.

**B. PTF-MUSIC**

In the presence of time-varying polarized sources, the auto- and cross-polarized SPTFD, defined in Eqs. (18) and (20), respectively, can be expressed as

\[
\begin{align*}
\mathbf{D}_{\mathbf{x}[i]\mathbf{x}[k]}(t,f) & = \int \int \phi(t-u,\tau)\mathbf{x}[i](u+\frac{T}{2})(\mathbf{x}[k](u-\frac{T}{2}))^H e^{-j2\pi f \tau} dud\tau \\
& = \mathbf{A}[i] \left[ \int \int \phi(t-u,\tau) \left( \mathbf{q}[i](u+\frac{T}{2})(\mathbf{q}[k](u-\frac{T}{2}))^H \right) \circ \left( \mathbf{s}(u+\frac{T}{2})\mathbf{s}^H(u-\frac{T}{2}) \right) e^{-j2\pi f \tau} dud\tau \right] (\mathbf{A}[k]^H \\
& = \mathbf{A}[i] \mathbf{D}_{\mathbf{s}[i]\mathbf{s}[k]}(t,f) (\mathbf{A}[k]^H, \quad (43)
\end{align*}
\]

where \( \mathbf{G}[ik](t,\tau) = \mathbf{q}[i](t+\frac{T}{2})(\mathbf{q}[k](t-\frac{T}{2}))^H \), and \( \mathbf{K}(t,\tau) = \mathbf{s}(t+\frac{T}{2})\mathbf{s}^H(t-\frac{T}{2}) \). We assume that the frequency and the polarization signatures of the sources change almost linearly within the temporal span of the t-f kernel. Then, using the first-order Taylor-series expansion, the polarization-dependent terms can be approximated as \( \gamma_i(t + \frac{T}{2}) = \gamma_i(t) + \frac{\sigma^2}{2} \gamma_i(t) \), where \( \dot{\gamma_i}(t) = \frac{d}{dt} \gamma_i(t) \). The autoterms of the source polarization information, which reside on the diagonals of \( \mathbf{G}^{[ve]}(t,\tau), \mathbf{G}^{[vh]}(t,\tau), \mathbf{G}^{[hv]}(t,\tau) \) and \( \mathbf{G}^{[hh]}(t,\tau) \), are given by

\[
\begin{align*}
[\mathbf{G}^{[ve]}(t,\tau)]_{ii} & = \frac{1}{2} \left[ \cos(2\gamma_i(t)) + \cos(\tau \dot{\gamma}_i(t)) \right] \quad (44) \\
[\mathbf{G}^{[vh]}(t,\tau)]_{ii} & = \frac{1}{2} \left[ \sin(2\gamma_i(t)) - \sin(\tau \dot{\gamma}_i(t)) \right] \quad (45) \\
[\mathbf{G}^{[hv]}(t,\tau)]_{ii} & = \frac{1}{2} \left[ \sin(2\gamma_i(t)) + \sin(\tau \dot{\gamma}_i(t)) \right] \quad (46) \\
[\mathbf{G}^{[hh]}(t,\tau)]_{ii} & = \frac{1}{2} \left[ - \cos(2\gamma_i(t)) + \cos(\tau \dot{\gamma}_i(t)) \right], \quad (47)
\end{align*}
\]

respectively. For symmetric t-f kernels, \( \phi(t,\tau) \), the second sinusoidal terms in Eqs. (45)
and (46) assume zero values in the TFD. Therefore, \( D_{s_i|s_i}(t,f) \) can be expressed at the autoterm points as

\[
D_{s_i|s_i}(t,f) = \frac{1}{2} \cos(2\gamma_i(t)) D_{s_is_i}(t,f) + c_{ii}(t,f)
\]

(48)

\[
D_{s_i|s_i}(t,f) = -\frac{1}{2} \cos(2\gamma_i(t)) D_{s_is_i}(t,f) + c_{ii}(t,f)
\]

(49)

\[
D_{s_i|v_i|v_i}(t,f) = D_{s_i|s_i}(t,f) = \frac{1}{2} \sin(2\gamma_i(t)) D_{s_is_i}(t,f)
\]

(50)

with

\[
c_{ii}(t,f) = \frac{1}{2} \int \int \cos(\tau \dot{\gamma}_i(t)) \phi(t-u,\tau) [K(t,\tau)]_{ii} e^{-j2\pi f t} dud\tau.
\]

(51)

When only the t-f points located in the autoterm region of the \( i \)th source are used in constructing the SPTFD matrix, then

\[
D_{xx}(t,f) = \begin{bmatrix} a^{[v]}(\phi_i) & 0 \\ 0 & a^{[h]}(\phi_i) \end{bmatrix} M \begin{bmatrix} a^{[v]}(\phi_i) & 0 \\ 0 & a^{[h]}(\phi_i) \end{bmatrix}^H,
\]

(52)

where

\[
M = \frac{1}{2} D_{s_is_i}(t,f) \begin{bmatrix} \cos(2\gamma_i(t)) & \sin(2\gamma_i(t)) \\ \sin(2\gamma_i(t)) & -\cos(2\gamma_i(t)) \end{bmatrix} + \begin{bmatrix} c_{ii}(t,f) & 0 \\ 0 & c_{ii}(t,f) \end{bmatrix}.
\]

In the new structure of the SPTFD matrix of Eq. (52), the source time-varying polarization has the effect of loading the diagonal elements with \( c_{ii}(t,f) \) and, as such, alters the eigenvalues of the above 2 \( \times \) 2 matrix. However, the eigenvector of \( M \) remain unchanged. The eigenvalues are \( \lambda_{1,2} = c(t,f) \pm \frac{1}{2} D_{ss}(t,f) \), and the signal polarization signature, i.e., the eigenvector corresponding to the maximum eigenvalue, is \( v_1 = \begin{bmatrix} \cos(\gamma(t)) & \sin(\gamma(t)) \end{bmatrix}^T \).

Therefore, in the context of PTF-MUSIC, the instantaneous polarization characteristics can be utilized to distinct different sources. It is worth noting that, when multiple t-f points are incorporated to construct the SPTFD matrices for improved subspace estimation, the t-f points should be chosen in a small vicinity of each time instant so that maximum polarization distinctions can be utilized.

VII. Subarray and Polarimetric Averaging

In coherent signal environments, spatial smoothing [24] and polarization averaging [25] methods are commonly applied in the MUSIC algorithms to restore the rank of the source matrix, prior to signal and noise subspace estimations. While spatial smoothing has a drawback of reducing the array aperture, polarization averaging eliminates pertinent
source polarization information. In a combined spatial and polarization averaging approach, signal polarizations can be used to limit the reduction in array aperture. This, in turn, increases the number of coherent sources resolved by the array over the case where only spatial averaging is performed.

In this section, these methods are considered for the PTF-MUSIC for estimating DOAs of coherent sources in the context of TFDs, using dual-polarized double-feed arrays. For subarray averaging, uniform linear arrays (ULAs) are assumed and, therefore, the array manifolds are identical in both polarizations, i.e. \( a_v(\phi) = a_h(\phi) = a(\phi) \). For polarization averaging, only the latter assumption (identical manifolds for both polarizations) is required.

A. Subarray Averaging

Subarray averaging involves dividing the \( m \) dual-polarized antenna array into \( p \) overlapping subarrays of \( m_1 = m - p + 1 \) antennas, and averaging the respective \( p \) subarray SPTFD matrices. Define \( A_1(\Phi) \) as the new \( m_1 \times n \) steering matrix for the first subarray which consists of the first \( m_1 \) rows of matrix \( A = A_v = A_h \). The data vector at the \( k \)th subarray is expressed as

\[
x^{(k)}(t) = \begin{bmatrix} x^{(k)v}(t) \\ x^{(k)h}(t) \end{bmatrix} = B_1^{(k)}(\Phi) \begin{bmatrix} s^{(v)}(t) \\ s^{(h)}(t) \end{bmatrix} + \begin{bmatrix} n^{(k)v}(t) \\ n^{(k)h}(t) \end{bmatrix}, \quad k = 1, 2, \ldots, m_1, \tag{53}
\]

where \( n^{(k)i}(t) \) is the noise vector at the subarray for polarization \( i, i = v \) or \( h \),

\[
B_1^{(k)}(\Phi) = \begin{bmatrix} A_1(\Phi)\Lambda^{-1}(\Phi) & 0 \\ 0 & A_1(\Phi)\Lambda^{-1}(\Phi) \end{bmatrix}, \tag{54}
\]

\[
\Lambda(\Phi) = \text{diag} \left[ e^{-j2\pi \frac{d}{\lambda} \sin(\phi_1)}, \ldots, e^{-j2\pi \frac{d}{\lambda} \sin(\phi_n)} \right]. \tag{55}
\]

Denoting \( D_{xx}^{(k)}(t, f) \) as the SPTFD matrix corresponding to \( x^{(k)}(t) \) of the \( k \)th subarray, the spatially smoothed SPTFD matrix is defined by averaging \( D_{xx}^{(k)}(t, f) \) over the \( p \) subarrays, i.e.,

\[
D_{xxSA}(t, f) = \frac{1}{p} \sum_{k=1}^{p} D_{xx}^{(k)}(t, f). \tag{56}
\]

The averaged SPTFD matrix can be written as the augmentation of four spatially-smoothed auto- and cross-polarized SPTFD matrices, expressed as

\[
D_{xxSA}(t, f) = \begin{bmatrix} D_{xxSA}^{[vv]}(t, f) & D_{xxSA}^{[vh]}(t, f) \\ D_{xxSA}^{[hv]}(t, f) & D_{xxSA}^{[hh]}(t, f) \end{bmatrix}. \tag{57}
\]
It has been shown in [29] that, in a t-f array processing context, averaging the TFD of the received signal across array sensors reduces source signal crossterms and noise effect. Therefore, it is a straightforward extension to show that subarray averaging not only decorrelates coherent sources, but also results in reduced crossterms and noise effect as well. It is obvious that the higher the number of subarrays is, the more effective the crossterm suppression will be. Therefore, a byproduct of this operation is an enhanced source t-f signature, which can lead to improved t-f autoterm selection, and subsequently improved subspace estimation.

B. Polarimetric Averaging

Similar to subarray averaging, polarimetric averaging aims at combating the rank deficiency of the source SPTFD matrix, \( D_{ss}(t, f) \), provided that the sources have different polarization states. The polarimetric averaged SPTFD matrix is defined as

\[
D_{xxPA}(t, f) = \frac{1}{2} \left[ D_{x[v]x[v]}(t, f) + D_{x[h]x[h]}(t, f) \right].
\]  

As with subarray averaging, polarization averaging also reduces source signal crossterms depending on the polarization correlation between them, as was shown in [30].

C. Combined Spatial and Polarimetric Averaging

Polarization averaging can also be used in conjunction with subarray averaging. Denote \( D_{x[i]v[i]}^{(i)}(t, f) \) and \( D_{x[i]h[i]}^{(i)}(t, f) \) as the STFDs corresponding to \( x^{(i)[v]}(t) \) and \( x^{(i)[h]}(t) \), respectively. Then, the combined subarray and polarization averaged SPTFD matrix becomes

\[
D_{xxSPA}(t, f) = \frac{1}{2p} \sum_{i=1}^{p} \left[ D_{x[i]v[i]}^{(i)}(t, f) + D_{x[i]h[i]}^{(i)}(t, f) \right].
\]  

It is implicit in Eqs. (56)–(59) that whether it is polarization and/or subarray averaging, source decorrelation is performed for each t-f point. Once the rank deficiency in the SPTFD matrices corresponding to multiple t-f points is restored, one can estimate the DOAs through PTF-MUSIC (for subarray averaging) or TF-MUSIC (for polarimetric or combined spatial and polarimetric averaging since the polarimetric information is lost in the process of averaging).

D. Decorrelation Requirements

Consider that \( n_0 \) sources are selected in the t-f domain, out of which a maximum number of \( n_c \) sources are coherent with each other. It is well-known that to decorrelate \( n_c \) coherent
sources using spatial averaging, the minimum number of subarrays must be \( p \geq n_c \). In addition, the condition \( m_1 > n_0 \) is required so that the DOAs of all \( n_0 \) sources can be identified. However, when polarization averaging is used in addition to subarray averaging, only half the number of subarrays is needed, i.e. \( p \geq \lceil n_c/2 \rceil \), given that the polarization states of the coherent sources are not identical. Accordingly, to decorrelate two coherent sources with different polarization states, polarization averaging alone will suffice. To decorrelate four coherent sources with different polarization states, polarization averaging accompanied with two subarrays will then be required.

The proof is provided in [25] for non-time-frequency based methods. The extension to the t-f based methods is rather straightforward by substituting the covariance matrix with a STFD or SPTFD matrix, and noting the fact that

\[
D_{xxSPA}(t, f) = \frac{1}{2p} A_1 \left\{ \sum_{l=1}^{p} A_1^{-1} \left[ \sum_{i=v}^{h} D_{s[i]s[i]}(t, f) \right] A_1^{1-l} \right\} A_1^H
\]

\[
= \frac{1}{2p} A_1 \left\{ \sum_{l=1}^{p} A_1^{-1} \left[ \sum_{i=v}^{h} Q[i] D_{ss}(t, f) (Q[i])^H \right] A_1^{1-l} \right\} A_1^H. \tag{60}
\]

Therefore, when the coherent signals have different polarization states, the Cholesky factorization of the source signal TFD matrices at the two polarizations will yield a matrix of twice the number of columns. As a result, only half the number of subarrays is needed to assure the full rank of the Cholesky factor.

\textbf{E. Remarks}

From the above discussion, the following remarks are in order.

1. Polarization averaging does not require a ULA, a condition that has to be satisfied in subarray averaging. However, the dual-polarized sensors must be identically polarized and both polarizations have the identical array manifolds.

2. Polarization averaging is beneficial for matrix rank restoration only when the coherent sources have different polarization states. Polarization averaging sacrifices the polarization information and, therefore, signal polarization parameters can not be estimated.

3. In some cases, polarization averaging must be utilized along with subarray averaging. For example, when three sources impinge on a five-sensors ULA, while polarization averaging combined with two subarrays can resolve the source DOAs, subarray averaging alone would fail.
VIII. Simulations

A. Uncorrelated Source Scenarios

We consider two sources (sources 1 and 2) with chirp waveforms in the presence of an undesired sinusoidal signal (source 3) which impinge on a ULA of four \( m = 4 \) dual-polarization cross-dipoles with half-wavelength interelement spacing. The vertical and horizontal array manifolds are set to be equal. Table 1 shows the sources’ respective normalized starting and end frequencies, DOAs (measured from the broadside), and the two polarization parameters, \( \gamma \) and \( \phi \). All signals have the same signal power (SNR=13dB). The task is to find the DOAs of the chirp signals. The data length is 256 samples and the length of the rectangular window used in the pseudo Wigner-Ville distribution (PWVD) is 65 samples.

The PWVDs averaged over the four sensors are shown in Figs. 2(a) and 2(b), respectively, for the vertical and horizontal polarizations. Because the sources are closely spaced, the crossterm suppression through array averaging is limited. To further suppress the crossterms, we utilize the combined spatial and polarization information. Fig. 2(c) shows the PWVD averaged over the four sensors as well as both polarizations. In this case, since source 1 and source 2 have orthogonal polarizations, the crossterms between the two chirp signals are completely suppressed, revealing the true chirp signatures. The t-f points along the chirp instantaneous frequencies can, therefore, be considered for STFD and SPTFD matrix constructions.

The PTF-MUSIC spectrum is computed and the results are compared with the conventional MUSIC, polarimetric MUSIC, and TF-MUSIC. The MUSIC spectra for three independent trials are shown in Fig. 3. For the conventional and TF-MUSIC, only the vertical polarization components are used. For TF- and PTF-MUSIC, on the other hand, the sinusoidal signal is eliminated from consideration through proper selection of the t-f points. The TF-MUSIC benefits from fewer sources and increased SNR, whereas the polarimetric MUSIC utilizes the distinction in the source polarization properties. Both attributes are enjoyed by the PTF-MUSIC. It is evident that only the proposed PTF-MUSIC accurately estimates the DOAs of the two chirp sources.

Figure 4 shows the root mean square error (RMSE) performance of estimated DOA of source 2 for the four MUSIC methods. The results are obtained using 50 independent trials for each value of SNR. The RMSE performance of the conventional MUSIC with twice the number of sensors (i.e., 8 sensors) is also included for comparison. It is seen that the PTF-MUSIC outperforms all other methods. The PTF-MUSIC enjoys about 5dB gain.
over the polarimetric MUSIC due to the source selection/discrimination capability and the localization of the source signal energy.

### TABLE I

**Signal Parameters (Uncorrelated Source Scenario)**

<table>
<thead>
<tr>
<th></th>
<th>start freq.</th>
<th>end freq.</th>
<th>DOA (deg.)</th>
<th>γ (deg.)</th>
<th>φ (deg.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>source 1</td>
<td>0.20</td>
<td>0.40</td>
<td>−3</td>
<td>45</td>
<td>0</td>
</tr>
<tr>
<td>source 2</td>
<td>0.22</td>
<td>0.42</td>
<td>3</td>
<td>45</td>
<td>180</td>
</tr>
<tr>
<td>source 3</td>
<td>0.10</td>
<td>0.10</td>
<td>9</td>
<td>20</td>
<td>0</td>
</tr>
</tbody>
</table>

### B. Coherent Source Scenarios

In the second set of simulations, we consider a ULA of five ($m = 5$) dual-polarized cross-dipoles with half-wavelength interelement spacing. Three sources are considered. The first two sources (sources 1 and 2) are coherent and of identical chirp signatures, whereas the third one is an undesired sinusoidal signal (source 3). Table II shows the signal parameters. All signals have the same signal power (SNR=10dB). The data length is 256 samples. The PWVD averaged over the five cross-polarization sensors is shown in Fig. 5.

### TABLE II

**Signal Parameters (Coherent Source Scenario)**

<table>
<thead>
<tr>
<th></th>
<th>start freq.</th>
<th>end freq.</th>
<th>DOA (deg.)</th>
<th>γ (deg.)</th>
<th>φ (deg.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>source 1</td>
<td>0.20</td>
<td>0.50</td>
<td>−6</td>
<td>35</td>
<td>5</td>
</tr>
<tr>
<td>source 2</td>
<td>0.20</td>
<td>0.50</td>
<td>6</td>
<td>45</td>
<td>170</td>
</tr>
<tr>
<td>source 3</td>
<td>0.10</td>
<td>0.10</td>
<td>12</td>
<td>25</td>
<td>−90</td>
</tr>
</tbody>
</table>

### 1. Polarimetric Averaging

Polarimetric averaging of the STFD matrices of the data samples across the vertical and the horizontal polarizations can successfully decorrelate coherent sources. Fig. 6 shows the spectra of the conventional MUSIC and TF-MUSIC, respectively, over three independent trials, where polarimetric averaging was employed on the five vertical and five horizontal antennas. For the TF-MUSIC method, only the two coherent sources (i.e., sources 1 and
are selected. It can be seen that both methods successfully decorrelate the two coherent sources. However, it is only the TF-MUSIC that clearly shows an exemplary performance due to the source selection capability.

2. Subarray and Polarization Averaging

In this simulation, in addition to the polarimetric averaging, spatial smoothing preprocessing is performed with two subarrays, each of four sensors. The spectra of the MUSIC and the TF-MUSIC utilizing the combined polarization and subarray averaging are shown in Fig. 7. In this case, the number of subarrays is 2. For comparison, we plotted in Fig. 8 the spectra using the conventional MUSIC method, applied to 10 vertically-polarized antenna array. Due to the close spatial separation between sources 2 and 3, the performance of all non-time-frequency based methods is not satisfactory. Only the TF-MUSIC spectrum, which isolates the third signal from consideration, shows sharp and less biased peaks at the DOAs of the two coherent sources.

C. Sources with Time-Varying Polarization

Figure 9 shows the PWVD of two chirp signals impinging on a ULA of five cross-dipoles. The signal parameters are listed in Table III. Fig. 10 shows the time-varying polarization angles (\(\gamma\)) of the two chirp signals. The five pairs \((a_1, a_2)-(e_1, e_2)\) show the polarization angles at the selected time instants \((c_1 = c_2 \text{ is shown as } c \text{ in Fig. 10})\). The first element of each pair corresponds to the source 1, whereas the second element describes source 2 for a given time instant. The SPTFD matrices constructed from each pair along with its three consecutive t-f points are utilized for DOA estimation.

The performance of the polarimetric MUSIC is shown in Fig. 11(a), where both sources appear to have the same polarization, due to the fact that the two sources have the same second-order moment of the polarization signature over the observation period and, therefore, the covariance matrix based polarimetric MUSIC method cannot distinguish their instantaneous polarization differences. The PTF-MUSIC, on the other hand, can utilize the distinctions in their instantaneous polarization signatures for improved DOA estimation, and its performance is a function of the location of the t-f point pairs used in the subspace estimation. As expected, when only t-f points corresponding to point c are used, the performance of the PTF-MUSIC becomes similar to that of the polarimetric MUSIC (see Fig. 11(d)), since both signals have the same polarization at that t-f point. However, improved performance can be achieved, when t-f points corresponding to pairs \((a_1, a_2), (b_1, b_2), (d_1, d_2), \text{ or } (e_1, e_2)\) are selected, due to the different polarization signatures of the two source signals at these points, as can be seen in Figs. 11(b), 11(c), 11(e) and
TABLE III

Signal Parameters (Time-Varying Polarization Scenario)

<table>
<thead>
<tr>
<th></th>
<th>start freq.</th>
<th>end freq.</th>
<th>DOA (deg.)</th>
<th>$\gamma$ (deg.)</th>
<th>$\phi$ (deg.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>source 1</td>
<td>0.20</td>
<td>0.60</td>
<td>6</td>
<td>0 to 90</td>
<td>0</td>
</tr>
<tr>
<td>source 2</td>
<td>0.40</td>
<td>0.80</td>
<td>9</td>
<td>90 to 0</td>
<td>0</td>
</tr>
</tbody>
</table>

11(f).

IX. Conclusion

The platform to deal with diversely polarized sources emitting nonstationary signals with clear time-frequency (t-f) signatures has been introduced. This platform, which is termed Spatial polarimetric time-frequency distributions (SPTFDs), utilizes the polarimetric, spatial, and temporal signatures of signals impinging on an array of sensors. Each sensor is of dual-feed, double-polarization antennas. The SPTFD incorporates the time-frequency distributions (TFD) of the received data across the polarization and spatial variables. It allows the distinction of sources based on their respective direction-of-arrival as well as their polarization and t-f signal characteristics. The use of TFD reveals the source time-varying frequency natures, and as such, permits the consideration of those t-f points of high signal energy concentrations. The eigen-decomposition of SPTFDs constructed from a portion of, or the entire, t-f signatures of all or a subset of the incoming signals is used to define the polarimetric time-frequency MUSIC (PTF-MUSIC) algorithm. This algorithm outperforms other existing MUSIC methods, including conventional MUSIC, time-frequency MUSIC, and polarimetric MUSIC. For coherent signal environments, collecting the data from the horizontal and vertical polarized antenna arrays, separately, provides the flexibility to trade off subarray and polarization averaging for source matrix rank restoration, and as such, can be used to limit the reduction in array aperture necessary for source decorrelations. The paper considered the application of TFDs to sources with a time-varying polarization in the context of array processing. It has been shown that the difference in the instantaneous polarizations of the sources can be uniquely utilized by the proposed approach to maintain polarization diversity, specifically, in the cases when the source polarizations have similar span of polarization angles over the observation period.
References


Fig. 1. Dual-polarized array.
(a) PWVD averaged over the vertically-polarized sensors.

(b) PWVD averaged over the horizontally-polarized sensors.

(c) PWVD averaged over sensors and polarizations.

Fig. 2. Averaged PWVD results.
Fig. 3. Comparison of MUSIC spectra.
Fig. 4. RMSE performance of the MUSIC methods.

Fig. 5. PWVD averaged over all sensors and polarizations.
Fig. 6. Conventional MUSIC and TF-MUSIC spectra with polarization averaging.
Fig. 7. Conventional and TF-MUSIC spectra with spatial smoothing and polarization averaging.

Fig. 8. 10-sensor conventional MUSIC with spatial smoothing.
Fig. 9. PWVD of two chirp signals arriving at the reference sensor.

Fig. 10. Source time-varying polarization signatures.
Fig. 11. Polarimetric MUSIC and PTF-MUSIC spectra for time-varying polarization sources.