A Dual Function Radar-Communications System Using Sidelobe Control and Waveform Diversity

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***Abstract*—In this paper, we develop a new technique for dual-function radar-communications in a transmit multi-sensor array where information embedding is achieved using sidelobe control in tandem with waveform diversity. A set of Q orthogonal waveforms is exploited to embed a sequence of Q bits during each radar pulse. All waveforms are transmitted simultaneously where one bit is embedded in each waveform. We design two transmit weight vectors to achieve two distinct transmit power distribution patterns which share the same main radar beam but have different sidelobe levels towards the communication direction. The receiver interprets the bit associated with a certain waveform as binary information based on whether that waveform is radiated over the transmit beam associated with the first or the second weight vector. The proposed technique enables information delivering to a single or multiple communication directions located outside the mainlobe of the radar. The communication message has low probability of intercept from directions other than the preassigned communication directions. Additionally, the waveform diversity enables the radar to operate in multiple-input multiple-output (MIMO) mode. The performance of the proposed technique is investigated in terms of the bit error rate (BER).**

I. INTRODUCTION

The coexistence of radar and wireless communication systems is becoming increasingly necessary owing to greater demand on bandwidth against strictly finite RF electromagnetic spectrum resource [1], [2]. This has motivated the communications research community to turn to cognitive radio as means to increase efficiency of the spectrum usage (see [3] and references therein). Similar research efforts have been exerted to bringing cognition into radar (see [4] and references therein). The cooperative operation of radar and communication systems has also received considerable attention [2], [5]–[7].

Incorporating communication as secondary to the primary radar function is reported in a number of papers [8]–[10]. Embedding information into radar emission has been achieved in [9] via radiating one waveform during each radar pulse from a set of pre-designed waveforms. Based on which waveform is transmitted and reliably received, the receiver can decode the corresponding information. However, changing the waveform from pulse to pulse may affect the radar operation even within a coherent processing interval. More recently, a dual-function radar communication approach using time modulated arrays was proposed in [10]. The phases of the transmit array elements are adjusted from pulse to pulse in order to introduce variations in the sidelobe levels (SLLs) towards the communication direction. During each radar pulse, the communication receiver detects the SLL and interpret the associated bit sequence. Although the waveform is fixed and remains the same over each pulse repetition period, it is not always easy to design multiple transmit power distribution patterns with the same mainlobe using time modulated arrays as the optimization criterion involved is highly nonlinear and computationally demanding.

In this paper, we develop a dual-function radar-communications technique using sidelobe control in tandem with waveform diversity. A set of Q orthogonal waveforms are used in order to embed a sequence of Q bits of information. The SLLs towards the communication direction(s) are controlled to have two distinct levels via designing two transmit beamforming wight vectors. All waveforms are transmitted simultaneously where one bit is embedded in each waveform. The receiver interprets the bit associated with a certain waveform as '0' or '1' based on whether that waveform is radiated over the transmit beam associated with the first or the second weight transmit vector, respectively. The proposed technique enjoys the following advantages: (i) The communication system can serve multiple users as long as they are located outside the mainlobe where the main radar function of the system takes place. (ii) The communication process is inherently secure against intercepts from directions other than the preassigned communication directions. (iii) The decoding of each bit at the receiver is independent of other neighboring bits in the sequence. (iv) The same set of Q waveforms are transmitted within every radar pulse thus enabling the radar to perform coherent processing. (v) The waveform diversity enables the radar to operate in a multiple-input multiple-output (MIMO) mode. (vi) The communication direction(s) can be adjusted adaptively if the receiver is mounted on a moving platform. The superiority of the proposed method over the recently developed method in [10] is validated using simulation examples.

II. PROBLEM FORMULATIONS

Consider a radar system equipped with an M-antenna transmit array of an arbitrary linear shape. The objective of the dual-function radar-communication system is to deliver a communication message to a receiver (or multiple receivers) located within the sidelobe region as a secondary task without affecting the radar operation being the main task of the system. Therefore, one key requirement is to keep the magnitude of the main beam of the radar the same during the entire processing interval. On the other hand, in order to embed information in the beamformer, the SLL at the communication direction(s) should be permitted to assume different values over the region of interest. These two key requirements can be achieved via appropriate transmit beamforming designs.

Assume that the main beam where the main radar task takes place is defined by the spatial sector $\boldsymbol{\Theta} = [\theta_{\mathtt{min}}\ \theta_{\mathtt{max}}]$. The special case where the radar main beam is focused towards a single spatial direction corresponds to $\theta_{\mathtt{min}} = \theta_{\mathtt{max}}$. The sidelobe area is denoted as $\bar{\boldsymbol{\Theta}}$. Assume that the number of communication receivers is L and the corresponding communication directions are $\theta_l,\ l = 1, \ldots, L$. Note that the number of waveforms, Q, determines the number of information bits that can be embedded in each radar pulse. This means that a data rate in the range of Q bits per pulse can be achieved. Most modern pulsed radar systems support pulse repetition frequency in the range of kHz [9]. Therefore, for moderate values of Q, an overall data rate in the range of kbits per sec can be achieved.

Let $\mathbf{w}_k,\ k = 1, \ldots, K$, be the $M\times 1$ transmit beamforming weight vectors. One way to design the transmit weight vectors is to formulate the following optimization problem,

$$\min_{\mathbf{w}_k} \quad \max_{\theta} \quad \left| G_{\mathrm{d}}(\theta) - \left|\mathbf{w}_k^H \mathbf{a}(\theta)\right| \right|, \quad \theta \in \boldsymbol{\Theta} \tag{1}$$

$$\text{subject to} \quad \left|\mathbf{w}_k^H \mathbf{a}(\theta\right| \leq \varepsilon, \quad \theta \in \bar{\boldsymbol{\Theta}}, \tag{2}$$

$$\mathbf{w}_k^H \mathbf{a}(\theta_l) = \delta_{l,k}, \quad l = 1, \ldots, L, \tag{3}$$

where $G_{\mathrm{d}}(\theta)$ is the magnitude of the desired beampattern within the main radar beam, ε is a positive number of user's choice used for controlling the SLLs, $\delta_{l,k}$ is a positive number used to determine the SLL associated with the kth transmit beam towards the lth communication direction,

$$\mathbf{a}(\theta) = \left[1, e^{-j2\pi h_1 \sin\theta}, \ldots, e^{-j2\pi h_{M-1}\sin\theta}\right]^T, \tag{4}$$

is the $M \times 1$ steering vector of the transmit array toward the spatial angle θ, h_m is the displacement measured in wavelength between the mth and $(m+1)$th antennas, respectively, and $(\cdot)^T$ and $(\cdot)^H$ stand for the transpose and the Hermitian operations, respectively. In (1)–(3), the objective function fits the actual transmit pattern associated with each transmit beam which is mandated by the radar operation. The set of constraints in (2) is used to upper-bound the transmit power leakage within the sidelobe areas, which is also mandated by the radar operation. Note that the upper bound determined by the parameter ε is the same for all transmit beams. The set of linear constraints (3) is associated with the secondary function of the system which is to embed information via different SLLs towards the communication directions. It is worth-noting that the parameter $\delta_{l,k}$ which determines the SLL is different for each transmit beam. Since ε is the highest sidelobe level as mandated by the main radar operation of the system, the condition $\delta_{l,k} \leq \varepsilon,\ k = 1, \ldots, K$ should be satisfied. However, a tradeoff between the primary radar and the secondary communication operations can be achieved by allowing the SLLs towards the communications directions to be higher than the rest of the sidelobe region which means more transmit power is assigned to the communication operation at the price of a decreased transmit gain within the main radar beam. In such a case, the set of constraints in (2) should cover the sidelobe region excluding the communications directions.

The optimization problem (1)–(3) is difficult to solve due to the non-convex objective function. Therefore, we reformulate the problem by slightly modifying the objective function. We set $G_{\mathrm{d}}(\theta) = e^{j\varphi(\theta)}$, yielding the following optimization problem

$$\min_{\mathbf{w}_k} \max_{\theta_i} \left| e^{j\varphi(\theta_i)} - \mathbf{w}_k^H \mathbf{a}(\theta_i) \right|, \ \theta_i \in \boldsymbol{\Theta},\ i = 1, \ldots, I \tag{5}$$

$$\text{subject to} \ \left|\mathbf{w}_k^H \mathbf{a}(\theta_p)\right| \leq \varepsilon, \quad \theta_p \in \bar{\boldsymbol{\Theta}},\ p = 1, \ldots, P, \tag{6}$$

$$\mathbf{w}_k^H \mathbf{a}(\theta_l) = \delta_{l,k}, \quad l = 1, \ldots, L, \tag{7}$$

where $\theta_i,\ i = 1, \ldots, I$, and $\theta_p,\ p = 1, \ldots, P$, are discrete grids of angles used to approximate $\boldsymbol{\Theta}$ and $\bar{\boldsymbol{\Theta}}$, respectively, and $\varphi(\theta)$ is a phase profile of user's choice. The optimization problem (5)–(7) is convex and can be solved in a computationally efficient manner [11]. It is worth-noting that the transmit beamforming weight vector obtained by solving (5)–(7) yields a unity magnitude within the main radar beam. However, in practice the transmit weight vector can be scaled up to the desired transmit gain as long as the total transmit power budget does not exceed the maximum allowed power of the actual system. Note that scaling up the transmit weight vector results in magnifying the transmit power distribution at all angles equally, i.e., the relative SLLs with respect to the mainlobe remains unchanged.

III. Proposed Information-Embedding Technique

Assume that, during each radar pulse, a Q-bit information message composed of 1's and 0's is denoted as the binary sequence $B_q,\ q = 1, \ldots, Q$. Let $\boldsymbol{\psi}(t) = [\psi_1(t), \ldots, \psi_Q(t)]$ be the $Q \times 1$ vector of transmit orthogonal waveforms, where t denotes the fast-time index. The waveforms are assumed to be known to the communication receiver. Let $\mathbf{w}_1$ and $\mathbf{w}_2$ be two $M\times 1$ transmit beamforming weight vectors which can be optimized offline using (5)–(7). The baseband representation of the signals at the input of the transmit antennas is given by

$$\mathbf{x}(t) = \mathbf{W}\boldsymbol{\psi}(t), \tag{8}$$

where

$$\mathbf{W} = \left[B_1\mathbf{w}_1^* + (1-B_1)\mathbf{w}_2^*, \ldots, B_Q\mathbf{w}_1^* + (1-B_Q)\mathbf{w}_2^*\right] \tag{9}$$

is the $M \times Q$ transmit weight matrix and $(\cdot)^*$ stands for the conjugate.

Assume that the lth communication receiver located at the relative spatial direction ϕ_l is equipped with an arbitrary shaped array of N receive antennas. As the subsequent discussion is focused on the lth receiver, all subscripts l associated with the receiver parameters will be dropped hereafter for the sake of simplicity. Then, the baseband representation of $N\times 1$ signal vector at the output of the receive array is given by

$$\mathbf{y}(t) = \beta\Big(\mathbf{a}^T(\theta_l)\mathbf{x}(t)\Big)\mathbf{b}(\phi) + \mathbf{z}(t), \tag{10}$$

where β is the channel coefficient which summarises the propagation environment between the transmit array and the communication receiver, $\mathbf{b}(\phi)$ is the steering vector of the the receive array, and $\mathbf{z}(t)$ is the $N \times 1$ vector of additive white Gaussian noise with zero-mean and covariance $\sigma_z^2\mathbf{I}_N$, and $\mathbf{I}_N$ denotes the $N \times N$ identity matrix.

Matched-filtering the received data (10) to each of the transmitted orthogonal waveforms yields the $N \times 1$ data vectors $\mathbf{y}_q,\ q = 1, \ldots, Q$, defined as,

$$\mathbf{y}_q = \begin{cases} \beta\Big(\mathbf{w}_1^H \mathbf{a}(\theta_l)\Big)\mathbf{b}(\phi) + \mathbf{z}_q, & B_q = 1, \\ \beta\Big(\mathbf{w}_2^H \mathbf{a}(\theta_l)\Big)\mathbf{b}(\phi) + \mathbf{z}_q, & B_q = 0, \end{cases} \tag{11}$$

where $\mathbf{z}_q$ is the $N \times 1$ additive noise vector at the output of the qth matched-filter with the same statistics as that of $\mathbf{z}(t)$.

To detect the transmitted bits, we first apply a simple receive beamforming step, that is

$$y_q = \mathbf{b}^H(\phi)\mathbf{y}_q, \qquad q = 1, \ldots, Q. \tag{12}$$

Then, by performing a simple ratio test, we obtain

$$\hat{B}_q = \begin{cases} 1, & \text{if } |y_q| \geq T, \\ 0, & \text{if } |y_q| < T, \end{cases} \tag{13}$$

where T is a threshold.

IV. Simulation Results

We consider two simulation examples. In both examples, a transmit uniform linear transmit array consisting of $M = 10$ antennas spaced half a wavelength apart is considered.

Example 1. In the first example, we investigate the possibility of synthesizing transmit power distribution patterns with a fixed mainbeam towards a specific spatial direction and variable SLLs towards multiple spatial communication directions. We design two transmit weight vectors which focus their mainbeam towards the direction $\theta_{\text{radar}} = 20°$. Three communication receivers are assumed to be located at directions $\theta_1 = -50°$, $\theta_2 = -20°$, and $\theta_3 = 40°$, respectively. The communication SLLs associated with the first and second transmit weight vectors are constrained to be at -20 dB and -30 dB with respect to the mainbeam, respectively. For all other sidelobe directions, the SLLs are controlled by choosing $\varepsilon = 0.1$. Fig. 1 shows the normalized transmit power distribution versus the spatial angle for both transmit weight vectors. The figure shows that the two transmit weight vectors have almost the same pattern within the mainbeam. This means that if the waveform is radiated via either transmit beam, the radar operation will not be affected. The figure also shows that the SLLs towards the communication directions are separated from each other with 10 dB difference. Therefore, information embedding can proceed by choosing a certain waveform to be radiated over either of the transmit beams.

Example 2. In the second example, we investigate the performance of the proposed algorithm in terms of the bit error rate (BER). We also demonstrate secure communications by calculating the probability of intercept from directions other than the intended user directions. The performance of the proposed method is compared with the recent technique proposed in [10]. Note that the method described in [10] does not employ waveform diversity. Instead, it employs a single waveform in tandem with 2^Q SSLs towards the communication direction to deliver Q bits of information. In this example, we assume that the main radar operation takes place within the sector of $\mathbf{\Theta} = [-10° \ 10°]$. A single communication direction towards $\theta = -50°$ is assumed. It is also assumed that the radar operation requires that the power level emitted in the sidelobe areas to be at least 20 dB lower than the mainlobe. We consider $Q = 2$ bits of information is needed to be transmitted during every radar pulse. For the proposed method, two transmit wight vectors are used along with two orthogonal waveforms. For the method in [10], four transmit weight vectors are required. We design all four transmit weight vectors by solving (5)–(7) using $\delta_1 = 10^{-1},\ \delta_2 = \sqrt{10^{-3}},\ \delta_3 = 10^{-2}$, and $\delta_4 = \sqrt{10^{-5}}$, respectively. The first two of the designed vectors are used for the proposed method while all four are used to implement the method of [10]. For comparison, Fig. 2 shows the transmit power distribution patterns for all four transmit weight vectors. It can be observed from the figure that all transmit power distribution patterns are almost the same within the sector $\mathbf{\Theta}$, while the SLLs towards the communication directions are separated from each other by 10 dB.

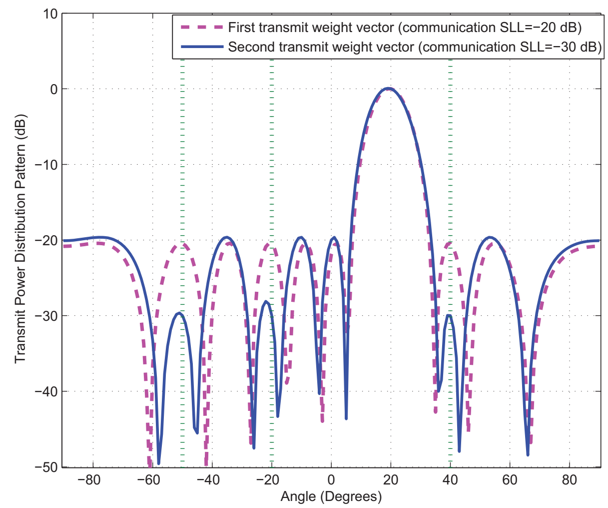

Fig. 1. Transmit power distribution versus spatial angle; first example.

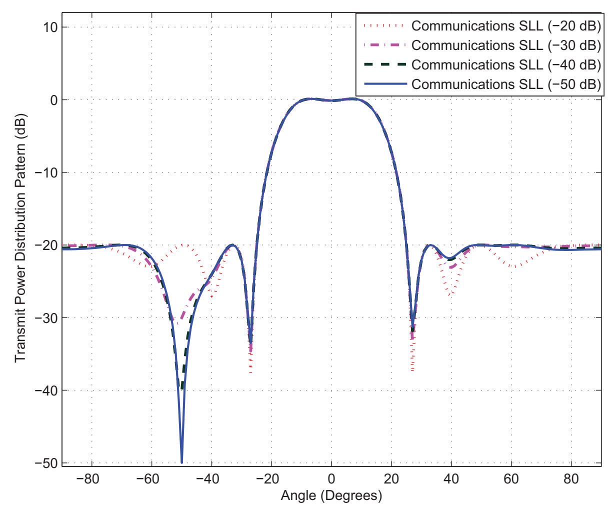

Fig. 2. Transmit power distribution versus spatial angle; second example.

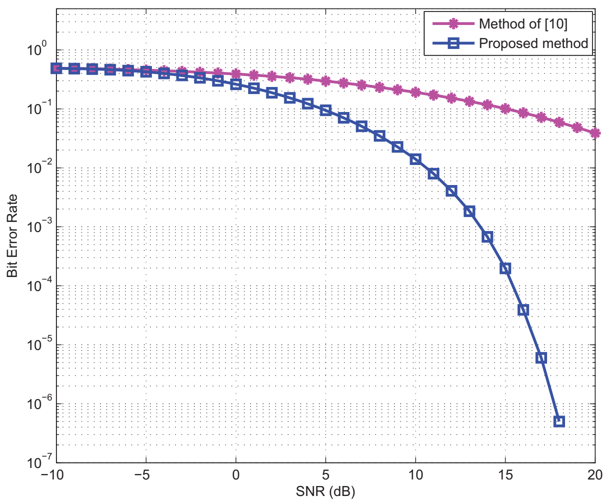

Fig. 3. BER versus SNR; second example.

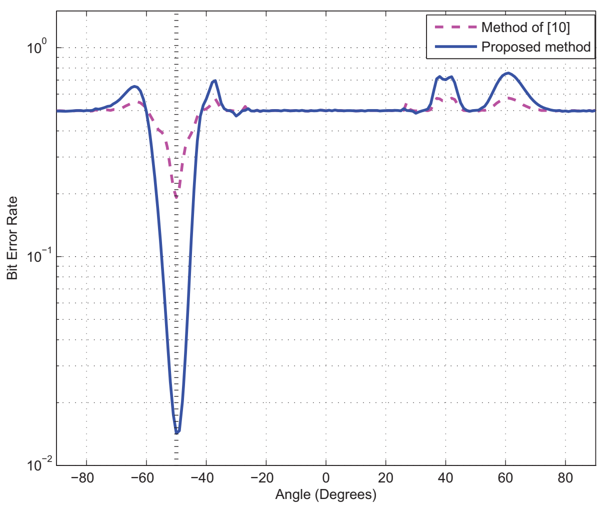

Fig. 4. BER versus angle of departure; second example.

To test the BER, 10^6 symbols (two bits each) are transmitted. The communication receiver is assumed to be equipped with $N = 15$ receive antennas arranged in a non-uniform linear array. The total aperture of the receive array is fixed to 7 wavelength. Fig. 3 shows the BERs for the two methods tested versus the signal-to-noise ratio (SNR). It is clear that the proposed technique achieves a superior BER performance as compared to the method of [10]. Note that the method of [10] transmits 25% of the information via each of the four beams. The very low SLLs associated with the third and fourth beams at -40 dB and -50 dB, respectively, result in poor SNR at the receiver and cause symbol detection to fail. This phenomenon is expected to be worse if longer binary sequences are used.

Finally, Fig. 4 depicts the BER versus the spatial angle with the SNR fixed to 10 dB for both methods. It can be observed from the figure that, for both methods, the BER becomes quite high for directions other than the intended user direction. This means that both methods have inherent security against information interception from directions other than the communication direction. It can be also confirmed from the figure that the proposed method has better BER towards the intended user direction as compared to the method of [10].

V. Conclusion

A new technique for dual-function radar-communications system using sidelobe control together with waveform diversity has been developed. In order to deliver a sequence of Q bits during each radar pulse, a set of Q orthogonal waveforms is needed in tandem with appropriate transmit beamforming. For each waveform, we designed two transmit weight vectors to achieve distinct transmit power distribution patterns, which share the same radar main beam patterns but have different sidelobe levels towards the communication directions. The latter enables a communication receiver to interpret the received waveform to an information bit in each orthogonal waveform. The proposed technique permits information delivery to a single or multiple communication directions as long as they are located outside the mainlobe of the radar. The communication process is inherently secure against intercept from directions other than the preassigned communication directions. The effectiveness of the proposed technique and its superiority over existing techniques was verified through simulations in terms of the bit error rate (BER) performance.

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