

FM Jammer Suppression in GPS Receivers Exploiting Sparsity and Chirp Signal Approximations

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BIOGRAPHY

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ABSTRACT

Anti-jamming capability is critical for GPS receivers operated in challenging environments as GPS reception is vulnerable to jammers. An important class of jammer is frequency modulated (FM) signals whose waveforms exhibit nonstationarity by the virtue of linearly or nonlinearly sweeping the signal bandwidth over time. These FM jammers display clear signatures in the time-frequency (TF) domain. As such, suppression of FM jammers is typically performed by utilizing the joint time-frequency domain for signal representations. The jammer instantaneous frequency (IF) is first estimated through linear or bilinear TF analysis methods, and then this estimate is used to remove the jammer through a variety of filtering and projection techniques.

In this paper, we consider an effective method for complex jammer IF signature estimation. Our focus is on the case where the received signals are incomplete, i.e., the received GPS signals, contaminated by FM jammers, do not follow the typical assumption of uniform Nyquist sampling. Rather, some data samples are missing due to, for example, impulsive noise removal, line-of-sight obstruction, and/or multipath fading.

The method proposed in this paper is based on sparse TF signature reconstruction for jammer IF estimation. It utilizes the fact that the FM jammer signals have continuous IF signatures that are well approximated by piece-wise chirps. Such approximation calls for using chirp atoms in sparse signal TF representation. It is demonstrated that the use of chirp dictionary is deemed to enhance IF estimation with improved IF signature continuity compared to the conventional sinusoid dictionary.

INTRODUCTION

Ubiquitous use of Global Navigation Satellite Systems (GNSSs) including GPS in civilian, security, and defense applications, and the growing dependence in many

aspects of life have created a vulnerability resulting from intentional or unintentional interference sources. GNSS receivers are currently used in variety of critical infrastructural services including communications, power grid distribution, finance, emergency services, aviation, active sensing, high-precision surveying, and a number of other industries. Financial corporations rely on GNSS receivers to provide precise timing for high-frequency trading. Wireless networks and cell phone base stations use GNSS timing to coordinate signal handshakes that render communications possible. Bi-static and multi-static radars as well as multiple-input multiple-output (MIMO) system configurations rely on GPS for transmitter-receiver synchronizations. Accurate satellite signal parameter estimation is crucial for Position, Navigation and Timing (PNT) for GNSS applications. However, achieving this level of accuracy can be compromised by various interfering sources, which reduces the satellite signal acquisition and tracking accuracy. The interference problem is compounded by the fact that the desired GNSS signal is extremely weak with highly negative dB signal-to-noise and interference ratio.

An important class of jamming waveforms has frequency modulated (FM) signal structures, and is defined by their instantaneous frequencies (IFs). Therefore, these jammers assume clear signatures in the time-frequency (TF) domain which can be used for jammer characterization and suppression [1-5]. The jammer IF is first estimated through linear or bilinear TF analysis methods, and then used to remove the jammer signal through a variety of filtering and projection techniques [6].

In this paper, we consider an effective method for jammer IF signature estimations. We focus on the situations where the received signals are labeled as incomplete, owing to missing data samples resulting from, for example, impulsive noise removal, line-of-sight obstruction, and/or multipath fading. In these situations, conventional TF analysis techniques of nonstationary jammer signals, e.g., FM jammer waveforms fail, as they result in a high level of noise-like artifacts that clutter the TF domain, obscuring the FM jammer, and disabling a reliable estimation of its IF signatures.

Our earlier work shows that, in missing sample situations, FM jammer suppression can be achieved by using emerging compressive sensing and sparse reconstruction techniques [7, 8]. In essence, we exploit the fact that FM jammers are sparse in the joint-variable domain, that is, only a small number of frequencies at any given time are occupied, whereas the rest of the local frequency spectrum assumes zero or negligible values. This jammer sparsity property exploitation can be captured in two steps. First, the GPS receiver data is represented in the ambiguity domain, where a data-dependent time-frequency kernel is applied to suppress cross-terms and noise-like artifacts. Second, the kernelled ambiguity function (AF) is transformed to the sparse TF domain by

performing compressive sensing techniques. While there are many ways to reconstruct the sparse TF signature of the jammer its ambiguity domain information, we advocated the use of one-dimensional (1-D) Fourier dictionary in lieu of two-dimensional (2-D) based reconstruction. This was made possible by invoking the intermediate domain separating the TF domain and ambiguity domain, referred to as the instantaneous autocorrelation function (IAF) domain. Sparse reconstruction was made simpler by utilizing the 1-D relationship between the TF domain and IAF.

A noticeable drawback of the above approach is that it treats each time instant separately and thus does not factor in the fact that the FM jammer signals have continuous IF signatures. Such continuity can be enforced using a prior in the context of sparse Bayesian learning [9]. Alternatively, the FM jammer can be approximated by piece-wise chirps in this paper [10]. Naturally, this approximation becomes exact for linear FM jammers and less accurate for jammers signals with higher order polynomial phase structures. The piece-wise chirp approximation calls for using chirp atoms and a dictionary consistent with the signal TF representation. In this case, the dictionary, which linearly relates the sparse chirp parameters to the corresponding windowed observations, is overcomplete and accounts for possible jammer sweeping rates. The use of chirp dictionary is deemed to enhance IF estimation with improved IF signature continuity compared to the conventional sinusoid dictionary.

We use orthogonal matching pursuit (OMP) algorithm to perform the sparse FM jammer IF estimation over partially overlapping time segments. The reconstructed local chirps that correspond to overlapping windows are fused to improve the FM jammer IF estimation accuracy [11]. Following sparse reconstruction, removal of FM jammer is achieved through stationarization to direct-current (DC) component, applying notch filtering and then restoring the GPS signal through remodulation that reverse the stationarization operation [7, 12].

In this paper, we first introduce the signal model of the jammer GPS signal with missing samples. Jammer IF estimation based on sparse reconstruction exploiting chirp atoms is then described. Simulation results are presented to verify the effectiveness of the proposed techniques.

SIGNAL MODEL

We consider the reception of a GPS signal that is contaminated by jammer and noise. The signal at the receiver antenna is expressed as:

$$y(t) = s(t) + u(t) + n(t), \quad (1)$$

where $s(t)$ and $u(t)$ are, respectively, the GPS signal and the jammer signal, and $n(t)$ is the additive white Gaussian noise with zero mean and variance σ^2 .

Assume that signal $y(t)$ is sampled at the GPS chip

rate into T samples $1 \leq t \leq T$. Consider the thinned sampling of the observation with a random pattern, where the number of missing samples is $0 \leq M < T$. As such, the thinned or compressed observations $x(t)$ are expressed as the product of $y(t)$ in (1) and the following observation mask, i.e.,

$$x(t) = b(t)y(t), \quad (2)$$

where

$$b(t) = \begin{cases} 1, & \text{if } t \in \mathbb{S}, \\ 0, & \text{if } t \notin \mathbb{S}, \end{cases} \quad (3)$$

where $\mathbb{S} \subset \{1, \dots, T\}$ is the set of observed time instants and its cardinality is $|\mathbb{S}| = T - M$.

JAMMER IF ESTIMATION THROUGH SPARSE RECONSTRUCTION

The FM jammer signals $j(t)$ can be expressed as:

$$u(t) = A \exp(j\phi(t)), \quad t = 1, \dots, T, \quad (4)$$

where A is the magnitude, and $\phi(t)$ is time-varying phase.

The proposed approach builds on the local approximation of each signal component as a chirp. That is, by dividing the observation time interval into (possibly overlapping) time windows of a judiciously chosen duration, T_w , the discrete-time signal over each window is approximated by:

$$x_m(n) \approx C_m \exp \left\{ j2\pi \left[\frac{1}{2} \alpha_m n^2 + \beta_m n \right] \right\} + v(n), \quad 0 \leq n \leq N_w - 1, \quad (5)$$

where m is the window index, C_m , α_m , and β_m are the complex amplitude, the chirp rate, and the initial frequency of over the m th window, $x_m(n) = x(mL+n)$ and $v_m(n) = v(mL+n)$, with L being the shift between two consecutive windows in terms of number of samples, and $N_w = \lfloor T_w / T_s \rfloor$.

To apply sparse reconstruction, we first discretize the 2-D search space Ω of α_m and β_m . Denote I as the total number of chirp rates in the discrete dictionary. Denote the i th chirp rate in the dictionary as α_i , and let $\beta_{i,j}$ denote the corresponding possible values for the initial frequency, where $j = 1, \dots, J_i$.

In a vector form, the signal over the m th window can be expressed as:

$$\mathbf{x}_m = \mathbf{\Psi} \mathbf{s}_m + \mathbf{v}_m, \quad (6)$$

where $\mathbf{x}_m = [x_m(0), \dots, x_m(N_w - 1)]^T$, $\mathbf{v}_m = [v_m(0), \dots, v_m(N_w - 1)]^T$, $(\cdot)^T$ denotes the transpose operation, and \mathbf{s}_m is a K -sparse amplitude vector of length $J = \sum_{i=1}^I J_i$. In addition, $\mathbf{\Psi}$ is the dictionary matrix, defined as:

$$\mathbf{\Psi} = [\mathbf{\Psi}_1, \mathbf{\Psi}_2, \dots, \mathbf{\Psi}_I], \quad (7)$$

$$\mathbf{\Psi}_i = [\boldsymbol{\psi}_{i,1}, \boldsymbol{\psi}_{i,2}, \dots, \boldsymbol{\psi}_{i,J_i}]^T, \quad (8)$$

where the n th element of $\boldsymbol{\psi}_{i,j}$ is given as

$$[\boldsymbol{\psi}_{i,j}]_n = \exp[j\pi(\alpha_i n^2 + 2\beta_{i,j} n)]. \quad (9)$$

Equation (5) is a typical sparse reconstruction problem that can be solved by various compressive sensing algorithms, such as OMP, LASSO and Bayesian compressive sensing [13-16]. In this paper, the orthogonal matching pursuit (OMP) method [13] is used.

The jammer IF can be estimated once the coefficients of the chirp components are obtained from sparse reconstruction as described above. The estimated local chirps of different overlapping windows that include a given time sample are fused to further improve the FM jammer IF estimation accuracy. A 50% overlapping implies that each time sample witnesses two reconstructed chirps which can be fused or averaged for better estimation.

Once the IF of the jammer signals is estimated, the jammer signal can be reconstructed up to the initial phase. As described in [7], we perform the signal stationarization, DC component removal, and re-modulation of the GPS signal. This is equivalent to performing time-varying filtering, which would be difficult to directly apply in the underlying problem due to missing data. The required phase accuracy can be relaxed by dividing the entire data into multiple segments for separated processing [12].

TIME-FREQUENCY DISTRIBUTIONS

We use TF distributions (TFDs) to analyze and visualize the FM jammer contamination in the received GPS signal. A nonstationary signal can be quadratically represented as joint variables in the TF domain, IAF domain, and the AF domain [17, 18]. The IAF of signal $x(t)$ is defined for time lag τ as

$$C_{xx}(t, \tau) = x(t + \tau)x^*(t - \tau). \quad (10)$$

The Wigner-Ville distribution (WVD) is known as the simplest form of TFD. The WVD is the Fourier transform of the IAF with respect to τ , defined as

$$W_{xx}(t, f) = \mathcal{F}_\tau[C_{xx}(t, \tau)] = \sum_\tau C_{xx}(t, \tau) e^{-i4\pi f \tau}, \quad (11)$$

where f represents the frequency. Note that 4π is used in the DFT instead of 2π because the time-lag τ takes integer values in (10). It is clear that WVD maps 1-D signal $x(t)$ in the time domain into 2-D signal representations in the TF domain. The fundamental TFD property of concentrating the FM jammer energy at and around its IF, while spreading the GPS signal and noise energy over the entire TF domain, enables effective jammer and signal separations when considering the time and frequency variables jointly.

SIMULATION RESULTS

Without loss of generality, we consider an FM jammer

impinging on the receiver along with a GPS signal. The normalized IF law of the jammer signal is chosen as

$$f(t) = 0.05 + 0.05t/T + 0.3t^2/T^2, \quad (12)$$

for $t = 1, \dots, T$, where the number of the sampling data is chosen as $T=256$. We assume that 50% of the data samples are randomly missed. In the following simulations, we set the input signal-to-noise ratio (SNR) of the GPS signal as -16dB, and the input jammer-to-noise ratio (JNR) of the FM jammer signal as 25dB.

One realization of the real-part waveform of the jammed GPS signal is shown in Fig. 1. The red dots depict the positions of the missing data samples. Because of the high JNR and jammer-to-signal ratio (JSR), the waveform is dominated by the jammer. The corresponding WVD is depicted in Fig. 2. Affected by the missing data samples, the WVD is highly cluttered by the artifacts which make it difficult to estimate the IF of the jammer signal.

Fig. 3 shows the sparse reconstruction result utilizing the proposed method where chirp atoms are used to build the sparse reconstruction dictionary. Fig. 4 shows that the estimated jammer IF is accurate when compared with the true IF, except in the two edges due to zero-padding. The WVD of the signal after the jammer suppression is shown in Fig. 5. It is clear that the jammer is effectively suppressed.

For comparison, and to demonstrate the appropriateness of the proposed chirp dictionary, we depict in Fig. 6 the sparse reconstruction result when a sinusoidal dictionary is used. Clearly this dictionary does not lend itself to provide accurate jammer IF signatures. It should be mentioned that a sinusoidal dictionary implicitly assumes staircase piece-wise approximation of the IF, rather than chirp-like.

CONCLUSIONS

We have considered the FM jammer suppression in a GPS receiver, where the observed data samples are incomplete. A sparse reconstruction method was used to exploit the sparseness of the jammer FM signature in the joint-variable TF domain. The employed dictionary is of a chirp atoms which are more appropriate than sinusoidal atoms for the reconstruction of a TF signature characterized by IF. The jammer phase information was obtained from the IF estimate and used to to stationarize the jammed signal, leading to its effective suppression.

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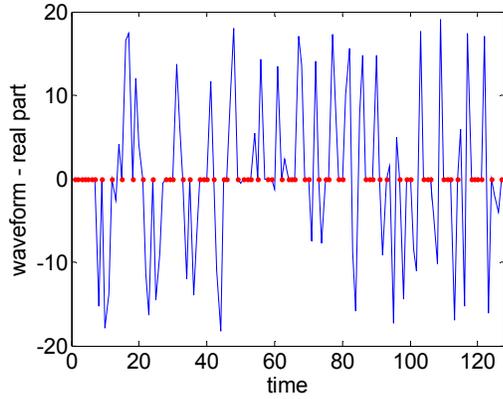


Fig. 1. Real-part waveform of jammed signal

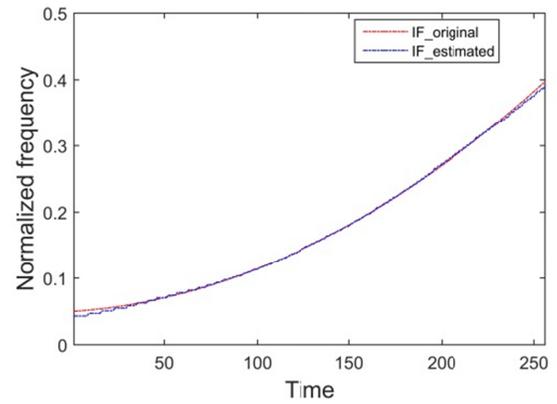


Fig. 4. Comparison of the true and the estimated jammer IF signatures.

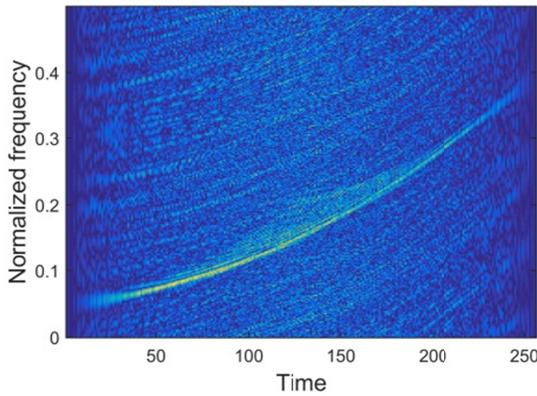


Fig. 2. WVD of the jammed GPS signal

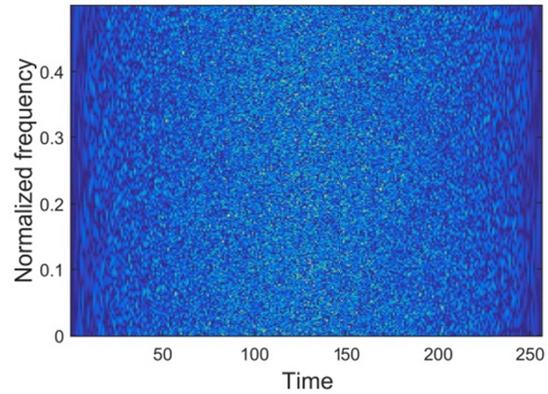


Fig. 5. WVD of the signal after jammer suppression

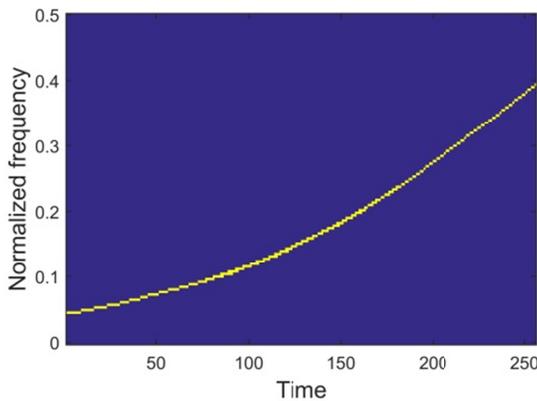


Fig. 3. Sparse reconstruction using chirp dictionary

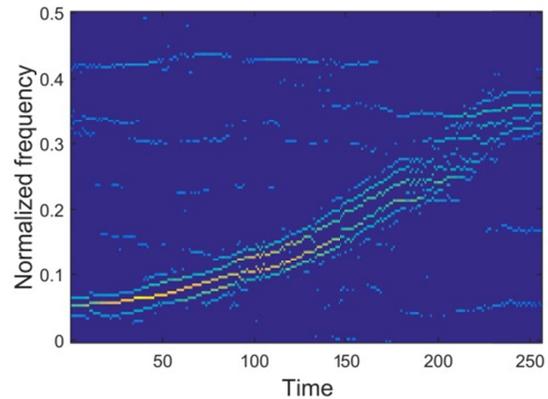


Fig. 6. Sparse reconstruction using sinusoid dictionary