Multi-Sensor Excision of Sparsely Sampled Nonstationary Jammers for GPS Receivers

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BIOGRAPHY

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ABSTRACT

Anti-jamming capability is essential for reliable GPS operations in hostile environments. A large class of “smart” jammers assumes nonstationary frequency modulated (FM) waveforms. This makes jammer suppression difficult when using conventional jammer suppression methods, such as frequency-domain notch filters or time-domain gaiting. We utilize two important spatio-temporal properties of the jamming FM signals. The first property is the sparsity of the waveforms in the joint time-frequency domain in which the jammer energy is concentrated in narrow ridges across the time and frequency variables. The second property is the highly correlated jammer waveforms received by multi-antenna receivers, enabling the use of spatial degrees of freedom for effective jammer suppression and GPS signal preservation.

In this paper, we develop novel techniques for FM jammer suppression of sparsely sampled GPS signals in the context of multi-sensor GPS receivers. The following important issues are addressed for the first time in this context: (a) We develop a multi-sensor data-dependent time-frequency kernel design method for suppression of noise-like artifacts arising from incomplete observations; (b) We utilize proper sparse compressive sensing techniques for FM jammer signature estimation that preserves phase information at each array sensor; (c) We develop multi-sensor array processing methodology that achieves effective jammer suppression and GPS signal preservation based on reconstructed group sparse data estimates and through the exploitation of spatial degrees of freedom offered by the sensor array; and (d) We examine the effect of different sparse sampling patterns and reveal that sensor-dependent missing samples is key to improved jammer signature estimation and jammer removal capability.

INTRODUCTION

Anti-jamming capability is essential for reliable GPS operations in hostile environments. A large class of “smart” jammers assumes nonstationary frequency modulated (FM) waveforms. Such waveforms obscure the GPS signal across the entire GPS frequency band and over consecutive time periods. This makes jammer suppression difficult when using conventional jammer suppression methods, such as frequency-domain notch filters or time-domain gaiting. It is more effective to exploit the sparsity of jammer waveforms in the joint time-frequency (TF) domain in which their energy is concentrated in narrow ridges across the time and frequency variables [1-5]. With an accurate estimation of the jammer signal TF behavior, nonstationary jammer suppression methods, including time-varying notch filtering, can be successfully applied. A number of methods have been developed for parametric and nonparametric estimations of FM jammer signals [6-9]. It is noted that, irrespective of the employed suppression method, the use of spatial degrees of freedom through multi-antenna receiver configuration has invariably led to more effective jammer suppression and GPS signal preservation [4, 10].

Traditional anti-jamming GPS receiver signal processing assumes that the following two conditions are satisfied: (a) The propagation channel conditions are time-invariant; and (b) The GPS signals contaminated by the jammers are uniformly sampled above the Nyquist sampling rate. In practical operations, however, GPS and jammer signals often experience random time-varying channel conditions due to, for example, multipath fading and line-of-sight (LOS) obstruction. Such phenomena are frequently observed in ground GPS operation in heavy urban environments, such as “city canyon”. Missing data may also be resulted from discarding samples contaminated by impulsive noise. Sources of impulsive noise may, for example, include motor ignition noise and wideband radar emissions using narrow pulse, frequency-hopping, and other types of TF-selective waveforms. As a result, the observed data may be sparsely sampled described by missing samples from Nyquist sampled data. In these situations, parametric jammer estimation neither yields accurate characterization nor does it lend itself to effective removal of jammer signals. On the other hand, nonparametric TF analysis becomes unreliable as missing samples yields a high level of noise-like artifacts.

Recently, we have developed a novel method for effective jammer suppression by exploiting the local sparsity of FM jammer signals [11]. Reconstruction of FM jammer signals from sparsely sampled observations falls under the emerging area of compressive sensing and sparse reconstruction where, in this case, sparsity is exhibited in the frequency domain when the signal is locally viewed through a short window. In such approaches, FM jammer instantaneous frequency estimation based on sparse signal reconstruction is built upon the linear Fourier relationship between the TF domain and other joint-variable domains [12-14]. By exploiting data-dependent kernel and applying sparse signal reconstruction, high-accuracy instantaneous frequency (IF) estimation is achieved even when a high number of data samples is missing.

Considering the fact that many existing anti-jamming GPS receivers are equipped with multi-sensor array processing capability, the objective of this paper is to develop novel techniques for FM jammer suppression of sparsely sampled GPS signals in the context of multi-sensor GPS receivers. The output of the anti-jamming processing yields “clean” GPS signals for geolocation through standard dispreading and discrimination operations. The added spatial domain degrees of freedom benefit jammer TF signature estimation, jammer removal, and GPS signal preservation. Reaping these benefits, however, requires the developed anti-jamming algorithm to properly utilize the multi-sensor array GPS receiver platform. In particular, the following important issues are addressed in this paper: (a) In achieving effective suppression of noise-like TF domain artifacts due to missing data samples, adaptive TF kernels are designed to incorporate multi-sensor observations; (b) Group sparsity of the FM jammer signature over multiple array sensors, that is, the data observed in each sensor corresponds to identical time-varying instantaneous frequency signature but with different complex coefficients, should be exploited for enhanced FM jammer signature estimation. The applied group sparse compressive sensing method should generate individual complex signature estimate corresponding to each sensor. In particular, the phase information of the FM jammer in each sensor should be preserved; (c) Using multiple sensors enables reliable jammer signature estimation and effective jammer removal even under a large number of missing samples; and (d) We examine the effect of identical or different sparse sampling patterns across the array sensors, and reveal that sensor-dependent sparse sampling enhances jammer signature estimation and jammer removal capability.

In this paper, after presenting the signal model, we first review the concept of jammer suppression, and the joint space-time subspace projection is exploited as a means to perform jammer suppression. We then provide a detailed description of the estimation of the temporal and spatial jammer signatures, including the sparsely sampled multi-sensor signal model, TF analysis, multi-sensor TF kernel design, and group sparsity based IF signature estimation. The effectiveness of the proposed work is validated using simulation results.

NOTATIONS

We use lower-case (upper-case) bold characters to denote vectors (matrices). In particular, $\mathbf{I}_N$ denotes the $N \times N$

identity matrix. $(\cdot)^*$ denotes complex conjugation, and $(\cdot)^T$ and $(\cdot)^H$, respectively, denote the transpose and Hermitian (conjugate transpose) operations. $\mathcal{F}_x(\cdot)$ and $\mathcal{F}_x^{-1}(\cdot)$ respectively represent the discrete Fourier transform (DFT) and inverse DFT (IDFT) with respect to x. $\|\cdot\|_1$ and $\|\cdot\|_2$ respectively denote the l_1 and l_2 norm operations. In addition, $\otimes$ denotes the Kronecker product.

SIGNAL MODEL

GPS signals and the associated jammers adhere to the narrowband signal model. Considering an N-elements array that receives GPS signals contaminated by a jammer and noise, the discrete-time received signal vector can be expressed as

$$\mathbf{y}(t)=\sum_{i=1}^{Q_s}\boldsymbol{a}(\theta_i)s_i(t)+\boldsymbol{a}(\theta_j)s_j(t)+\mathbf{n}(t), \tag{1}$$

where $t\in[1,\cdots,T]$, and Q_s is the number of the GPS signals. In addition, $\boldsymbol{a}(\theta)$ is the $N\times 1$ steering vector of the array corresponding to a direction of signal arrival θ, and $s_i(t)$ and $s_j(t)$ are waveforms of the i-th GPS signal and the jammer, respectively. The jammer signal is assumed to be an FM signal. $\mathbf{n}(t)$ is the $N\times 1$ additive white Gaussian noise vector with zero mean and covariance matrix $\sigma^2\mathbf{I}_N$.

Consider the thinned sampling of the array observations with a random pattern applied to each array sensor, where the number of missing samples is $0\le M_n<T$ for n = 1, …, N. As such, for the n-th array sensor, the thinned observation $x_n(t)$ can be expressed as the product of $y_n(t)$, the n-th element of $\mathbf{y}(t)$ in (1), and the following observation mask,

$$b_n(t)=\begin{cases}1, & \text{if } t\in\mathbb{S}_n,\\ 0, & \text{if } t\notin\mathbb{S}_n,\end{cases} \tag{2}$$

where $\mathbb{S}_n\subset\{1,\cdots,T\}$ is the set of observed time instants and its cardinality is $|\mathbb{S}_n|=T-M_n$. For simplicity but without loss of generality, we assume M_n to be the same for all sensors, i.e., $M_n=C$, for $n=1,\cdots,N$, whereas $b_n(t)$ may or may not be the same.

JAMMER SUPPRESSION BASED ON JONT SPACE-TIME DOMAIN PROJECTION

In this paper, we use the orthogonal projection scheme for effective jammer suppression. That is, the received signal vector, defined over the joint space and time domain, is projected into the orthogonal subspace of the jammer [4].

Consider that the temporal signature of the jammer is considered over L time samples, expressed as

$$\mathbf{s}_j=[s_j(0),...,s_j(L-1)]^T. \tag{3}$$

The Kronecker product of the temporal signature and its spatial signature, $\boldsymbol{a}(\theta)$, yields its overall subspace defined in the joint space-time domain, denoted as $\mathbf{v}_j=\mathbf{s}_j\otimes\mathbf{a}(\theta_j)$. The projection matrix into the orthogonal subspace of the jammers is given by

$$\mathbf{P}=\mathbf{I}_{LN}-(\mathbf{v}_j^H\mathbf{v}_j)^{-1}\mathbf{v}_j\mathbf{v}_j^H. \tag{4}$$

In such an approach, the temporal domain can typically exploits a large dimension to ensure insignificant GPS signal loss in the projection-based jammer suppression, whereas the spatial domain degrees-of-freedom provide robust jammer suppression capability and additional GPS signal protection. The challenges underlying the proposed work lie in the requirement of accurate estimates of the jammer spatial and temporal signatures in the presence of missing samples in the observed data.

To achieve this objective, we consider the recently developed multi-sensor sparsity-aware TF analyses and signal filtering MUSIC (SF-MUSIC) techniques that are summarized below [14, 15]. We separately address the TF analysis of the jammer signals based on sparse data observations through adaptive TF kernel design and sparse signal reconstruction. Their IF laws, and subsequently their phase signatures, are then estimated. The jammer spatial signature is estimated using the SF-MUSIC.

TIME FREQUENCY REPRESENTATION USING MODIFIED ADAPTIVE OPTIMAL-KERNAL

The simplest bilinear TF distribution (TFD) is the Wigner-Ville distribution (WVD),

$$D(t,f)=\mathcal{F}_\theta^{-1}\{\mathcal{F}_\tau\ (A(\theta,\tau))\}, \tag{5}$$

which is defined as a two-dimensional (2-D) Fourier transform of the following ambiguity function (AF):

$$A(\theta,\tau)=\sum_t x(t+\tau)x^*(t+\tau)e^{-j4\pi\theta t}, \tag{6}$$

with θ and τ respectively denoting the frequency shift and the time lag.

WVD is known to provide the best TF resolution for single-component linear FM signals, but it is contaminated by cross-terms when the IF law is nonlinear or when a multi-component signal is present. A number of reduced interference distributions (RIDs) have been developed to reduce the cross terms [16, 17]. These RIDs apply a multiplicative kernel to the AF and function like a low-pass filter in the ambiguity domain. As the signal auto-terms are likely to be localized around the low-frequency region, whereas the cross-terms are away from the Doppler and lag axes, RID kernels generally preserve the signal auto-terms and suppress cross-terms. In addition, because white noise spreads over the entire TF domain, applying RID kernels also reduce the noise power. RID kernels can be classified into two major classes, i.e., data-independent and data-dependent. An example for the former is the Choi-Williams [18], whereas the adaptive optimal kernel (AOK) [19] is popularly used example for the latter.

When the observations are sparsely sampled, missing samples cause noise-like artifacts produced by the missing data. The effect of the artifacts due to missing data samples resembles that due to noise in the TF domain in the sense that they respectively spread over the entire TF region. Therefore, such effects can be mitigated using a proper TF kernel. The data-dependent kernels generally outperform the data-independent counterparts because they are adaptively optimized based on the signals.

It is shown in [14] that, in a multi-sensor platform, the design of AOK should not be separately performed at each array sensor, but rather it should utilize the data observed at all sensors to produce a single optimum kernel. Let $A_n(r,\psi)$ denote the AF of the n-th sensor in the polar coordinate with respect to the radius r and angle ψ, where $n=1,\cdots,N$. The modified AOK solves the following optimization problem which replaces the sensor AF by the average AF, i.e.,

$$\begin{aligned} &\max_{\Phi} \quad \int_0^{2\pi}\int_0^{\infty} | A_\Sigma(r,\psi)\Phi(r,\psi)|^2 \; rdrd\psi \\ &\text{subject to} \quad \Phi(r,\psi)=\exp\left(-\frac{r^2}{2\sigma(\psi)}\right), \\ &\frac{1}{2\pi}\int_0^{2\pi} |\sigma(\psi)|^2 \; rd\psi \le \alpha, \end{aligned} \tag{7}$$

where the averaged AF over all sensors is obtained as

$$A_\Sigma(r,\psi)=\frac{1}{N}\sum_{n=1}^{N} A_n(r,\psi), \tag{8}$$

and α is a constant which influences the tradeoff between cross-term suppression and auto-term preser-vation.

The improvement of kernel design through such average is due to the phase invariance of the jammer auto-terms with respect to the sensors, whereas the cross-terms between different jammer signal components as well as between the jammer signals and noise change phases with respect to the sensors [7, 20, 21]. That is, averaging the AFs across all array sensors enhances the jammer auto-terms and mitigates the effects of cross-terms, noise, and missing samples artifacts. The effectiveness of artifact suppression is particularly pronounced when the different sensors have distinct missing sample patterns [14].

INSTANTANEOUS FREQUENCY ESTIMATION THROUGH SPARSE RCONSTRUCTION

Once the TF kernel is obtained from (8) for a multi-sensor platform, the TFD can be computed as the 2-D Fourier transform of the kernelled average AF, $A_\Sigma(r,\psi)\Phi(r,\psi)$, i.e.,

$$D(t,f)=\mathcal{F}_x^{-1}\{\mathcal{F}_x\;(\tilde{A}_\Sigma(\theta,\tau))\}. \tag{9}$$

Alternative to the Fourier transform, we can obtain the TF signal representation (TFSR) through sparse recons-truction from the same kernelled average AF. While earlier sparse TFSR reconstructions were based on the 2-D Fourier transform relationship between the AF and the TFSR [22], it is shown in [12, 13] that the 1-D Fourier transform relationship between the instantaneous autocorrelation function (IAF) and the TFSR yields simpler computations and enables the exploitation of local sparsity in the TF domain.

The 1-D inverse Fourier transform of $\tilde{A}_\Sigma(\theta,\tau)$ with respect to θ yielding the kernelled IAF,

$$C(t,\tau)=\mathcal{F}_\theta^{-1}\{\tilde{A}_\Sigma(\theta,\tau)\}. \tag{10}$$

Denote $\mathbf{c}^{[t]}$ as a vector that consist of all IAF entries along the τ dimension corresponding to time t , and $\mathbf{w}^{[t]}$ as a vector contains all the TFSR entries with respect to the frequency for the same time t . According to the relationship between AIF and TFSR, the following equation can be obtained

$$\mathbf{c}^{[t]}=\mathbf{\Psi}\mathbf{w}^{[t]}+\boldsymbol{\varepsilon}^{[t]}, \qquad t\in\left[1,\cdots,T\right], \tag{11}$$

where the dictionary matrix $\mathbf{\Psi}$ is an inverse Fourier matrix. Because the IF of the jammers is sparse in the frequency domain corresponding to a constant t , the non-zero entries of $\mathbf{w}^{[t]}$ in (11) can be obtained through sparse reconstruction, formulated as

$$\begin{aligned} &\min_{\mathbf{w}^{[t]}} \|\mathbf{w}^{[t]}\|_1 \\ &\text{subject to } \mathbf{c}^{[t]}-\mathbf{\Psi}\mathbf{w}^{[t]}=0, \;\; \forall t. \end{aligned} \tag{12}$$

Many sparse reconstruction algorithms can solve the above problem, such as OMP, LASSO and Bayesian compressive sensing [23-26]. In this paper, we utilize the OMP algorithm to perform sparse reconstruction of the TFSR, which is repeated for each time instant. Then, we can attain the frequency estimation $\hat{f}(t)$ of the jammer signal.

SF-MUSIC FOR SPATIAL JAMMER SIGNATURE ESTIMATON

In order to estimate the spatial jammer signature using the SF-MUSIC algorithm, we stationarize the array data using the IF estimated from the above sparse reconstruction approach.

Denote $\hat{f}(t)$ as the estimated jammer IF, the corresponding phase trajectory ca be estimated as

$$\hat{\phi}(t)=2\pi\int_0^t \hat{f}(u)du. \tag{13}$$

Note the estimated phase trajectory is shared by all the antennas, up to the ambiguity of the initial phase which differs for each antenna.

The stationarization of the jammer signal is achieved by multiplying the received signal vector to the conjugated jammer signal, i.e.,

$$\tilde{\mathbf{y}}(t) = \mathbf{y}(t)\,\mathrm{e}^{-j\hat{\phi}(t)}. \tag{14}$$

As such, the stationarized jammer signal lies in or around the direct-current (DC) or zero-Doppler region. Performing the Fourier transform and selecting the DC component will accumulate the jammer power while de-emphasizing the other components. Utilizing the MUSIC algorithm to estimate the jammer DOA yields the signal filtering MUSIC (SF-MUSIC) [15] that outperforms the conventional MUSIC because of the underlying signal discrimination capability. In the underlying problem where a portion of data samples are missing, the proposed approach ensures that all the measured data are utilized. On the contrary, the conventional MUSIC can only obtain samples to compute the sensor-array covariance matrix when none of the sensors have missing samples. As such, the advantage of the SF-MUSIC becomes more pronounced when different missing patterns are assumed in each antenna.

It is noted that, when the estimated jammer IF is inaccurate due to noise perturbation or frequency discretization error, such integral may yield a high phase error. In practice, the entire time period can be partitioned into multiple segments so that the phase deviation in each segment is insignificant.

For the stationatized signal vector $\tilde{\mathbf{y}}(t)$, the temporal signature is changed to $\tilde{\mathbf{s}}_j = [1,\cdots,1]^{\mathrm{T}}$, and the corresponding spatio-temporal signature is expressed as

$$\tilde{\mathbf{v}}_j = \tilde{\mathbf{s}}_j \otimes \boldsymbol{a}(\hat{\theta}_i) = [1,\cdots,1]^{\mathrm{T}} \otimes \boldsymbol{a}(\hat{\theta}_i). \tag{15}$$

The orthogonal projection follows the same expression of (4) except that $\tilde{\mathbf{v}}_j$ is used in lieu of $\mathbf{v}_j$.

SIMULATION RESULTS

Simulations are carried out to verify the effectiveness of the proposed method. Without loss of generality, we only consider one GPS signal because different GPS signals are well isolated due to their orthogonal spreading codes. The array consists of two antenna elements (N=2) which are separated by a half-wavelength interelement spacing. We set the input signal-to-noise ratio (SNR) of the GPS waveform as –16 dB, and the input jammer-to-noise ratio (JNR) as 25 dB.

The normalized IF law of the FM jammer is expressed as,

$$f(t) = 0.025 + 0.025t/T + 0.15t^2/T^2, \tag{20}$$

for $t = 1, \ldots, T$, where the block size of the signal is chosen to be T = 512. The DOAs of the GPS waveform and the jammer signal are $0°$ and $5°$, respectively. We assume that 50% of the received data samples are randomly missing and the missing sample positions are uniformly distributed. In the simulations, the entire data observation period is divided into half-overlapping segments of length 32 for phase trajectory estimation and jammer suppression.

Identical Sampling Patterns

First, we show the simulation where identical sparse sampling patterns across the array are utilized. Fig.1 depicts the WVD of the received data samples from the first sensor. Due to the missing data samples, the WVD is cluttered by the artifacts which make it difficult to accurately estimate the IF of the jammer signal. To suppress the effect of the artifacts, the AOK is applied and the corresponding TFD is presented in Fig. 2(a), whereas the estimated IF signature utilizing the OMP is shown in Fig. 2(b). It is clear that the reconstructed IF closely follows the true one.

Fig. 3 compares the estimated spatial spectra of the jammer obtained from the SF-MUSIC and MUSIC. The conventional MUSIC does not provide accurate DOA estimate, whereas the SF-MUSIC achieves an accurate result. The WVD of the received signal in the first antenna after orthogonal projection operation is given in Fig. 4. It is clear from this figure that the jammer is effectively suppressed.

Different Sampling Patterns

In the following simulation, we examine the situation with different sparse sampling patterns in the two antennas. Due to the diversity in the sampling patterns, the effect of the artifacts due to missing data samples becomes less significant in the reconstructed TFD through the proposed sparse reconstruction. On the other hand, for the conventional MUSIC, a less number of data pairs can be obtained for the computation of the sensor-array covariance matrix, yielding degradation in the performance of the estimated spatial spectrum. The estimated jammer IF and spatial spectrum are respectively shown in Figs. 5 and 6.

CONCLUSIONS

In this paper, we developed a sparse sampling FM jammer suppression method in the multi-sensor GPS receiver platform. Due to the effect of the missing data samplings, the conventional time-frequency techniques fail to estimate the IF of the jammers which makes it difficult to suppress their FM signals. The proposed method first utilized the multi-sensor data-dependent kernel to suppress the artifact. Then, the IF of the jammer signals based on the sparse reconstruction algorithm is recovered and used to stationarize the jammer, enabling high esolution DOA estimnation to be undertaken. Utilizing the spatial degree of freedom offered by the multi-sensor array, a spatio-temporal subspace projection technique is proposed to achieve an effective jammer excision.

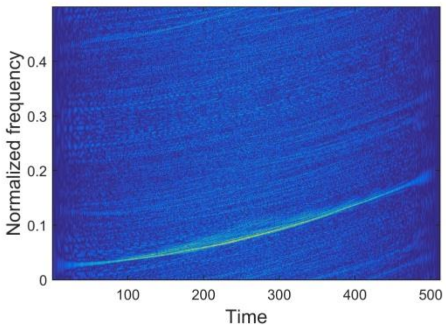
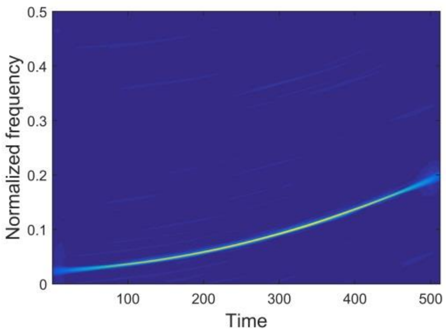
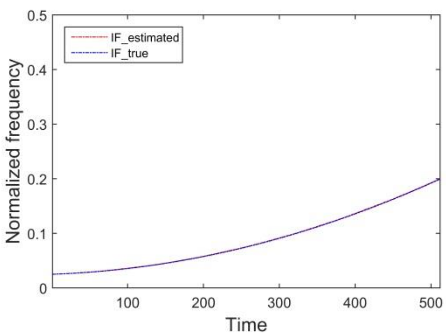


Fig. 1 WVD of the received data from the first sensor



(a) TFD using the modified AOK



(b) Estimated jammer IF signature

Fig. 2 TFD utilizing modified AOK and the estimated jammer IF signature (same missing pattern)

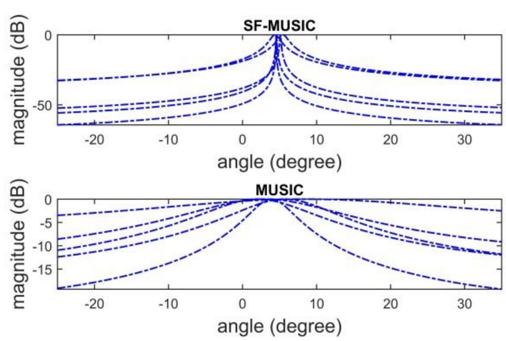


Fig. 3 Estimated spectra from SF-MUSIC and MUSIC (same missing pattern)

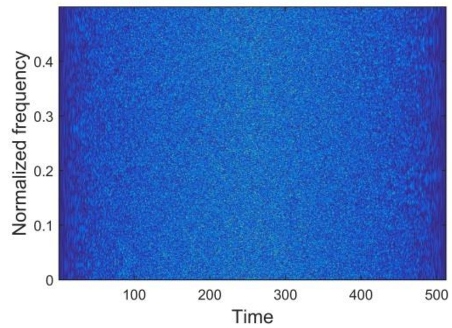
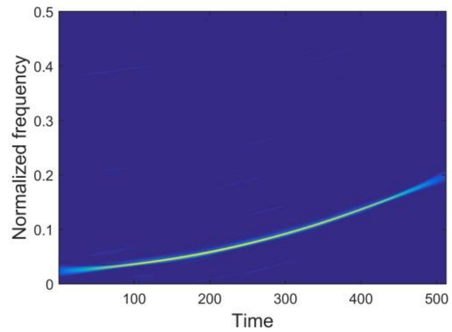
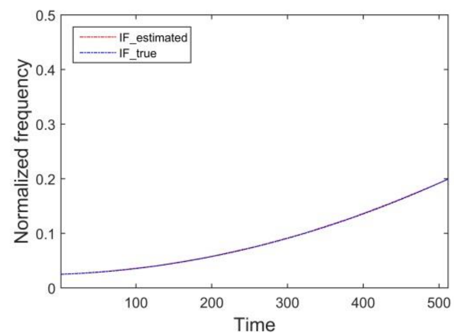


Fig.4 WVD of the signal after jammer suppression



(a) TFD using the modified AOK



(b) Estimated jammer IF signature

Fig. 5 TFD utilizing modified AOK and the estimated jammer IF signature (different missing patterns)

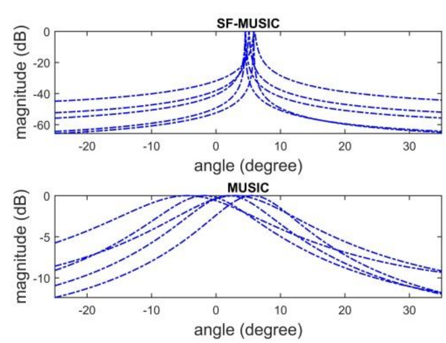


Fig.6 Estimated spectra from SF-MUSIC and MUSIC (different missing patterns)

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