Nonstationary Jammer Excision for GPS Receivers Using Sparse Reconstruction Techniques

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ABSTRACT

Nonstationary jammer excision has been the subject of interest for anti-jam GPS receivers for many years. A large class of "smart" jammers assumes frequency modulated (FM) waveforms that are characterized by their instantaneous frequencies (IFs). Several anti-jam techniques rely on the instantaneously narrowband signal characteristic and use joint-variable signal representations to reveal the jammer signature in the time-frequency (TF) domain. In this case, jammer excision becomes a two-step process, i.e., estimation of the TF signature or the IF of the jammer, and then removal of the jammer signal based on such estimate and with minimum distortion of the desired signal.

Jammer excision approaches commonly assume that the data is sampled uniformly at the Nyquist or higher sampling rate with all data samples available. There are situations, however, where the availability of the jammer signal sampled at the Nyquist rate is either unnecessary or impossible. For example, mitigating impulsive noise through discarding the high amplitude data samples renders the data to be "incomplete" or randomly sampled. In this case, FM jammer signals with missing data samples exhibit a high level of noise-like artifacts which clutter the entire TF domain, rendering TF signature and IF estimations difficult.

This paper provides a novel sparse reconstruction-based approach for effective jammer excision from incomplete signal observations. Jammer TF signature estimation is achieved by exploiting the fact that the FM jammers are locally sparse in the TF domain. Reconstruction of such signals from few random observations falls under the emerging area of compressive sensing (CS) and sparse reconstruction. Within the CS framework, jammer TF signature estimation is built upon the linear Fourier relationship between the TF domain and other joint-variable domains. In particular, the instantaneous auto-correlation function and the TF representations are associated by a one-dimensional Fourier relationship. This paper also describes TF kernels and its effect in the sparse reconstruction and mitigation of jammers.

INTRODUCTION

Nonstationary jammer excision has been the subject of interest for anti-jam GPS receivers for many years [1, 2]. A large class of nonstationary jammers assumes frequency modulated (FM) or polynomial phase signal (PPS) waveforms. These wideband "smart" jammers are characterized by their instantaneous frequencies (IFs). Several anti-jam techniques have been proposed which rely on the above instantaneously narrowband signal characteristic [1–3]. They use joint-variable signal representations to reveal the jammer signature in the time-frequency (TF) domain. In this case, jammer excision becomes a two-step process: the first step is to estimate the TF signature or the IF of the jammer, whereas the second step is to perform excision based on the estimate of the jammer signal. Both steps can be performed as a pre-processing prior to the correlation and dispreading loops of the GPS receivers. For the second step, some of the single-antenna temporal anti-jam techniques proceed to subtract the jammer from the data, whereas others prefer to perform data projection on the null space of the jammer to avoid performance degradation with signal subtraction when there are considerable phase estimation errors [3–6]. Towards the same goal of providing excision robust to jammer amplitude and phase, techniques have been devised which implement notch filters to remove the local jammer as its frequency changes over time or is translated to the origin through a startionarization process. Two important observations can be made regarding the above techniques: 1) The property that the jammer occupies very few TF points in the TF domain, i.e., sparse, is not fully exploited; 2) The data is assumed sampled uniformly at Nyquist or higher sampling rate with all data samples available. Whereas the first observation suggests the use of sparse signal reconstruction techniques, the second one calls for the application of compressive sensing when uniform Nyquist sampling is not attainable. When multiple sensors are available, the spatial degree of freedom serves to further separate the jammer from the GPS signal above and beyond what offered by the distinction in the respective TF signature characteristics [3, 7, 8].

There are situation, however, where the availability of the jammer signal sampled at the Nyquist rate is either unnecessary or impossible. Consider an impulsive noise present in the data in conjunction with a narrowband or a wideband jammer [9]. In this case, it becomes difficult to provide an accurate estimate of the jammer with the highly contaminating impulsive noise. Discarding the high amplitude data samples can remove most of the impulsive noise, rendering the data "incomplete" or randomly sampled [8, 10]. The GPS receivers are required to operate in a close proximity to various noise sources which, when acting alone, may impede signal acquisition and increase jitter errors and the bit error rates. Sources of non-Gaussian noise may, for example, include motor ignition noise, which is generated by spark plugs used in internal combustion engines, impulsive and noise-like waveforms generated by radar systems, and ultra-wideband emitters.

In addition to demonstrating the offering of sparse reconstruction in the area of FM jammer excision, the problem of non-uniform sampling or missing samples and their effects on jammer excision and correlation loops is considered and is an important part of this paper. FM jammer signals with missing data samples exhibit a high level of noise-like artifacts that clutter the entire TF domain, making the direct estimation of jammer TF signature and their IFs from their joint-variable representations difficult [10, 11].

This paper provides a novel compressive sensing-based approach for accurate jammer IF estimation and excision from incomplete signal observations at a single-antenna receiver. Jammer TF signature estimation is achieved by exploiting the fact that the FM jammers are locally sparse in the TF domain due to their power localizations at and around their IFs. Reconstruction of such jammer signals from few random observations falls under the emerging area of compressive sensing where the compressed observations can be in time, time-lag, or ambiguity domains [10–14].

Jammer TF sparse signature reconstruction builds upon the linear Fourier relationship between the TF domain and compressed observation domain. Depending on the specific domain representing the observation, the linear Fourier relationship may be one-dimensional or two-dimensional [12, 15]. In particular, the instantaneous auto-correlation function (IAF) and the TF representations are related by a one-dimensional Fourier transform relationship. With this underlying linear model, a number of methods become available for the reconstruction of sparse FM jammer signals after proper TF kernels are applied. Orthogonal matching pursuit (OMP) is one method that allows specification of the number of jammer components in each time instant [16]. In a different approach reminiscent of multiple window spectrograms, a reduced interference TF kernel is approximated by a small number of dominant eigen-components, each weighs, as a window, the compressed data prior to sparse reconstruction [13, 17]. This fits multiple measurement vector signal model, in which the signature support of the time-frequency is common among all windows. The problem can be effectively solved using group sparsity-based techniques, such as block OMP and multi-task Bayesian compressive sensing [18–22]. Recently, enhanced reconstruction of the FM signals with missing data is achieved by exploiting the structure of the FM signatures [22]. The proposed technique for jammer suppression under incomplete data builds on recent advances in TF analyses within the compressive sensing paradigm.

Note that compressive sensing and sparse reconstruction have been used for GPS receivers in the presence of missing data (e.g., [23]). The problem described in our paper, however, differs from that work as the underlying objective is to use compressive sensing techniques for effective jammer suppression.

NOTATIONS

A lower (upper) case bold letter denotes a vector (matrix). $(\cdot)^*$ denotes complex conjugation. $\mathcal{F}_x(\cdot)$ and $\mathcal{F}_x^{-1}(\cdot)$ respectively represent the discrete Fourier transform (DFT) and inverse DFT (IDFT) with respect to x. $\|\cdot\|_1$ and $\|\cdot\|_2$ respectively denote the l_1 and l_2 norm operations. In addition, $\delta(t)$ denotes the Kronecker delta function, and $\mathrm{var}(\cdot)$ denotes the variance.

SIGNAL MODEL

Without loss of generality and for the convenience of description, we consider the reception of one GPS satellite signal. The GPS signal, along with jammers and noise, is received at the receiver antenna, down-converted using a phase-synchronized down-converter and digitized using an analog-to-digital converter. The signals received by the receive antenna can be expressed, using complex baseband representations, as

$$x(t) = s(t) + j(t) + n(t), \quad (1)$$

where $s(t)$ is the GPS waveform, $j(t)$ is the jamming signal, and $n(t)$ is the additive noise which is assumed to be circular symmetric complex Gaussian with zero mean and variance σ_n^2. In practice, the GPS signals have a very low power before despreading, whereas a strong jammer is assumed, as the use of spreading spectrum signal equips standard GPS receiver with certain protection against weak jammers. The jammer signal is assumed to comprises a single or multiple components of FM signals.

Consider that the signal $x(t)$ is discretized with T samples for t = 1, ..., T. Denote $r(t)$ as its observation data with N missing samples, where $1 \le N < T$. The missing sample positions are assumed to be randomly and uniformly distributed over time. As such, $r(t)$ is the product of $x(t)$ and an "observation mask", $R(t)$, i.e.,

$$r(t) = x(t) \cdot R(t), \quad (2)$$

where

$$R(t) = \begin{cases} 1, & \text{if } t \in \mathbb{S}, \\ 0, & \text{if } t \notin \mathbb{S}. \end{cases} \quad (3)$$

$\mathbb{S} \subset \{1, \ ..., \ T\}$ is the set of observed time instants and its cardinality is $|\mathbb{S}| = T - N$. The observed waveform with missing samples can be expressed as the difference between the original waveform and the "missing samples", i.e.,

$$r(t) = x(t) - m(t), \quad (4)$$

where the missing data is expressed as

$$m(t) = x(t) \cdot M(t), \quad (5)$$

with $M(t)$ denoting the missing mask. To facilitate the analysis, we express the missing data mask as

$$M(t) = \sum_{i=1}^{N} \delta(t - t_i), \quad t_i \notin \mathbb{S}. \quad (6)$$

Accordingly, the missing signal is expressed as

$$m(t) = x(t) \cdot M(t) = \sum_{i=1}^{N} x(t)\delta(t - t_i) = \sum_{i=1}^{N} x(t_i)\delta(t - t_i), \ t_i \notin \mathbb{S}, \quad (7)$$

and the observed data with the missing samples is expressed as

$$r(t) = x(t) - m(t) = x(t) - \sum_{i=1}^{N} x(t_i)\delta(t - t_i), \ t_i \notin \mathbb{S}. \quad (8)$$

TIME-FREQUENCY REPRESENTATIONS

In this section, we consider the quadratic TF representation of the signals in the presence of entire data observations. The effect of missing samples is considered in the next section.

A signal can be quadratically represented as joint-variable in the TF domain, IAF domain, and the ambiguity function (AF) domain [24, 25]. The IAF of signal $x(t)$ is defined for time lag τ as

$$C_{xx}(t, \tau) = x(t + \tau)x^*(t - \tau). \quad (9)$$

The Wigner-Ville distribution (WVD) is known as the simplest form of TF distribution (TFD). The WVD is the Fourier transform of the IAF with respect to τ, defined as

$$W_{xx}(t, f) = \mathcal{F}_\tau[C_{xx}(t, \tau)] = \sum_\tau C_{xx}(t, \tau)e^{-i4\pi f \tau}, \quad (10)$$

where f represents the frequency. Note that 4π is used in the DFT instead of 2π because the time-lag τ takes integer values in (9). On the other hand, the IDFT of the IAF with respect to t yields the AF, expressed as

$$A_{xx}(\theta, \tau) = \mathcal{F}_t^{-1}[C_{xx}(t, \tau)] = \sum_t C_{xx}(t, \tau)e^{i2\pi ft}, \quad (11)$$

where θ is the frequency shift or Doppler.

It is clear that WVD maps 1-D signal $x(t)$ in the time domain into 2-D signal representations in the TF domain. The fundamental TFD property of concentrating the FM jammer energy at and around its IF, while spreading the GPS signal and noise energy over the entire TF domain, enables effective jammer and signal separations when considering the time and frequency variables jointly.

WVD is often regarded as the basic or prototype quadratic TFDs, since the other quadratic TFDs can be described as filtered versions of the WVD. WVD is known to provide the best TF resolution for single-component linear FM signals, but it yields high cross-terms when the frequency law is nonlinear or when a multi-component signal is considered. Various reduced interference kernels have been developed to reduce the cross-term interference [24, 25]. As such, the properties of a TFD can be characterized by the constraints on the kernel. Different kernels have been designed and used to generate TFDs with prescribed, desirable properties. While some kernels, e.g., the pseudo WVD and the Choi-Williams [26], assume fixed (signal-independent) parameters, other kernels, such as the adaptive optimal kernel (AOK), provide signal-adaptive filtering capability [27].

A TF kernel can be generally represented as a 2-D window function applied to the AF. As the AF and the IAF are associated with a 1-D Fourier relationship, the kernel operation can be treated as a mixed multiplication/ convolution in the IAF domain as

$$\begin{aligned} D_{xx}(t, f) &= \mathcal{F}_\tau[\sum_u \phi(t - u, \tau) C_{xx}(t, \tau)] \\ &= \sum_u \sum_\tau \phi(t - u, \tau) C_{xx}(t, \tau)e^{-i4\pi f \tau}. \end{aligned} \quad (12)$$

EFFECT OF MISSING DATA SAMPLES

It is demonstrated in [2] that, when full samples are present, the jammer auto-term will dominant and, thereby, a simple masking of strong TFD will lead to effective jammer suppression. When there are missing samples, as described in [10, 11], however, such missing samples generate a substantial presence of noise-like artifacts, cluttering the yielding TFDs. As such, the identification and masking of jammer components in the TFD domain become infeasible.

The received signal with missing samples can be decomposed into GPS signal, jammer, and noise as

$$\begin{aligned} r(t) &= x(t) - m(t) \\ &= s(t) + j(t) - n(t) - m_s(t) - m_j(t) - m_n(t), \end{aligned} \tag{13}$$

where $m_s(t)$, $m_j(t)$, and $m_n(t)$, respectively, represent the GPS signal, jammer, and noise components in the missing signal $m(t)$.

As the jammer signal is much stronger than the GPS signal and the noise, we focus on the study of the jammer component of (13) for TFD estimation and suppression. In this regards, the GPS signal and noise are considered weak interfering components. As a result, we rewrite (13) as

$$r(t) = j(t) - m_j(t) + \rho(t), \tag{14}$$

with $\rho(t) = s(t) - n(t) - m_s(t) - m_n(t)$ denoting the contribution of all other components.

The strong jammer-to-signal-plus-noise ratio makes the missing samples analysis of FM signal TF representation readily applicable to the underlying problem [10]. That is, the WVD of $r(t)$, $W_{rr}(t,\tau)$, represents an unbiased estimate of $W_{xx}(t,\tau)$ with scaling (corresponding to signal power reduction due to missing samples) and noise-like artifacts whose strength are determined by $W_{xx}(t,\tau)$ and the number of missing samples. More specifically, the missing data samples yield spreading artifacts that are randomly distributed over the entire TF domain, and the overall variance increases as the number of missing data samples increases. For TF points that are away from the IFs, the variance of the artifacts is uniformly distributed over f, whereas the variance depends on t due to the zero-padding effect.

ARTIFACTS MITIGATION USING TF KERNELS

The effect of the artifacts due to missing jammer data samples resembles that due to noise in the WVD domain in the sense that they respectively spread over the entire TF region, but with much stronger magnitudes. Therefore, such effects can be mitigated through a proper mask, or TF kernel. The application of TF kernels also mitigates undesired TF cross-terms. The best kernel in this case is one that only keeps the jammer signature while filtering out the other regions. One of the best choices for this purpose is the AOK [27], which is known to provide signal-adaptive filtering capability. The jammer TFD reconstruction performance using AOK will be compared to that using the Choi-William kernel as an example of signal-independent kernels.

JAMMER RECONSTRUCTION AND SUPPRESSION

As the jammer is sparsely presented in the TF domain, its TFD can be reconstructed based on their sparsity. For notation convenience, we denote $\mathbf{c}_x(t) = [C_{xx}(t,\tau_1),..., C_{xx}(t,\tau_T)]^T$ as a vector that contains all IAF entries along the τ dimension corresponding to time t, and $\mathbf{d}_x(t) = [D_{xx}(t, f_1),..., D_{xx}(t, f_T)]^T$ as a vector collecting all the TFD entries for the same time t. Note that $\mathbf{c}_x(t)$ may denote the original IAF, which corresponds to the WVD, or its smoothed version as a result of applying a kernel. Then, the two vectors $\mathbf{c}_x(t)$ and $\mathbf{d}_x(t)$ are related by the IDFT with respect to f, expressed as

$$\mathbf{c}_x(t) = \mathbf{G}_f \mathbf{d}_x(t), \ \forall t, \tag{15}$$

where $\mathbf{G}_f$ is a matrix performing the IDFT with respect to f. Because the signals are sparsely represented in the TF domain, the non-zero entries of $\mathbf{d}_x(t)$ can be reconstructed through sparse signal recovery techniques. The problem is formulated as

$$\min \| \mathbf{d}_x(t) \|_1 \text{ s.t. } \mathbf{c}_x(t) - \mathbf{G}_f \mathbf{d}_x(t) = 0, \ \forall t. \tag{16}$$

In this paper, we use the OMP [16] for each time instant and, as such, we can specify the number of non-zero entries (i.e., iterations) in each time instant.

Once the IF of the jammer signals is estimated, the jammer signal can be reconstructed except the initial phase. While the subspace projection can be used for jammer suppression, the performance is sensitive to the errors in the IF estimation error as well as that due to the frequency quantization accuracy. Instead, we perform the signal stationarization, DC component removal, and re-modulating the signal back. This is equivalent to performing time-varying filtering, which is difficult to directly perform in the underlying problem due to missing data. The required phase accuracy can be relaxed by dividing the entire data into multiple segments for separated processing [28]. After jammer removal, the GPS signal can be processed with matched filtering or sparse reconstruction.

SIMULATION RESULTS

For illustration purposes, we consider an FM jammer impinging on the receiver along with a GPS signal. The instantaneous phase law of the FM jammer is expressed as,

$$\varphi(t) = 0.05t + 0.05t^2/T + 0.1t^3/T^2, \tag{17}$$

for $t = 1, ..., T$, where the block size of the signal is chosen to be $T = 256$. The input signal-to-noise ratio (SNR) of the GPS signal is –16 dB, and the input jammer-to-noise ratio (JNR) is 25 dB.

One realization of the real-part waveform of the jammed GPS signal is shown in Fig. 1, where 50% of missing data samples is assumed. The red dots depict the positions of the missing data samples. The waveform is dominated by the jammer. The corresponding WVD is depicted in Fig. 2. Due to the presence of the strong jammer, the output signal-to-jammer-plus-noise ratio (SJNR) averaged over 10 independent trails, evaluated in each GPS symbol, is 1.08 dB.

Fig. 3 shows the estimated IF signature of the jammer from AOK processing and OMP-based sparse reconstruction, and Fig. 4 depicts the waveform after the jammer is suppressed through demodulation, notch, and re-modulation. The 128-sample data was divided into 4 segments in performing the jammer suppression. It is evident that the jammer signal is substantially mitigated. The yielding output SJNR averaged over the same 10 independent trails is 19.81 dB.

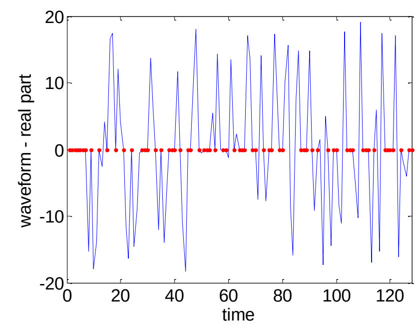

Fig. 1. Real-part waveform of jammed signal

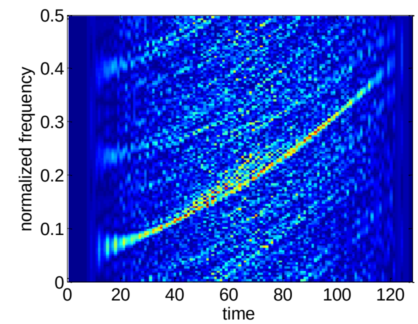

Fig. 2. WVD of jammed signal

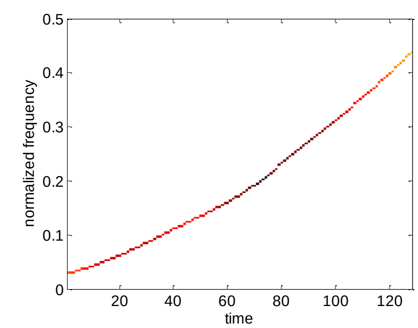

Fig. 3. Estimated IF signature from OMP

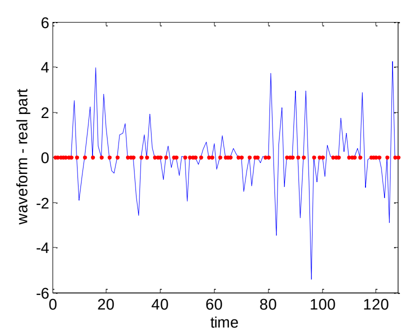

Fig. 4. Real-part waveform of jammed signal after proposed jammer removal

CONCLUSIONS

In this paper, we proposed a novel FM jammer suppression technique when the observed data include missing samples. The conventional time-frequency analysis techniques fail to effectively suppress jammer signals because of the high level of artifacts due to the missing data samples. The proposed technique, which is based on recent advances in time-frequency analyses and compressive sensing techniques, recovers the time-frequency distribution of the jammer signal and estimates the instantaneous frequency. This information was used for the demodulation of the jammer signal, enabling its mitigation through simple DC component removal.

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