DOA ESTIMATION OF MIXED COHERENT AND UNCORRELATED SIGNALS EXPLOITING A NESTED MIMO SYSTEM

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ABSTRACT

We propose a new scheme to estimate the directions-of-arrival of mixed coherent and uncorrelated signals exploiting a nested multiple-input multiple-output (MIMO) system. In the proposed scheme, the DOAs of the uncorrelated sources are first estimated using subspace-based methods, whereas those of the coherent sources are resolved using compressive sensing techniques. The proposed approach works for nonuniform linear sum coarrays and may resolve more sources than the number of coarray elements.

Index Terms— Nested array, sum coarray, MIMO, direction-of-arrival estimation, compressive sensing

1. INTRODUCTION

Direction-of-arrival (DOA) estimation is an important array processing technique that determines the spatial spectra of the impinging electromagnetic waves. Such techniques find broad applications in wireless communications and radar systems. Among the many DOA estimation methods that have been developed, subspace-based approaches, such as MUSIC [1] and ESPRIT [2], are broadly used due to their low complexity and superior performance. A problem with these methods is that they often fail to perform reliable DOA estimation when multiple signal arrivals are coherent. Coherent signal arrivals are commonly encountered in practice due to, for example, multipath propagation. Some techniques have been developed to decorrelate coherent signals at the expense of reduced number of degrees-of-freedom. The problem becomes even more challenging when a mix of coherent and uncorrelated signals is present.

In [3], a deflation method is proposed for DOA estimation in such a scenario with both uncorrelated and coherent signals appear in the filed of view. In this method, a uniform linear array (ULA) is considered, and the DOAs of the uncorrelated sources are estimated by directly applying the MUSIC algorithm. The uncorrelated signals are then eliminated from the received data by exploiting the symmetry property of the ULA. Elimination of uncorrelated signals can also be achieved using oblique projection. A Toeplitz matrix is then constructed to estimate the DOAs of the remaining coherent signals.

In this paper, we consider the DOA estimation exploiting a multiple-input multiple-output (MIMO) system configuration and dealing with both uncorrelated and coherent signals. The MIMO system is assumed to have a symmetric sum coarray [4]. In particular, a nested MIMO structure [5] is demonstrated in this paper, where the transmit array is a ULA with half-wavelength spacing, whereas the receive array is a sparsely located ULA. The DOA estimation is performed at the sum coarray of the MIMO system evaluated at its matched filter outputs. In our approach, while the DOA estimation of the uncorrelated signals and their elimination follow the same step as in [3], but compressive sensing (CS) techniques are used in the DOA estimation of the remaining coherent signals. Because CS methods can handle coherent sources with a general array structure [6, 7, 8], the Toelitz matrix structure becomes unnecessary, thereby facilitating the use of general symmetric sum coarray structures, instead of the ULA requirement. The proposed approach achieves better use of the covariance matrix entries for improved DOA estimation performance.

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Notations: We use lower-case (upper-case) bold characters to denote vectors (matrices). In particular, $\mathbf{I}_N$ denotes the $N \times N$ identity matrix. $(.)^*$ implies complex conjugation, whereas $(.)^T$ and $(.)^H$, respectively, denote transpose and conjugate transpose; $\|\cdot\|_2$ and $\|\cdot\|_1$ respectively represent the Euclidean (l_2) and l_1 norms, and $\mathrm{E}(\cdot)$ is the statistical expectation operator. $\bigotimes$ denotes the Kronecker product.

2. SIGNAL MODEL FOR NESTED MIMO SYSTEM

Consider a nested MIMO system consisting of an M-element ULA with an interelement spacing $d = \lambda/2$ for transmit array and an N-element ULA with interelement spacing Xd for receive array, where $X \geq M$, as shown in Fig. 1, and λ denotes the wavelength. The transmit and receive arrays are colocated, i.e., targets are located in the far-field are observed at the same direction by both arrays.

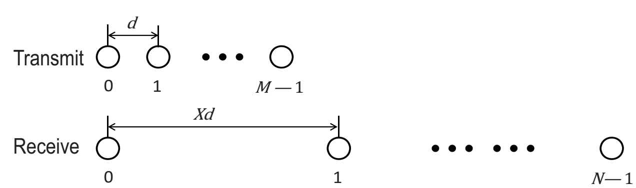


Fig. 1. A nested MIMO system.

Assume that Q narrowband far-field signals impinge on the array from angles $\Theta = [\theta_1, ..., \theta_Q]^T$, and their discritized baseband waveforms are expressed as $s_q(t), t = 1, ..., T$, for $q = 1, ..., Q$. The M transmit antennas transmit M orthogonal waveforms. The matched filter at the output of the nth receive antenna, corresponding to the mth transmit waveform, generates

$$\tilde{x}_{m,n}(t) = \sum_{q=1}^{Q} a_{t_{(m)}}(\theta_q) a_{r_{(n)}}(\theta_q) s_q(t) + n_{m,n}(t), \qquad (1)$$

where

$$a_{t_{(m)}}(\theta_q) = e^{-j\pi m d \sin(\theta_q)},$$
$$a_{r_{(n)}}(\theta_q) = e^{-j\pi n X d \sin(\theta_q)},$$

and $n_{m,n}(t)$ is assumed to be independent and identically distributed (i.i.d.) random variables following the complex Gaussian distribution $\mathcal{N}C(0, \sigma_n^2)$. Stacking them into a vector yields

$$\begin{aligned}\tilde{\mathbf{x}}(t) =& [\tilde{x}_{1,1}(t), \tilde{x}_{2,1}(t), \ldots, \tilde{x}_{M,1}(t), \tilde{x}_{1,2}(t), \tilde{x}_{2,2}(t), \ldots, \\ & \tilde{x}_{M,2}(t), \ldots, \tilde{x}_{1,N}(t), \tilde{x}_{2,N}(t), \ldots, \tilde{x}_{M,N}(t)]^T \\ =& \sum_{q=1}^{Q} \mathbf{a}_t(\theta_q) \bigotimes \mathbf{a}_r(\theta_q) s_q(t) + \mathbf{n}(t), \end{aligned} \quad (2)$$

where

$$\mathbf{a}_t(\theta_q) = [a_{t_{(1)}}(\theta_q), a_{t_{(2)}}(\theta_q), \ldots, a_{t_{(M)}}(\theta_q)]^T,$$
$$\mathbf{a}_r(\theta_q) = [a_{r_{(1)}}(\theta_q), a_{r_{(2)}}(\theta_q), \ldots, a_{r_{(N)}}(\theta_q)]^T,$$

are the transmit and receive steering vectors corresponding to θ_q, respectively, and $\mathbf{n}(t) = [n_{1,1}(t), n_{2,1}(t), ..., n_{M,N}(t)]^T$ is the noise vector. As a result of Eqn. (1), an MN-element ULA sum coarray is formed at the output of the receiver matched filters, as illustrated in Fig. 2.

Fig. 2. The sum coarray of the nested MIMO system.

3. FORMULATION OF THE PROBLEM

The coherent sources are grouped together and the number of groups is denoted as P. The sources within each group are coherent to each other and are uncorrelated to signals in other groups. In the cth group, there are L_c coherent arrivals and the DOA corresponding to the lth arrival of the signal $s_c(t)$ is denoted as θ_{cl} for $l = 1, \ldots, L_c$. The total number of coherent arrivals is $L = \Sigma_{c=1}^{P} L_c$ and the remaining $D = Q - L$ sources $s_u(t), u = L+1, \ldots, Q$, are uncorrelated to each other. For the convenience of presentation, we assume MN to be odd and express it as $MN = 2K + 1$. However, MN can take an even value and the problem can be similarly formulated.

Let the index of the central element of the array to be 0. The $(2K + 1) \times 1$ signal vector received at the sum coarray can be expressed as

$$\begin{aligned}\mathbf{x}(t) =& [x_{-K}(t), \ldots, x_0(t), \ldots, x_K(t)]^T \\ =& \sum_{c=1}^{P} \sum_{l=1}^{L_c} \mathbf{a}(\theta_{cl}) \rho_{cl} s_c(t) + \sum_{u=L+1}^{Q} \mathbf{a}(\theta_u) s_u(t) + \mathbf{n}(t) \\ =& \mathbf{A}_C \mathbf{s}_C(t) + \mathbf{A}_U \mathbf{s}_U(t) + \mathbf{n}(t), \end{aligned} \quad (3)$$

where $\mathbf{a}(\theta)$ is the steering vector corresponding to θ and ρ_{cl} is the complex fading coefficient of the lth arrival in the cth group. In addition, $\boldsymbol{\rho}_c = [\rho_{c1}, \ldots, \rho_{L_c}]^T$, $\mathbf{A}_c = [\mathbf{a}(\theta_{c1}), \ldots, \mathbf{a}(\theta_{cL_c})]$, $\mathbf{A}_C = [\mathbf{A}_1 \boldsymbol{\rho}_1, \ldots, \mathbf{A}_P \boldsymbol{\rho}_P]$, $\mathbf{A}_U = [\mathbf{a}(\theta_{L+1}), \ldots, \mathbf{a}(\theta_Q)]$, $\mathbf{s}_C = [s_1(t), \ldots, s_P(t)]^T$, and $\mathbf{s}_U = [s_{L+1}(t), \ldots, s_Q(t)]^T$.

As a consequence, the covariance matrix of the coarray can be expressed as

$$\mathbf{R_{xx}} = \mathbf{E}\{\mathbf{x}(t)\mathbf{x}^H(t)\} = \mathbf{A}_C \mathbf{R}_C \mathbf{A}_C^H + \mathbf{A}_U \mathbf{R}_U \mathbf{A}_U^H + \sigma_n^2 \mathbf{I}_{2K+1}, \quad (4)$$

where $\mathbf{R}_C$ and $\mathbf{R}_U$ are the covariance matrix of $\mathbf{s}_C(t)$ and $\mathbf{s}_U(t)$, respectively. Note that $\mathbf{R}_U$ is a diagonal matrix, whereas $\mathbf{R}_C$ is a block diagonal matrix with P blocks, and the rank is P.

4. DOA ESTIMATION

The DOA estimation is separately performed for coherent and uncorrelated sources in two steps. Similar to [3], the uncorrelated signals are estimated by directly appling the MUSIC algorithm. To estimate the DOAs of the coherent signals, we first eliminate the presence of uncorrelated sources by exploiting the property of a symmetric array that the received signal vector is conjugate symmetric. The CS technique is then applied, which outperforms the subspace-based DOA estimation techniques. In addition, the CS-based approach only requires symmetric sum coarray structure instead of a ULA one.

4.1. The proposed approach

All arrivals corresponding to the same coherent source yield a single rank. Therefore, the covariance matrix $\mathbf{R}_{xx}$ can be written as

$$\mathbf{R_{xx}} = \mathbf{U}_s \Lambda_s \mathbf{U}_s^H + \mathbf{U}_n \Lambda_n \mathbf{U}_n^H, \quad (5)$$

where $\mathbf{U}_s$ represents the signal subspace consisting of the $D + P$ dominant eigenvectors of $\mathbf{R_{xx}}$ and $\mathbf{U}_n$ represents the noise subspace with the remaining $2K + 1 - D - P$ eigenvectors. Λ_s and Λ_n are diagonal matrices containing corresponding eigenvalues. The uncorrelated sources can be estimated by exploiting the MUSIC algorithm directly. Because the combined array steering vector from multiple coherent arrivals, denoted as $\mathbf{a}_c = \sum_{l=1}^{L_c} \mathbf{a}(\theta_{cl})\rho_{cl}$, does not represent a valid array manifold, the coherent sources generally do not demonstrate a strong presence in the estimated MUSIC spectra.

Next, the estimation of the coherent sources is performed by using the symmetric configuration of array. As in [3], define

$$\mathbf{R} = \mathbf{R_{xx}} - \sigma_n^2 \mathbf{I}_{2K+1}, \quad (6)$$

whose (i, k)th element is expressed as

$$r(i,k) = \sum_{c=1}^{P} \sum_{l=1}^{L_c} b_{i,c}^l e^{j\pi k \sin(\theta_{cl})} + \sum_{u=L+1}^{Q} d_{i,u} e^{j\pi k \sin(\theta_u)}, \quad (7)$$

where $b_{i,c}^l = \sigma_c^2 \rho_{cl}^* \sum_{p=1}^{L_c} \rho_{cp} e^{-j\pi i \sin(\theta_{cp})}$ and $d_{i,u} = \sigma_u^2 e^{-j\pi i \sin(\theta_u)}$. It is noted that the uncorrelated components in $r(i, k)$ are conjugate symmetric, with respect to i and k, while the coherent components are not because of the complex fading coefficient ρ.

Define

$$g(i,k) = r(i,k) - r^*(-i,-k) = \sum_{c=1}^{P} \sum_{l=1}^{L_c} q_{i,c}^l e^{j\pi k \sin(\theta_{cl})}, \quad (8)$$

for the symmetric configuration $i, k = -K, \ldots, K$, where $q_{i,c}^l = \sigma_c^2 \sum_{p=1}^{L_c} (\rho_{cl}^* \rho_{cp} - \rho_{cl} \rho_{cp}^*) e^{-j\pi i \sin(\theta_{cp})}$. Note that only the coherent part remains in $g(i, k)$, whereas the uncorrelated components are eliminated. Stacking $g(i, k)$ for all k results in $\mathbf{g}(i)$, denoted as

$$\mathbf{g}(i) = \mathbf{A}_c \mathbf{r}, \quad (9)$$

where $\mathbf{r} = [q_{i,c}^1, \ldots, q_{i,c}^{L_c}]$. Eqn. (9) can be solved using the CS approaches [6]. The desired result of $\mathbf{r}$ is represented as the solution to the following constrained l_1-norm minimization problem

$$\hat{\mathbf{r}}^\circ = \arg\min_{\mathbf{r}^\circ} ||\mathbf{r}^\circ||_1 \quad \text{s.t.} \quad ||\mathbf{g}(i) - \mathbf{A}_\mathbf{c}^\circ \mathbf{r}^\circ||_2 < \epsilon, \tag{10}$$

where ϵ is a user-specific bound, $\mathbf{A}_\mathbf{c}^\circ$ is a dictionary matrix consisting of the searching steering vectors and $\mathbf{r}^\circ$ is the sparse entries to be determined. The positions of nonzero entries in $\hat{\mathbf{r}}^\circ$ represent the DOAs of the coherent arrivals.

This type of problems has been the objective of intensive studies in the area of CS, and a number of effective numerical computation methods have been developed. We use the batch Lasso algorithm [10] in this paper, but other methods may also be used.

4.2. Analysis of degrees of freedom

In the first step, the uncorrelated signals are estimated by the MUSIC algorithm directly. Hence, the number of dominant eigenvectors in $\mathbf{U}_s$, $D + P$, must be valid under the requirement of $D + P \leq 2K$. The total number of coherent sources $L = \Sigma_{c=1}^P L_c$ is no more than the number of sensors, $2K + 1$, in the second step. Then the degrees of freedom, i.e., the maximum number of resolvable sources, can be as high as $4K + 1 = 2MN - 1$.

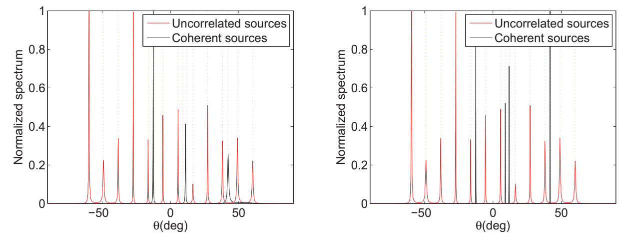
5. SIMULATION RESULTS

We first consider a nested MIMO system with $M = 5$, $N = 3$ and $X = 5$ in Fig. 3, yielding a uniform linear sum coarray of $MN = 15$ elements (i.e., $K = 7$). $D = 12$ uncorrelated sources are uniformly distributed between -60° and 60°. In addition, $P = 2$ sources with $L_1 = L_2 = 2$ coherent arrivals, whose fading coefficients are $\rho = [0.7085 + 0.5550i, -0.0487 - 0.9988i]$ and $[-0.6524 + 0.2537i, 0.8000 + 0.0039i]$, respectively, arrive from -12°, 9°, 12°, and 42°. As such, the total number of the sources is $Q = 16$, which is higher than the number of coarray elements as well as that of the physical sensors. 500 snapshots are used in the presence of noise with a 0 dB signal-to-noise ratio (SNR) for all signal arrivals. The results demonstrate that all 16 sources are resolved, whereas the MUSIC-based approach fails to resolve part of the coherent signal arrivals from 9° and 12°.

In the next example, we demonstrate that the proposed technique can exploit non-uniform linear sum coarrays whereas the method developed in [3] fails. We keep the same $M = 5$ and $N = 3$, but change X to 6. The signals are assumed the same as those in Fig. 3, but the directions of the coherent arrivals are changed to -40°, -12°, 9°, and 42°, so that the MUSIC-based technique in [3] would work if a uniform linear coarray were used. It is evident from Fig. 4 that, because of the non-uniform linear coarray structure, the approach developed in [3] yields incorrect estimates, whereas the proposed technique offers a good estimation performance.

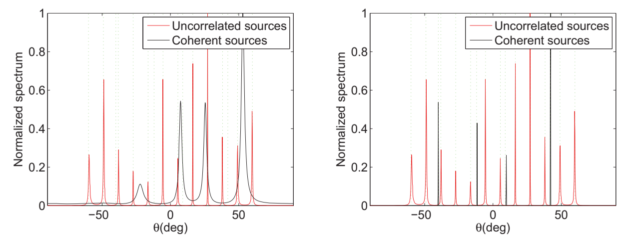
6. CONCLUSIONS

We have examined the direction-of-arrival (DOA) estimation problem of both uncorrelated and coherent signals in a multiple-input multiple-output (MIMO) platform. The proposed approach succeeds to separate coherent and noncoherent sources. Then, by applying the subspace-based approaches for uncorrelated signals and compressive sensing based methods for coherent signals, the proposed technique achieves superior performance with a flexible array configuration.



(a) MUSIC (b) CS

Fig. 3. Estimated spatial spectra using uniform linear sum coarray ($X = 5$).



(a) The approach in [3] (b) The proposed approach

Fig. 4. Estimated spatial spectra using non-uniform linear sum coarray ($X = 6$).

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