Adaptive Beamforming Based on Interference Covariance Matrix Estimation

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***Abstract*—In this paper, we propose a robust adaptive beamforming algorithm, where the interference-plus-noise covariance matrix is estimated by identifying and removing the desired signal component from the sample covariance matrix. For this purpose, we construct a desired signal subspace and its orthogonal subspace to identify the eigenvector of the sample covariance matrix corresponding to the desired signal. The adaptive beamformer is then designed using the estimated interference-plus-noise covariance matrix and the identified signal eigenvector. Because both are independent of the knowledge of the array geometry, the proposed adaptive beamformer is robust to array model mismatch. Simulation results demonstrate the effectiveness of the proposed robust adaptive beamforming algorithm.**

***Index Terms*—Adaptive beamforming, interference-plus-noise covariance matrix estimation, robust beamforming, signal identification.**

I. Introduction

Due to the superior capability of interference suppression, adaptive beamformers have been widely applied in radar, sonar, radio astronomy, wireless communications, seismology, microphone array, and medical imaging [1]. However, it is well known that, compared with data-independent beamformers (e.g., delay-and-sum beamformer), adaptive beamformers are very sensitive to model mismatches, such as look direction error and imperfect array calibration. The performance degradation is particularly severe when the desired signal is strong. Hence, achieving robustness of adaptive beamformers has been an enduring research topic in the past decades [2, 3].

Among various robust adaptive beamforming techniques, diagonal loading (DL) [4] is the most popular one, and many different loading factor selection criteria have been developed [5–9]. However, the performance of such beamformers deteriorates as the power of the desired signal increases. In practical applications, there is always some level of signal self-nulling as long as the desired signal component is included in the loaded sample covariance matrix. To address this issue, an interference covariance matrix reconstruction method was proposed by integrating the Capon spectral estimator over a region which is separated from the desired signal direction [10]. Such technique leads to a near-optimal beamformer performance regardless of the power of the desired signal. More recently, a sparse reconstruction method was proposed to obtain an improved estimation of the interference-plus-noise covariance matrix with a much lower computational complexity [11–13]. Considering the inaccurate prior information about the array structure, the interference covariance matrix is reconstructed by integrating the Capon spectrum over the surface of the annulus uncertainty set [14]. By sampling the spatial power spectrum, an efficient beamforming algorithm was proposed in [15] to reduce the computational complexity of reconstructing the interference covariance matrix. Unfortunately, all these beamformers require the knowledge of the interference steering vectors in the construction of the interference-plus-noise covariance matrix and, therefore, their performance degrades when the array is not well calibrated and/or when the interference spatial signatures deviate from the array manifold due to, e.g., the multipath propagation.

Another important factor that affects the performance of adaptive beamformers is the accuracy of the adopted desired signal steering vector. Beamforming algorithms based on worst-case performance optimization are popular when the nominal steering vector of the desired signal is known and the norm of the steering vector mismatch is bounded [5, 6, 16]. The steering vector of the desired signal can also be estimated by maximizing the beamformer output under the constraint that the estimated signal steering vector does not converge to that of the interference [10, 17, 18]. While these beamformers yield improved estimation of the steering vector of the desired signal, their performance still degrades at a high signal power due to the limited estimation accuracy of the interference-plus-noise covariance matrix.

In this paper, we propose a novel adaptive beamforming algorithm which is robust to array model mismatch and coherent multipath interference. With the *a priori* knowledge of an angular sector covering the desired signal only, we construct a signal subspace and its orthogonal subspace to identify the eigenvector of the sample covariance matrix corresponding to the desired signal. The interference-plus-noise covariance matrix is then estimated by removing the identified signal component from the sample covariance matrix. Using the identified signal eigenvector as the steering vector, the proposed adaptive beamformer achieves robust beamforming performance as it depends only on the received array data, and not on the knowledge of the array geometry. Simulation results verify the performance advantage of the proposed adaptive beamforming algorithm.

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II. The Signal Model

Assume multiple narrowband signals impinging on an array of M sensors. The array observation data vector $\mathbf{x}(k) \in \mathbb{C}^M$ at the k-th snapshot can be modeled as

$$\mathbf{x}(k) = \mathbf{x}_s(k) + \mathbf{x}_i(k) + \mathbf{x}_n(k), \tag{1}$$

where $\mathbf{x}_s(k)$, $\mathbf{x}_i(k)$ and $\mathbf{x}_n(k)$ are statistically independent components corresponding to the desired signal, interference, and noise, respectively. The desired signal component can be written as $\mathbf{x}_s(k) = \mathbf{a}_s s(k)$, where $\mathbf{a}_s \in \mathbb{C}^M$ is the steering vector associated with the signal waveform $s(k)$. Similarly, the interference component can be written as $\mathbf{x}_i(k) = \sum_{\ell=1}^{L} \mathbf{a}_\ell s_\ell(k)$, where $\mathbf{a}_\ell \in \mathbb{C}^M$ is the steering vector corresponding to the ℓ-th interference waveform $s_\ell(k)$ for $\ell = 1, \cdots, L$.

An adaptive beamformer determines the weight vector $\mathbf{w} = [w_1, \cdots, w_M]^{\mathrm{T}} \in \mathbb{C}^M$ such that the beamformer output,

$$y(k) = \mathbf{w}^{\mathrm{H}} \mathbf{x}(k), \tag{2}$$

leads to an accurate estimate of the desired signal waveform $s(k)$, where $(\,\cdot\,)^{\mathrm{T}}$ and $(\,\cdot\,)^{\mathrm{H}}$ denote the transpose and Hermitian transpose, respectively. As an effective performance evaluation metric, the beamformer output signal-to-interference-plus-noise ratio (SINR) is defined as

$$\text{SINR} = \frac{\mathbf{w}^{\mathrm{H}} \mathbf{R}_s \mathbf{w}}{\mathbf{w}^{\mathrm{H}} \mathbf{R}_{i+n} \mathbf{w}} = \frac{\sigma_s^2 \left|\mathbf{w}^{\mathrm{H}} \mathbf{a}_s\right|^2}{\mathbf{w}^{\mathrm{H}} \mathbf{R}_{i+n} \mathbf{w}}, \tag{3}$$

where $\mathbf{R}_{i+n} = \mathbb{E}\left\{[\mathbf{x}_i(k) + \mathbf{x}_n(k)][\mathbf{x}_i(k) + \mathbf{x}_n(k)]^{\mathrm{H}}\right\} = \sum_{\ell=1}^{L} \sigma_\ell^2 \mathbf{a}_\ell \mathbf{a}_\ell^{\mathrm{H}} + \sigma_n^2 \mathbf{I}$ and $\mathbf{R}_s = \mathbb{E}\{\mathbf{x}_s(k)\mathbf{x}_s^{\mathrm{H}}(k)\} = \sigma_s^2 \mathbf{a}_s \mathbf{a}_s^{\mathrm{H}}$ are the interference-plus-noise covariance matrix and the desired signal covariance matrix, respectively. Here, $\mathbb{E}\{\,\cdot\,\}$ denotes the statistical expectation operator, $\mathbf{I}$ is the identity matrix, and $\sigma_s^2 = \mathbb{E}\left\{s(k)s^{\mathrm{H}}(k)\right\}$, $\sigma_\ell^2 = \mathbb{E}\left\{s_\ell(k)s_\ell^{\mathrm{H}}(k)\right\}$ and σ_n^2 are the power of the desired signal, the ℓ-th interference and noise, respectively.

The weight vector maximizing the output SINR (3) leads to the minimum variance distortionless response (MVDR) beamformer

$$\mathbf{w}_{\text{MVDR}} = \frac{\mathbf{R}_{i+n}^{-1} \mathbf{a}_s}{\mathbf{a}_s^{\mathrm{H}} \mathbf{R}_{i+n}^{-1} \mathbf{a}_s}, \tag{4}$$

which, also known as the Capon beamformer, requires the exact interference-plus-noise covariance matrix $\mathbf{R}_{i+n}$ and the actual signal steering vector $\mathbf{a}_s$.

In practice, $\mathbf{R}_{i+n}$ is not readily available, and it is usually replaced by the sample covariance matrix

$$\hat{\mathbf{R}} = \frac{1}{K} \sum_{k=1}^{K} \mathbf{x}(k)\mathbf{x}^{\mathrm{H}}(k), \tag{5}$$

where K is the number of snapshots. The corresponding adaptive beamformer,

$$\mathbf{w}_{\text{SMI}} = \frac{\hat{\mathbf{R}}^{-1} \mathbf{a}(\theta_s)}{\mathbf{a}^{\mathrm{H}}(\theta_s) \hat{\mathbf{R}}^{-1} \mathbf{a}(\theta_s)}, \tag{6}$$

is referred to as the sample matrix inversion (SMI) beamformer, where $\mathbf{a}(\theta_s)$ is the signal steering vector with the assumed DOA θ_s.

By using the matrix inversion lemma, it can be shown [1] that the SMI beamformer is equivalent to the MVDR beamformer when the following two conditions are satisfied: 1) no mismatch between the presumed steering vector $\mathbf{a}(\theta_s)$ and the actual one $\mathbf{a}_s$ of the desired signal, and 2) infinite number of snapshots such that $\hat{\mathbf{R}}$ equals to the theoretical data covariance matrix $\mathbf{R} = \mathbb{E}\{\mathbf{x}(k)\mathbf{x}^{\mathrm{H}}(k)\}$. Otherwise, the SMI beamformer performs worse than the optimal MVDR beamformer, especially when the training data are contaminated by the strong desired signal.

In practice, it is considered possible to distinguish the signal-plus-interference subspace from the noise subspace by eigen-decomposing the sample covariance matrix, where the information theoretic criteria [19] (e.g., minimum description length (MDL) criterion, Akaike information criterion (AIC) and Bayesian information criterion (BIC)) can be adopted. Correspondingly, the eigenspace-based beamformer is robust against arbitrary steering vector mismatch [20]. However, it is not straightforward to further separate the interference subspace from the signal subspace, making it challenging to perform robust beamforming, especially when the desired signal is stronger than the interference.

III. The Proposed Algorithm

In this section, we describe the proposed robust adaptive beamforming algorithm. In this approach, with the *a priori* information that the impinging angle of the desired signal is limited within a known angular sector, the interference-plus-noise covariance matrix is estimated by removing the identified signal component from the sample covariance matrix, whereas the identified signal eigenvector is used as the signal steering vector.

The eigen-decomposition of the theoretical data covariance matrix $\mathbf{R} = \mathbf{R}_s + \mathbf{R}_{i+n}$ yields

$$\mathbf{R} = \mathbf{V}\mathbf{\Lambda}\mathbf{V}^{\mathrm{H}} = \sum_{m=1}^{M} \lambda_m \mathbf{v}_m \mathbf{v}_m^{\mathrm{H}}, \tag{7}$$

where the unitary matrix $\mathbf{V} = [\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_M]$ contains the eigenvectors, and the diagonal matrix $\mathbf{\Lambda} = \text{diag}\left[\lambda_1, \lambda_2, \cdots, \lambda_M\right]$ contains the corresponding eigenvalues. The eigen-decomposition (7) has the similar form as the expanded form of the covariance matrix $\mathbf{R} = \sigma_s^2 \mathbf{a}_s \mathbf{a}_s^{\mathrm{H}} + \sum_{\ell=1}^{L} \sigma_\ell^2 \mathbf{a}_\ell \mathbf{a}_\ell^{\mathrm{H}} + \sigma_n^2 \mathbf{I}$. Hence, the interference-plus-noise covariance matrix $\mathbf{R}_{i+n}$ can be estimated by removing the desired signal component from the sample covariance matrix as

$$\hat{\mathbf{R}}_{i+n} = \hat{\mathbf{R}} - \hat{\lambda}_s \hat{\mathbf{v}}_s \hat{\mathbf{v}}_s^{\mathrm{H}}, \tag{8}$$

where $\hat{\lambda}_s$ and $\hat{\mathbf{v}}_s$ are, respectively, the eigenvalue and eigenvector of the sample covariance matrix $\hat{\mathbf{R}}$ corresponding to the desired signal waveform $s(k)$ impinging from a specific angular sector. Hence, the rest work is to identify the eigenvector corresponding to the desired signal.

In order to identify the desired signal eigenvector, we reconstruct a signal covariance matrix as

$$\tilde{\mathbf{R}}_s = \int_{\Theta} \hat{p}(\theta)\mathbf{d}(\theta)\mathbf{d}^{\mathrm{H}}(\theta)d\theta, \tag{9}$$

where $\hat{p}(\theta) = 1/[\mathbf{d}^{\mathrm{H}}(\theta)\hat{\mathbf{R}}^{-1}\mathbf{d}(\theta)]$ is the Capon spatial spectrum estimator, and the angular sector Θ covers the desired signal direction θ_s only (i.e., $\theta_s \in \Theta$, while the DOAs of the interferers satisfy $\theta_\ell \notin \Theta$). Here, $\mathbf{d}(\theta)$ is the steering vector associated with a hypothetical direction θ based on the assumed array geometry. In so doing, $\tilde{\mathbf{R}}_s$ collects the information of the desired signal and noise in the angular sector Θ and, hence, the effect of the interference is excluded.

The signal subspace of the reconstructed signal covariance matrix $\tilde{\mathbf{R}}_s$ is spanned by the columns of the column orthogonal matrix $\mathbf{U}_s = \mathcal{P}\{\tilde{\mathbf{R}}_s\}$, where $\mathcal{P}\{\cdot\}$ stands for the principal eigenvectors of a matrix. The number of columns of $\mathbf{U}_s$ equals to the number of dominant eigenvalues of $\tilde{\mathbf{R}}_s$. As such, the actual steering vector of the desired signal belongs to the signal subspace of $\tilde{\mathbf{R}}_s$ spanned by $\mathbf{U}_s$.

Considering that the signal steering vector belongs to the signal subspace $\mathbf{P}_s = \mathbf{U}_s\mathbf{U}_s^{\mathrm{H}}$ and is orthogonal to the projection subspace $\mathbf{P}_s^{\perp} = \mathbf{I} - \mathbf{U}_s\mathbf{U}_s^{\mathrm{H}}$, the eigenvector associated with the desired signal can be identified according to

$$\hat{\mathbf{v}}_s = \arg\max_{\hat{\mathbf{v}}\in\hat{\mathbf{V}}} \frac{\|\mathbf{P}_s\hat{\mathbf{v}}\|}{\|\mathbf{P}_s^{\perp}\hat{\mathbf{v}}\|} = \arg\max_{\hat{\mathbf{v}}\in\hat{\mathbf{V}}} \frac{\|\mathbf{U}_s\mathbf{U}_s^{\mathrm{H}}\hat{\mathbf{v}}\|}{\|(\mathbf{I}-\mathbf{U}_s\mathbf{U}_s^{\mathrm{H}})\hat{\mathbf{v}}\|}, \tag{10}$$

where $\hat{\mathbf{V}} = [\hat{\mathbf{v}}_1, \hat{\mathbf{v}}_2, \cdots, \hat{\mathbf{v}}_M]$ is the matrix that contains all eigenvectors of the sample covariance matrix $\hat{\mathbf{R}}$. Substituting the identified signal eigenvector $\hat{\mathbf{v}}_s$ and the corresponding eigenvalue $\hat{\lambda}_s$ into (8) yields an estimate of the interference-plus-noise covariance matrix. In such a way, the desired signal component is effectively removed from the sample covariance matrix. In order to avoid the rank deficiency, we add an identity matrix scaled by $\lambda_{\min}$, the minimum eigenvalue of $\hat{\mathbf{R}}$, to the estimated interference-plus-noise covariance matrix $\hat{\mathbf{R}}_{i+n}$.

Substituting the estimated interference-plus-noise covariance matrix $\tilde{\mathbf{R}}_{i+n} = \hat{\mathbf{R}}_{i+n} + \lambda_{\min}\mathbf{I}$ into the MVDR beamformer (4), we have the proposed adaptive beamformer

$$\mathbf{w} = \frac{\tilde{\mathbf{R}}_{i+n}^{-1}\hat{\mathbf{v}}_s}{\hat{\mathbf{v}}_s^{\mathrm{H}}\tilde{\mathbf{R}}_{i+n}^{-1}\hat{\mathbf{v}}_s}, \tag{11}$$

where the signal steering vector $\mathbf{a}_s$ is replaced by the identified signal eigenvector $\hat{\mathbf{v}}_s$. Hence, the resulting adaptive beamformer depends only on the signal eigenvector via the array observation data and does not rely on the signal steering vector via the inaccurate knowledge of the array geometry.

Nevertheless, there is one exception that the signal eigenvector cannot be effectively identified when the signal power is close to the interference power or the noise power. In such a case, we prefer to directly use the sample covariance matrix to design the adaptive beamformer, such as the worst-case beamformer, to avoid the identification ambiguity. The weight vector of the worse-case beamformer is solved from the following constrained optimization problem [16],

Algorithm 1 : Proposed Adaptive Beamforming Algorithm

Input: Array received data vector $\{\boldsymbol{x}(k)\}_{k=1}^{K}$ and signal angular sector Θ;
Initialize: Compute the sample covariance matrix $\hat{\boldsymbol{R}} = \frac{1}{K}\sum_{k=1}^{K}\boldsymbol{x}(k)\boldsymbol{x}^{\mathrm{H}}(k)$;
Estimate signal covariance matrix $\tilde{\mathbf{R}}_s$ using (9);
Estimate signal subspace $\mathbf{P}_s = \mathbf{U}_s\mathbf{U}_s^{\mathrm{H}}$ and orthogonal subspace $\mathbf{P}_s^{\perp} = \mathbf{I} - \mathbf{U}_s\mathbf{U}_s^{\mathrm{H}}$ where $\mathbf{U}_s = \mathcal{P}\{\tilde{\mathbf{R}}\}$;
Perform eigen-decomposition $\hat{\boldsymbol{R}} = \sum_{m=1}^{M}\hat{\lambda}_m\hat{\mathbf{v}}_m\hat{\mathbf{v}}_m^{\mathrm{H}}$;
Identify signal eigenvector $\hat{\mathbf{v}}_s$ from (10) and the corresponding eigenvalue $\hat{\lambda}_s$;
if $\exists_{\hat{\lambda}_m \neq \hat{\lambda}_s}\hat{\lambda}_m \in \left[\frac{1}{\delta}\hat{\lambda}_s, \delta\hat{\lambda}_s\right]$, **then**
    Design worst-case beamformer using (12);
else
    Estimate the interference-plus-noise covariance matrix as $\hat{\mathbf{R}}_{i+n} = \hat{\mathbf{R}} - \hat{\lambda}_s\hat{\mathbf{v}}_s\hat{\mathbf{v}}_s^{\mathrm{H}} + \hat{\lambda}_{\min}\mathbf{I}$;
    Design proposed beamformer using (11);
end if
Output: Proposed beamforming weight vector $\mathbf{w}$

$$\min_{\mathbf{w}} \quad \mathbf{w}^H\hat{\mathbf{R}}\mathbf{w} \tag{12}$$
$$\text{subject to} \quad \mathbf{w}^H\mathbf{a} \le \varepsilon\,|\mathbf{w}| + 1,\ \Im\{\mathbf{w}^H\mathbf{a}\} = 0,$$

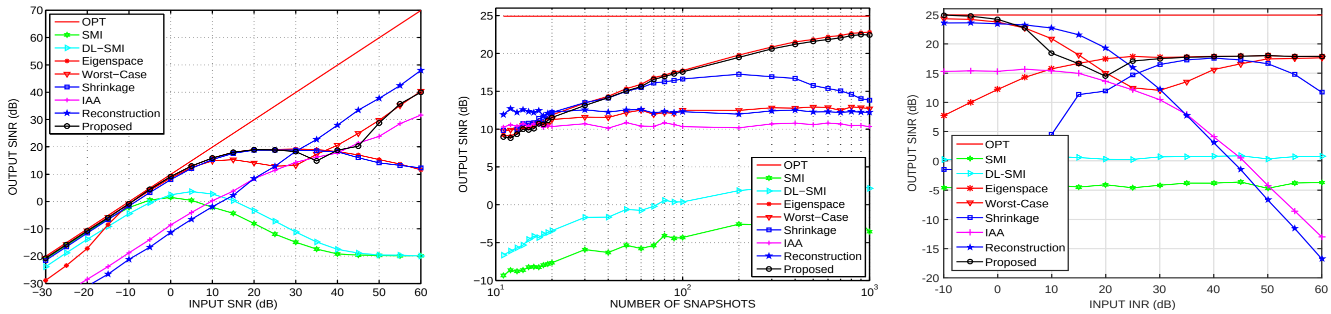
where $\Im\{\cdot\}$ denotes the imaginary part, and ε denotes the upper bound of the norm of the steering vector mismatch.

The selection between the proposed beamformer based on the robust interference-plus-noise covariance matrix estimation and the worse-case beamformer is determined by whether the identified signal eigenvalue clearly differs from the other eigenvalues. Specifically, the former is chosen when no other eigenvalue falls into the range between $(1/\delta)\hat{\lambda}_s$ and $\delta\hat{\lambda}_s$, where $\delta > 1$ is a constant. $\delta = 1.25$ is used in this paper. The proposed adaptive beamforming algorithm is summarized in Algorithm 1.

Considering that the signal angular sector Θ is typically much smaller than its complement sector $\bar{\Theta}$, the computational complexity for the reconstruction of the signal covariance matrix (9) is in the same order as that for the eigen-decomposition and matrix inversion. Meanwhile, the computational complexity of the worst-case beamforming algorithm is in the same order as that of the SMI beamforming algorithm [16]. Hence, the overall computational complexity of the proposed beamforming algorithm is a little higher than that of the SMI beamforming algorithm, but with the same order of $\mathcal{O}(M^3)$.

IV. Simulation Results

We consider a uniform linear array (ULA) with $M = 10$ omnidirectional sensors and half-wavelength inter-element spacing. The DOA of the desired signal is assumed to be $\theta_s = 5^\circ$, while those of the two interferers are -50° and -20°, respectively.



(a) (b) (c)

Fig. 1. Comparison of the output SINR in the presence of array mismatch. (a) Output SINR versus input SNR; (b) Output SINR versus the number of snapshots; (c) Output SINR versus input INR.

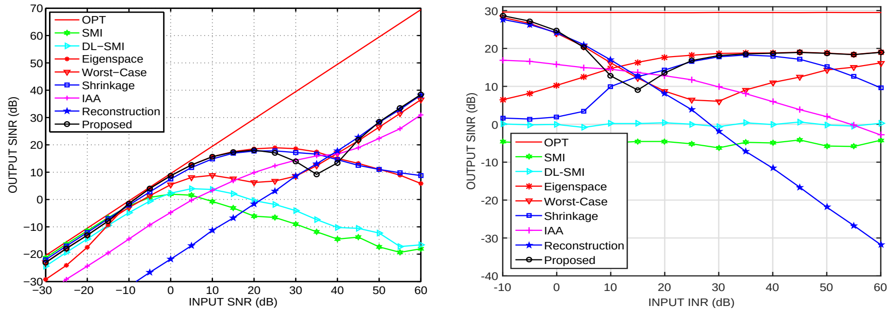
The proposed beamformer is compared to the SMI beamformer (6), the DL-SMI beamformer [4], the eigenspace-based beamformer [20], the worst-case beamformer [16], the shrinkage-based beamformer [7], the iterative adaptive approach (IAA) beamformer [21], and the reconstruction-based beamformer [10]. In the DL-SMI beamformer, the loading factor is taken to be ten times the noise power. In the eigenspace-based beamformer, the number of sources is assumed to be known. In the worst-case beamformer, the upper bound of the norm of the steering vector mismatch is set to $\varepsilon = 0.3M$. In the reconstruction-based beamformer and the proposed beamformer, the desired signal is assumed to be from an angular sector $\Theta = [\theta_s - 5^\circ, \theta_s + 5^\circ] = [0^\circ, 10^\circ]$. The optimal output SINR is also plotted for reference. The CVX toolbox [22] is used to solve all convex optimization problems. In the comparison of the output SINR performance of different beamforming algorithms, 1,000 Monte-Carlo trials are performed for each data point (i.e., an input signal-to-noise ratio (SNR) value or each snapshot).

In the first example, model mismatches due to random sensor position errors and look direction mismatch are considered. More specifically, each sensor is assumed to be randomly displaced from its original location and the displacement is drawn uniformly from $[-0.05, 0.05]$ wavelength. The look direction mismatch is assumed to be uniformly distributed in $[-4^\circ, 4^\circ]$ for the desired signal and the two interferers. The actual sensor positions and DOAs of the sources vary from run to run but remain fixed from snapshot to snapshot. The input interference-to-noise ratio (INR) is assumed to be the same 40 dB for both interferers (except in Fig. 1(c) where the input INR varies).

In Fig. 1(a), we compare the output SINR of the tested beamformers versus the input SNR, where the number of snapshots is set to $K = 100$. It is observed that, because of random sensor position errors, there is a constant performance loss for the reconstruction-based beamformer regardless of the input SNR. At a higher SNR, the IAA beamformer has an extra performance loss because of the look direction mismatch. The eigenspace-based beamformer has severe performance degradation when the power of the desired signal is higher than that of the interference. On the other hand, the proposed beamformer performs better than the others, especially when there is no identification ambiguity, e.g., input SNR $\in [0, 30]$ dB. As we mentioned before, there is a high swap probability between the signal subspace and the noise (or interference) subspace when the power of the desired signal is comparable to that of the noise or the interference. In Fig. 1(b), we compare the output SINR versus the number of snapshots at input SNR of 15 dB. It is evident that the proposed beamformer and the eigenspace-based beamformer benefit more from the increased number of snapshots, because the eigen-decomposition becomes more accurate when it is computed using more snapshots. In Fig. 1(c), we compare the output SINR versus the input INR by fixing the SNR at 15 dB. It is clear that the proposed beamformer guarantees an output SINR of about 15 dB regardless of the interference power. However, both the IAA beamformer and the reconstruction-based beamformer degrade as the interference power increases because of the array mismatch.

In the second example, we consider the effect of coherent multipath interference. The array is assumed to be perfectly calibrated, and the same random look direction mismatch as in the first example is assumed. Here, each interferer arrives with a direct path and four coherent multipaths. In this case, the spatial signature of the ℓ-th interference is expressed as $\mathbf{a}_\ell = \bar{\mathbf{a}}_\ell + \sum_{p=1}^{4} e^{j\psi_{\ell,p}} \mathbf{d}(\theta_{\ell,p})$, where $\bar{\mathbf{a}}_\ell$ and $\mathbf{d}(\theta_{\ell,p})$ are the steering vectors of the direct path and the ℓ-th multipath, respectively, and $\psi_{\ell,p}$ is uniformly distributed within $[0, 2\pi)$. The DOA of the direct path of the first interference is -50°, and those for the multipaths are $[-45^\circ, -40^\circ, -35^\circ, -32^\circ]$. For the second interferer, the DOAs of the direct path and multipaths are -20° and $[-15^\circ, -10^\circ, -5^\circ, -2^\circ]$, respectively.

In Fig. 2, we compare the output SINR of the tested beamformers versus the input SNR and versus the input INR, where the input INR is fixed to 30 dB and the input SNR is fixed to 20 dB respectively in Fig. 2(a) and Fig. 2(b). It is clear that, even without array mismatch, coherent multipath interference distorts the reconstruction of the interference-plus-noise covariance matrix in [10], thereby degrading the beamformer performance when the input interference power is high. From this point of view, beamformers that depend only on the array received data, including the proposed beamformer, are more robust than structure-dependent beamformers in the scenario of coherent multipath interference.



(a) (b)

Fig. 2. Comparison of the output SINR in the presence of coherent multipath interference. (a) Output SINR versus input SNR; (b) Output SINR versus input INR.

V. Conclusion

In this paper, we proposed a novel adaptive beamforming algorithm by desired signal identification, that is, the desired signal is identified by comparing the projection ratios of the eigenvectors of the sample covariance matrix onto the constructed signal subspace and its orthogonal subspace. The interference-plus-noise covariance matrix is then estimated by removing the identified signal component from the sample covariance matrix. The proposed adaptive beamformer is designed based on the estimated interference-plus-noise covariance matrix and the identified signal eigenvector, both of which rely on the knowledge of the array observation data rather than the array geometry. Simulation results clearly demonstrated the superiority of the proposed beamformer over existing beamformers when the array is not perfectly calibrated and/or there is coherent multipath interference.

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