OFDM-based Joint Radar-Communication System: Optimal Sub-carrier Allocation and Power Distribution by Exploiting Mutual Information

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Abstract—**In this paper, we present a novel joint radar-communication system which exploits orthogonal frequency division multiplexing (OFDM) waveforms for performing radar and communication operations simultaneously. A dual-purpose OFDM transmitter is exploited which optimizes the transmit power of different sub-carriers to fulfill the radar objectives. These OFDM sub-carriers used by the radar are also allocated to different communication receivers to achieve the communications objectives. The mutual information (MI) between the frequency-dependent target response and the transmit waveform is used as the optimization objective for radar performance. The communication performance is optimized by allocating the radar sub-carriers to different communication users by using MI maximization as the criterion. For the communication system, the problem has been discussed in terms of maximizing the overall MI as well as achieving the worst-case MI for each user. Two optimization strategies are discussed and compared which optimize the two systems respectively using a radar-centric design and a cooperative design. Simulation results illustrate the performance of the proposed strategies.**

keywords: Joint radar-communication systems, mutual information, spectrum sharing, waveform design.

I. INTRODUCTION

Spectrum sharing has attracted a lot of attention from several researchers due to the rapidly increasing demand of spectral resources [1–5]. In this context and to reduce the spectral congestion problem, co-existence of multiple platforms within the same frequency bands is extensively discussed in the literature [6–26]. Successful co-existence of radar and communication systems requires both systems to work collaboratively so as to mitigate the interference created among them. Such an objective can be simplified when both systems are operated by a joint control unit which performs both radar and communication operations simultaneously. Joint radar-communication (JRC) systems are examples of such systems where the waveforms are transmitted by a dual-purpose transmitter for both radar and communication operations [5, 9, 11, 12, 14–23].

The basic principle of a JRC system is to transmit waveforms for achieving radar and communication objectives by using the same physical platform as illustrated in Fig. 1. JRC systems achieve their objectives either by spatially multiplexing the signal transmission using smart antenna arrays [4, 5, 9–12, 14, 16–20, 22, 27] or by employing waveform diversity [1–3, 6, 15, 21, 23, 24, 26]. The communication operation is realized by either embedding the communication information in the radar waveform or by dedicating separate waveforms for radar and communication operation [1–3, 5–12, 14–24, 26].

Mutual information (MI) has been widely used as a performance metric for radar and communication systems [15, 28–31]. This is because MI maximization is related to the maximization of the probability of detection in radar systems for a fixed probability of false alarm [28]. From a communications perspective, MI maximization is analogous to maximizing the channel capacity of the communication systems [30]. Since MI maximization is a convex optimization problem by definition, it becomes an attractive measure for JRC system design as compared to other optimization criteria, like probability of detection and Cramer-Rao bound, which generally yield non-convex problems [23].

In this paper, we present an OFDM-based JRC system which exploits dual-purpose OFDM waveforms to perform radar and communication objectives. All the sub-carriers are primarily used by the radar and the secondary communication operation is enabled by embedding the information in OFDM waveforms. We discuss the optimal power distribution for the OFDM sub-carriers and their allocation to different communication users based on MI maximization. Simulation results illustrate the effectiveness of the proposed method.

II. SIGNAL MODEL

We consider a JRC system consisting of a single-antenna dual-purpose transmitter responsible for transmitting a dual-purpose radar-communication waveform in the presence of one radar target and R communication receivers. The target response and communication channels are assumed to vary with the frequency. The transmitter emits OFDM waveforms such that all the sub-carriers are used by the radar, whereas these sub-carriers are further allocated to different communication users so as to enable a secondary communication operation. Fig. 2 illustrates an example of power distribution among the sub-carriers and their allocation to the communication receivers.

The L-symbol OFDM vector $\mathbf{x}$ emitted from a dual-purpose transmitter, which consists of K sub-carriers with $K \leq L$, can be represented as:

$$\mathbf{x} = \mathbf{F}_{\mathrm{IDFT}}\mathbf{s}, \tag{1}$$

where $\mathbf{F}_{\mathrm{IDFT}}$ is the $L \times K$ inverse discrete Fourier transform (IDFT) matrix, and each column of $\mathbf{F}_{\mathrm{IDFT}}$ corresponds

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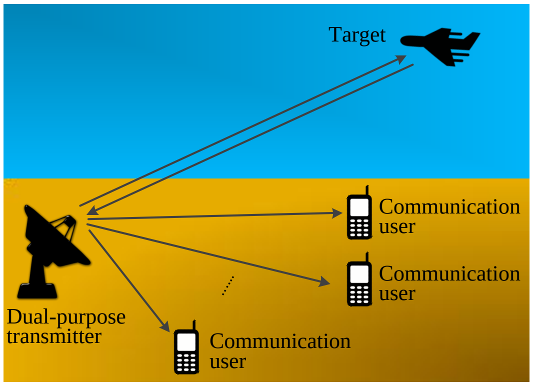


Fig. 1. Joint radar-communication system consisting of a dual-purpose transmitter performing radar and communication tasks simultaneously.

to an OFDM sub-carrier having a unique sub-carrier frequency. Note that the columns of $\mathbf{F}_{\text{IDFT}}$ are ortho-normal, i.e., $\mathbf{F}_{\text{IDFT}}^{\mathrm{H}}\mathbf{F}_{\text{IDFT}} = \mathbf{I}_K$, where $(\cdot)^{\mathrm{H}}$ denotes the Hermitian transpose operator and $\mathbf{I}_K$ is the $K \times K$ identity matrix. In addition, $\mathbf{s} = [s_1, \cdots, s_K]^{\mathrm{T}}$ is a $K \times 1$ vector whose elements correspond to the amplitudes and phases of the respective OFDM waveforms, where $(.)^{\mathrm{T}}$ denotes the transpose operator.

We use quadratic phase shift keying (QPSK) in each sub-carrier. As such, the phase of s_k carries the communication information in the kth sub-carrier whereas its magnitude determines the corresponding transmit power $p_k = |s_k|^2$, which will be optimized later. The total transmit power of the OFDM signal is given as:

$$P_{\text{total}} = \mathbf{x}^{\mathrm{H}}\mathbf{x} = \mathbf{s}^{\mathrm{H}}\mathbf{F}_{\text{IDFT}}^{\mathrm{H}}\mathbf{F}_{\text{IDFT}}\mathbf{s} = \mathbf{s}^{\mathrm{H}}\mathbf{s} = \sum_{k=1}^{K} p_k = \mathrm{tr}\{\mathbf{P}\}, \tag{2}$$

where $\mathrm{tr}(\cdot)$ denotes matrix trace, and $\mathbf{P} = \mathrm{diag}\{\mathbf{p}\}$ is a diagonal matrix with with $\mathbf{p} = [p_1, \cdots, p_K]^{\mathrm{T}}$. We denote the maximum possible transmit power for the kth sub-carrier by $p_{k,\max}$ and let $\mathbf{p}_{\max} = [p_{1,\max}, \cdots, p_{K,\max}]^{\mathrm{T}}$, whereas the maximum total transmit power is represented by $P_{\text{total,max}}$.

The transmitted OFDM signal is reflected by the target with frequency-dependent characteristics and reaches the radar receiver. Denote $\mathbf{h} = [h_1, \cdots, h_K]^{\mathrm{T}}$ as the radar channel coefficients, including the radar cross-section (RCS), for the K sub-carriers, and let $\tilde{\mathbf{h}} = \mathbf{F}_{\text{IDFT}}\mathbf{h}$ be the corresponding channel impulse response. Then, the received signal at the radar receiver is expressed as:

$$\tilde{\mathbf{y}}_{\text{rad}} = \tilde{\mathbf{h}} * \mathbf{x} + \tilde{\mathbf{n}}, \tag{3}$$

where $*$ denotes the convolution operator, and $\tilde{\mathbf{n}}$ is the zero-mean complex additive white Gaussian noise vector.

After performing the discrete Fourier transform, the K sub-carriers of the received OFDM signal are recovered as:

$$\mathbf{y}_{\text{rad}} = \mathbf{H}\mathbf{s} + \mathbf{n}, \tag{4}$$

where $\mathbf{H} = \mathrm{diag}(\mathbf{h})$, and $\mathbf{n}$ is the Fourier transform of $\tilde{\mathbf{n}}$ and denotes the zero-mean additive white complex Gaussian noise vector in the K sub-carriers. We assume that the noise components in the K sub-carriers are independent and identically distributed with known covariance matrix $\mathbf{\Sigma}_{\mathrm{n}} = \mathrm{diag}\{\sigma_{\mathrm{n},1}^2, \cdots, \sigma_{\mathrm{n},K}^2\}$.

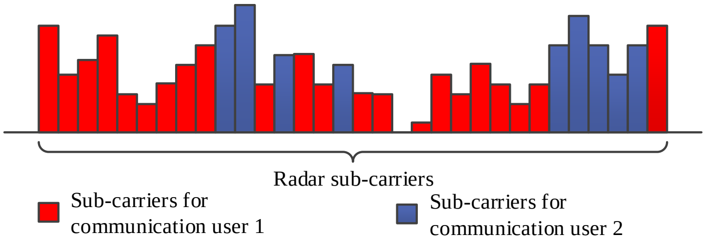


Fig. 2. Sub-carrier allocation and power distribution strategy for a JRC system. Two communication users are shown in the example.

Similarly, the OFDM sub-carriers reaching the communication receiver r can be jointly expressed as

$$\mathbf{y}_{\text{com},r} = \mathbf{G}_r\mathbf{s} + \mathbf{m}_r, \quad r = 1, \ldots, R, \tag{5}$$

where $\mathbf{G}_r = \mathrm{diag}(\mathbf{g}_r)$ and $\mathbf{g}_r = [g_{r,1}, \ldots, g_{r,K}]^{\mathrm{T}}$ denotes the channel coefficients of the K sub-carriers associated with the rth communication receiver. In addition, $\mathbf{m}_r$ is the zero-mean additive white complex Gaussian noise vector with a known covariance matrix $\mathbf{\Sigma}_{\mathrm{m}_r} = \mathrm{diag}\{\sigma_{\mathrm{m}_{r,1}}^2, \cdots, \sigma_{\mathrm{m}_{r,K}}^2\}$. Furthermore, the statistical properties of the radar and communication channels are known to be $\mathbf{h} \sim \mathcal{CN}(\mathbf{0}_K, \mathbf{\Sigma}_{\mathbf{h}})$ and $\mathbf{g}_r \sim \mathcal{CN}(\mathbf{0}_K, \mathbf{\Sigma}_{\mathrm{g}_r})$, where $\mathbf{\Sigma}_{\mathrm{h}} = \mathrm{diag}\{\sigma_{\mathrm{h}_1}^2, \cdots, \sigma_{\mathrm{h}_K}^2\}$ and $\mathbf{\Sigma}_{\mathrm{g}_r} = \mathrm{diag}\{\sigma_{\mathrm{g}_{r,1}}^2, \cdots, \sigma_{\mathrm{g}_{r,K}}^2\}$ are $K \times K$ diagonal matrices and $\mathbf{0}_K$ is the $K \times 1$ vector of all zeros. Moreover, we assume that $\mathbf{h}$ and $\mathbf{n}$ as well as $\mathbf{g}_r$ and $\mathbf{m}_r$, $r = 1, \cdots, R$, are mutually independent.

III. OPTIMIZATION CRITERIA

In this section, we develop the MI-based optimization criteria respectively for the radar and communication subsystem of the JRC system.

A. Radar sub-system

We consider the MI between the dual-purpose transmit waveform and the frequency-dependent target response $\mathbf{h}$ as the performance criterion for the radar sub-system which can be stated as [30]:

$$I(\mathbf{y}_{\text{rad}}; \mathbf{h}|\mathbf{s}) = h(\mathbf{y}_{\text{rad}}|\mathbf{s}) - h(\mathbf{y}_{\text{rad}}|\mathbf{h}, \mathbf{s}) = \mathbf{h}(\mathbf{y}_{\text{rad}}|\mathbf{s}) - \mathbf{h}(\mathbf{n}), \tag{6}$$

where $h(\cdot)$ denotes the differential entropy. Using Eq. (4), we can find the covariance matrix of $\mathbf{y}_{\text{rad}}$ as [29]:

$$\mathrm{E}\left[\mathbf{y}_{\text{rad}}\mathbf{y}_{\text{rad}}^{\mathrm{H}}\right] = \mathrm{E}\left[\mathbf{H}\mathbf{s}\mathbf{s}^{\mathrm{H}}\mathbf{H}^{\mathrm{H}} + \mathbf{n}\mathbf{n}^{\mathrm{H}}\right] = \mathbf{P}\mathbf{\Sigma}_{\mathrm{h}} + \mathbf{\Sigma}_{\mathrm{n}}, \tag{7}$$

where $\mathrm{E}[\cdot]$ stands for statistical expectation. Thus, $\mathbf{y}_{\text{rad}}|\mathbf{s} \sim \mathcal{CN}(\mathbf{0}, \mathbf{P}\mathbf{\Sigma}_{\mathrm{h}} + \mathbf{\Sigma}_{\mathrm{n}})$. Eq. (6) takes the following form [30]:

$$\begin{aligned} I(\mathbf{y}_{\text{rad}}; \mathbf{h}|\mathbf{s}) &= \log\left[(\pi e)^K \det(\mathbf{P}\mathbf{\Sigma}_{\mathrm{h}} + \mathbf{\Sigma}_{\mathrm{n}})\right] \\ &\quad - \log\left[(\pi e)^K \det(\mathbf{\Sigma}_{\mathrm{n}})\right] \\ &= \log\left(\det(\mathbf{P}\mathbf{\Sigma}_{\mathrm{h}} + \mathbf{\Sigma}_{\mathrm{n}})\right) - \log\det(\mathbf{\Sigma}_{\mathrm{n}}), \end{aligned} \tag{8}$$

where $\log(\cdot)$ represents the logarithm of base 2. Since $\mathbf{P}\mathbf{\Sigma}_{\mathrm{h}}$ is a diagonal matrix, we can express its determinant as the product of its diagonal elements, thus yielding

$$\begin{aligned} &I(\mathbf{y}_{\text{rad}}; \mathbf{h}|\mathbf{s}) \\ &= \log\left(\prod_{k=1}^{K} \frac{p_k\sigma_{\mathrm{h}_k}^2 + \sigma_{\mathrm{n},k}^2}{\sigma_{\mathrm{n},k}^2}\right) = \sum_{k=1}^{K} \log\left(1 + \frac{p_k\sigma_{\mathrm{h}_k}^2}{\sigma_{\mathrm{n},k}^2}\right). \end{aligned} \tag{9}$$

B. Communication Sub-system

Now we consider the MI between the communication receiver and the dual-purpose transmit waveform as the performance criteria for the communication sub-system because maximizing the MI is analogous to maximizing the data rate [30]. For the rth communication receiver, the MI between the transmitted OFDM signal $\mathbf{s}$ and the communication channel $\mathbf{g}_r$ can be written as [30]:

$$\begin{aligned} I\left(\mathbf{y}_{\mathrm{com},r};\mathbf{g}_r|\mathbf{s}\right) &= h(\mathbf{y}_{\mathrm{com},r}|\mathbf{s}) - h(\mathbf{y}_{\mathrm{com},r}|\mathbf{g}_r,\mathbf{s}) \\ &= h(\mathbf{y}_{\mathrm{com},r}|\mathbf{s}) - h(\mathbf{m}_r). \end{aligned} \tag{10}$$

Because $\mathbf{y}_{\mathrm{com},r}|\mathbf{s} \sim \mathcal{CN}\left(\mathbf{0}_K, \mathbf{P}\boldsymbol{\Sigma}_{\mathrm{g}_r} + \boldsymbol{\Sigma}_{\mathrm{m}_r}\right)$, we can re-write Eq. (10) as [30]:

$$I(\mathbf{y}_{\mathrm{com},r};\mathbf{g}_r|\mathbf{s}) = \log\left(\det\left(\mathbf{P}\boldsymbol{\Sigma}_{\mathrm{g}_r} + \boldsymbol{\Sigma}_{\mathrm{m}_r}\right)\right) - \log\left(\det(\boldsymbol{\Sigma}_{\mathrm{m}_r})\right). \tag{11}$$

Since $\mathbf{P}\boldsymbol{\Sigma}_{\mathrm{g}_r}$ is diagonal, Eq. (11) takes the following form:

$$\begin{aligned} &I(\mathbf{y}_{\mathrm{com},r};\mathbf{g}_r|\mathbf{s}) \\ &= \log\left[\prod_{k=1}^{K} \frac{p_k^{\mathrm{H}}\sigma^2_{\mathrm{g}_{r,k}} + \sigma^2_{\mathrm{m}_{r,k}}}{\sigma^2_{\mathrm{m}_{r,k}}}\right] = \sum_{k=1}^{K} \log\left(1 + \frac{p_k\sigma^2_{\mathrm{g}_{r,k}}}{\sigma^2_{\mathrm{m}_{r,k}}}\right). \end{aligned} \tag{12}$$

IV. OPTIMAL POWER DISTRIBUTION AND SUB-CARRIER ALLOCATION

In this section, we determine the optimal power for each sub-carrier and its allocation among the communication receivers for the optimal JRC operation. The radar sub-carriers are optimally allocated to the communication users to achieve the desired data rate such that an individual sub-carrier serves only one communication receiver. This enables interference-free multiple access by transmitting distinct data streams to different communication receivers over their dedicated sub-carriers. In the following, we discuss two optimization strategies for sub-carrier allocation and power distribution.

A. Radar-Centric Design

For this scenario, the optimization objective aims at maximizing the MI for radar as in Eq. (9). This design gives supreme precedence to radar objectives and the resulting sub-carrier power distribution of the dual-purpose OFDM transmitter provides maximum MI for the radar operation. However, it does not guarantee that the communication objectives are satisfied. The transmitted waveform can still be used by the communication receivers in the vicinity of the dual-purpose transmitter. The ODFM sub-carriers, whose individual powers for the optimal radar operation have already been determined, are allocated to different communication users.

1) Power distribution: Note that the MI in Eq. (9) is a concave function of $\mathbf{p}$ and the resulting convex optimization takes the following form:

$$\begin{aligned} \min_{\mathbf{p}} \quad & -\sum_{k=1}^{K} \log\left(1 + \frac{p_k\sigma^2_{\mathrm{h}_k}}{\sigma^2_{\mathrm{n},k}}\right) \\ \text{s.t.} \quad & \mathbf{1}^{\mathrm{T}}\mathbf{p} \leq P_{\mathrm{total,max}}, \\ & \mathbf{0} \leq \mathbf{p} \leq \mathbf{p}_{\mathrm{max}}, \end{aligned} \tag{13}$$

where $\mathbf{1}$ and $\mathbf{0}$ are $K \times 1$ vectors with all elements respectively being 1 and 0. The constraints emphasize the fact that the power of all OFDM sub-carriers is bounded by the total available power while the power of each sub-carrier is bounded by the maximum possible individual power.

2) Sub-carrier allocation: In the following, we formulate a mixed-integer linear program (MILP) which designates the OFDM sub-carriers to the individual communication receivers such that the communication MI is maximized. In order to ensure interference-free multiple access, each sub-carrier is dedicated to a single communication receiver. Note that the power of each sub-carrier is already determined in (13) and the following optimization only allocates the sub-carriers to the communication receivers. Two different optimization criteria are considered.

The first criterion maximizes the sum communication MI, expressed as:

$$\begin{aligned} \min_{\mathbf{w}_r} \quad & -\sum_{r=1}^{R}\sum_{k=1}^{K} w_{r,k} \log\left(1 + \frac{p_k\sigma^2_{\mathrm{g}_{r,k}}}{\sigma^2_{\mathrm{m}_{r,k}}}\right) \\ \text{s.t.} \quad & \mathbf{1}^{\mathrm{T}}\mathbf{w}_r = 1, \quad w_{r,k} \in \{0,1\}, \quad \forall r, \forall k, \end{aligned} \tag{14}$$

where $w_{r,k}$ is a binary selection variable, and $\mathbf{w}_r = [w_{r,1}, \cdots, w_{r,K}]^{\mathrm{T}}$. If $w_{r,k} = 1$, it means that the kth sub-carrier is assigned to the rth communication receiver. Note that, in the underlying scenario, it is possible that some communication users, which have poor channel conditions, are ignored.

To avoid this issue, the second optimization criterion maximizes the worst-case communication capacity to ensure that each communication user is served with a fair data rate, irrespective of their channel conditions. This is important for the communication users who cannot tolerate being ignored in case they have bad channel conditions. We address this worst-case optimization problem by exploiting the following min-max MILP:

$$\begin{aligned} \min_{\mathbf{w}_r} \max_{r} \quad & -\sum_{k=1}^{K} w_{r,k} \log\left(1 + \frac{p_k\sigma^2_{\mathrm{g}_{r,k}}}{\sigma^2_{\mathrm{m}_{r,k}}}\right) \\ \text{s.t.} \quad & \mathbf{1}^{\mathrm{T}}\mathbf{w}_r = 1, \quad w_{r,k} \in \{0,1\}, \quad \forall r, \forall k, \end{aligned} \tag{15}$$

which can be equivalently written as:

$$\begin{aligned} \min_{\mathbf{w}_r} \quad & t \\ \text{s.t.} \quad & -\sum_{k=1}^{K} w_{r,k} \log\left(1 + \frac{p_k\sigma^2_{\mathrm{g}_{r,k}}}{\sigma^2_{\mathrm{m}_{r,k}}}\right) \leq t, \quad \forall r, \\ & \mathbf{1}^{\mathrm{T}}\mathbf{w}_r = 1, \quad w_{r,k} \in \{0,1\}, \quad \forall r, \forall k. \end{aligned} \tag{16}$$

Note that the power p_k for each sub-carrier in the optimization (15) and (16) was obtained from (13). Although the optimization in (15) and (16) ensures the worst-case MI for the communication users, we should be careful that, if some communication users have an extremely low signal-to-noise ratio (SNR), a worst-case optimization might drain significant power in the poor communication channels, rendering the overall communication performance to be very low.

B. Cooperative Design

Unlike the radar-centric design where the power of each sub-carrier solely depends on the radar objectives, a cooperative design enables cooperation from the radar. In this case,

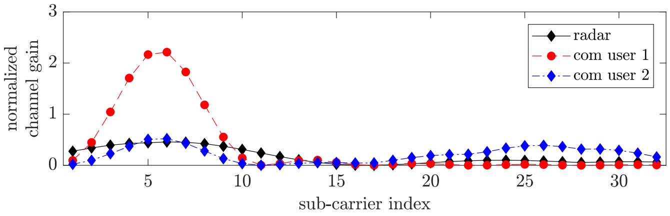
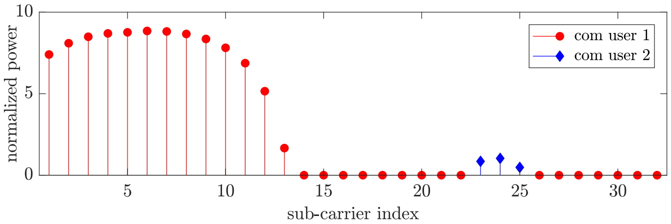
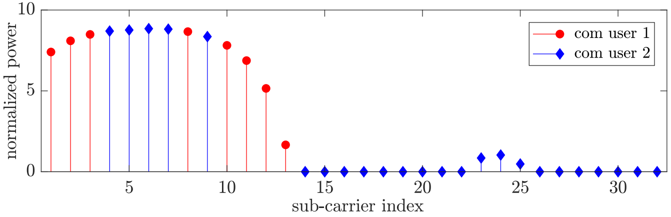


Fig. 3. Channel conditions for radar and communications.



(a) Maximum communication capacity optimization



(b) Worst-case communication capacity optimization

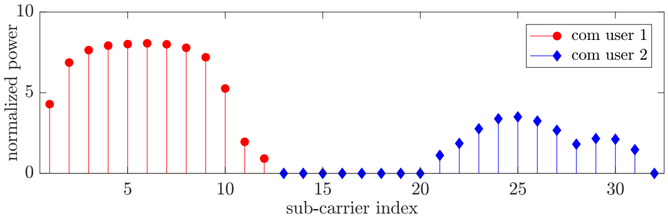
Fig. 4. Power allocation and sub-carrier distribution for radar-centric design.

radar shows some flexibility on the maximum possible MI it can achieve.

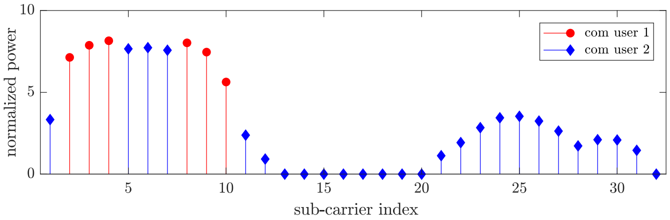
1) Power distribution: First, the optimization (13) is exploited to determine the maximum MI α_{opt} the radar can achieve. The radar then decides its flexibility parameter γ whose value varies between 0 and 1, where a higher γ favors the radar objectives. In this way, the radar function allows the dual-purpose transmitter to vary the power allocation such that the radar MI does not fall below $\gamma\alpha_{\text{opt}}$. The initial values of the sub-carrier allocation coefficients $w_{r,k}$ can be either randomly chosen, or optimized by (14) or (16). The following optimization then achieves the acceptable radar objective while maximizing the overall communication MI:

$$\begin{aligned} \min_{\mathbf{p}} \quad & -\sum_{r=1}^{R}\sum_{k=1}^{K} w_{r,k} \log\left(1+\frac{p_k \sigma^2_{\text{g}_{r,k}}}{\sigma^2_{\text{m}_{r,k}}}\right) \\ \text{s.t.} \quad & -\sum_{k=1}^{K} \log\left(1+\frac{p_k \sigma^2_{\text{h}_k}}{\sigma^2_{\text{n},k}}\right) \le -\gamma\alpha_{\text{opt}}, \\ & \mathbf{1}^{\text{T}}\mathbf{p} \le P_{\text{total,max}}, \\ & \mathbf{0} \le \mathbf{p} \le \mathbf{p}_{\max}. \end{aligned} \tag{17}$$

2) Sub-carrier allocation: The optimal value of p_k obtained from (17) is fed back to (14) or (16), depending upon which type of communication optimization criterion is required. The optimization for power distribution (17) and that for sub-carrier allocation (14) or (16) are repeated iteratively until there is no significant change in the achieved sub-carrier allocation and power distribution.



(a) Maximum communication capacity optimization



(b) Worst-case communication capacity optimization

Fig. 5. Power allocation and sub-carrier distribution for cooperative design ($\gamma = 0.95$).

V. NUMERICAL RESULTS

Consider a JRC transmitter exploiting 32 sub-carriers such that there is one radar target and two communication receivers. The normalized target channel gains and the normalized communication channel gains, respectively expressed as $\sigma_{\text{h}_k}/\sigma_{\text{n}_k}$ and $\sigma_{\text{g}_{r,k}}/\sigma_{\text{m}_{r,k}}$ are illustrated in Fig. 3. The maximum possible sub-carrier power and the total maximum power are normalized to 10 units and 100 units, respectively. We use the Gurobi solver [32] to solve for all the optimizations and the achieved MI for each case is listed in Table I.

First, we discuss the radar-centric design. Fig. 4(a) shows the power allocation for different sub-carriers using the radar-centric design (13) that maximizes the MI for radar. It can be observed that most of the power is allocated to the sub-carriers which have a high target RCS. The sub-carriers in the red and blue colors depict the OFDM sub-carriers respectively allocated to communication receivers 1 and 2 by maximizing the overall communication MI as in (14). It is observed that, although the overall communication MI is maximized, communication receiver 2 is allocated only three low-power sub-carriers to enable its communication operation. Fig. 4(b) depicts the optimized results using the worst-case optimization (16). We can see in Table I that more power is now allocated to the communication receiver 2 as it has poorer channel conditions than the communication receiver 1 in radar-favored sub-carriers. However, in Fig. 4(b), the overall communication MI is lower than that in Fig. 4(a) as we can observe in Table I.

Next, we discuss the cooperative radar-communication

TABLE I. ACHIEVED MUTUAL INFORMATION FOR THE PROPOSED STRATEGIES

	Radar-Centric Design		Cooperative Design ($\gamma = 0.95$)	
	Maximum Comm. MI	Worst-case Comm. MI	Maximum Comm. MI	Worst-case Comm. MI
$I(\mathbf{y}_{\text{rad}};\mathbf{h}\|\mathbf{s})$	15.77	15.77	11.26	11.26
$I(\mathbf{y}_{\text{com},1};\mathbf{g}_1\|\mathbf{s})$	15.29	5.66	14.23	7.89
$I(\mathbf{y}_{\text{com},2};\mathbf{g}_2\|\mathbf{s})$	0.48	5.67	4.30	7.71

design. For this purpose, the radar's objective is to achieve 95% of the maximum possible MI. Fig. 5(a) shows the power allocation and sub-carrier distribution for the case of maximum communication MI. We note in Table I that, although the radar MI is reduced, the overall communication MI is improved. Similarly, Fig. 5(b) illustrates the worst-case optimization which maximizes the worst-case communication MI for both communication receivers at the expense of reduced overall communication MI.

VI. CONCLUSION

In this paper, we present a novel JRC system which exploits OFDM waveforms for performing radar and communication operations simultaneously. A dual-purpose OFDM transmitter is exploited which optimizes the transmit power of different sub-carriers to fulfill the radar objectives. The same OFDM sub-carriers are allocated to different communication receivers to enable the communication objectives. The MI between frequency-sensitive radar and communication channels is used as the optimization objective for optimizing the system's performance. We discussed the problem for radar-centric and cooperative designs. Moreover, communication performance was discussed in terms of maximum overall mutual information as well as the worst-case communication mutual information. Simulation results show the comparison of the proposed strategies.

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