

# Imperfectly Synchronized Cooperative Network Using Distributed Space-Frequency Coding

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*Abstract*— In distributed multiple-input-multiple-output (MIMO) systems, imperfect synchronization causes a unique problem in a coded cooperative diversity system. In the presence of a fractional-symbol delay between the signals transmitted from different relay nodes, the channels become highly dispersive even at a flat-fading environment. Existing methods solve such problem based on time-domain approaches where adaptive equalization is required at the receivers for combining the information transmitted from distributed sources. In this paper, we propose the use of OFDM-based approaches using distributed space-frequency codes. The proposed schemes are insensitive to fractional-symbol delays and lead to higher data rate transmission and simplified implementation. In addition, the proposed schemes permit the use of relatively simple amplify-and-forward algorithm in multi-hop wireless networks without delay accumulations. The proposed methods remove the time delay in each relaying hop by reconstructing the prefix and, as such, improve the spectral efficiency, while keeping a simplified relaying structure.

## I. Introduction

In a wireless network, information transmission through relaying can be energy efficient. The use of multiple relaying nodes provides high diversity gain, improving robustness against channel impairments [1]. The concept of network cooperation or cooperative diversity has benefited from the recent advances of space-time codes, transmit diversity, and multi-input-multi-output (MIMO) technologies. Recent research work has shown the feasibility of cooperative protocols and provided various capacity and performance analyses of cooperative diversity systems (see for example, [2]–[5] and references therein).

To achieve effective data transmission in a cooperative network surrounded by a flat-fading environment, several coded cooperation schemes have been developed. However, to adopt the detection schemes established for MIMO technologies, a fundamental assumption of all these schemes is that the relaying terminals are perfectly synchronized. Unlike the MIMO systems, however, perfect synchronization in diversity systems is highly unlikely in practice, because of the following reasons [6]: (a) An accurate synchronization between the relaying terminals with different locations, which are probably subject to contin-

uous movements, is impractical. (b) Even for a network with fixed terminals (such as in a sensor network), changes in the parameters of electronic components may result in drifting and handshaking among transmitters is done infrequently so as to save energy and bandwidth. (c) Although nodes can synchronize the received signals as much as possible, low cost implementations may still make their timing and frequency slightly different, hence cause mismatch in the long run. (d) The major synchronization problem, however, lies in the delays of their signals when reaching at the receiving nodes in the next hop. Propagation delays may be unknown to them, while transmission times may be different. In fact, if the transmitting nodes try to synchronize toward one receiver, they may increase asynchronism toward other receivers because of different transmission distances. On the other hand, in most practical networks, nodes can (and are required) to maintain slot synchronization, which means that coarse slot synchronization is available. The difficulty lies in the fine transmission synchronization.

Most digital modulation signals guarantee no intersymbol interference (ISI) only when the signals are accurately sampled at the symbol center. When the signal arrivals from different relay terminals are not perfectly synchronized, the effect due to pulse tails resulted from sampling position error should be considered. This is particularly true when the receiver performs a symbol-based processing. To consider such effect, [6] proposed the use of an equivalent dispersive channel model, whether the channels are frequency-selective or frequency-nonsselective. It also proposed a useful scheme which employs distributed space-time block code (STBC) in a time-reverse manner to achieve full diversity while tolerating imperfect synchronization. The difficulty of using this method, nevertheless, lies in the fact that it requires a channel equalizer at each receiver that makes the receiver very complicated and expensive.

In this paper, we propose the use of multi-carrier modulation exploiting distributed space-frequency coding schemes. The proposed schemes allow higher data rates and simplified implementation, as well as the distribution of the system complexity to transmitters and receivers. In addition, the feasibility of distributed space-frequency

coding in a multi-hop relaying cooperative network is also considered. It is pointed out that the use of MIMO-OFDM methods allows the utilization of simple amplify-and-forward algorithm. While the time delay over different relay hops will be accumulated in the amplify-and-forward relaying process when space-time coding methods (e.g., time-reverse STBC, or TR-STBC [7]) are used, such delay accumulation problem can be easily avoided in the proposed method through the reconstruction of the cyclic prefix.

## II. Channel Model

Consider a wireless network where a source node transmits a data packet to a destination node through a multi-hop wireless network. In each intermediate hop, the data packet is received by multiple nodes, e.g., nodes 1 to  $J$  in hop 1. These nodes can retransmit the data in the next hop in a cooperative manner with proper space-time (ST) encoding.

Without loss generality, the noise-free signals received at the  $k$ th node is expressed as

$$x_k(t) = \sum_{i=1}^J \int_{-\infty}^{\infty} \sqrt{P_i} h_{i,k}(\tau) s_i(t - \tau - \tau_{D,i,k}) d\tau, \quad (1)$$

where  $s_i(t)$  is the signal waveform transmitted from node  $i$ ,  $h_{i,k}(t)$  and  $\tau_{D,i,k}$  are the channel coefficients and the time delay between transmit node  $i$  and receive node  $k$ , respectively. It is assumed that channels  $h_{i,k}$  are time-invariant during the transmission of a packet, but are randomly time-varying between packets. The channels are assumed spatially white, i.e.,  $h_{i,k}$  are independent for different indices  $(i, k)$ . In addition,  $\sqrt{P_i}$  is the transmit power of node  $i$ . In a distributed MIMO system, the transmit power of different nodes may be different, depending on the a priori information of the large-scale attenuation characteristics at the transmitters [3], [12]. The total power is assumed to be a constant, i.e.,  $\sum_{i=1}^J P_i = P$ .

Let  $I_i(n)$  as the discrete information sequence transmitted from at the  $i$ th node, and  $p(t)$  as the pulse shaping function. Then, (1) becomes

$$x_k(t) = \sum_{i=1}^J \sum_{m=-\infty}^{\infty} \sqrt{P_i} I_i(m) h'_{i,k}(t - mT_s), \quad (2)$$

where

$$h'_{i,k}(t) = \int_{-\infty}^{\infty} h_{i,k}(\tau) p(t - \tau - \tau_{D,i,k}) d\tau \quad (3)$$

is the equivalent channel response by taking the pulse shaping function into account. In general, the time delay consists of three factors: channel dispersion because of reflection and scattering ( $\tau_{C,i,k}$ ), delay due to different locations of the relaying nodes ( $\tau_{D,i,k}$ ), and the pulse

shaping function spreading due to sampling position errors ( $\tau_{W,i,k}$ ). Therefore, the significance of ISI depends not only on the channel length, but also on the node locations and the pulse shaping functions. Combining all these factors, we denote the upper bound of the effective channel length in terms of symbol period, as  $L = [\max(\tau_{C,i,k}) + \max(\tau_{D,i,k}) + \max(\tau_{W,i,k})]/T_s$  with  $T_s$  denoting the symbol period. We use  $L_C = \max(\tau_{C,i,k})/T_s$ ,  $L_D = \max(\tau_{D,i,k})/T_s$ , and  $L_W = \max(\tau_{W,i,k})/T_s$  for notational simplicity, and  $L = L_C + L_D + L_W$ .

Sampling at  $t = nT_s$ , and taking the receiver noise into account, we have

$$x_k(n) = x_k(nT_s) = \sum_{i=1}^J \sum_{l=0}^L \sqrt{P_i} I_i(n-l) h'_{i,k}(l) + v_k(n), \quad (4)$$

where  $h'_{i,k}(l) = h'_{i,k}(lT_s)$ . The noise is assumed to be of zero mean and unit variance, and is temporally and spatially white, i.e.,  $E[v_k(\tau)v_l^*(\nu)] = 0$  for  $k \neq l$  or  $\tau \neq \nu$ , where  $*$  denotes complex conjugation.

## III. Distributed MIMO-OFDM Schemes

### A. MIMO-OFDM Transmission

We first review the MIMO-OFDM transmission using space-frequency (SF) codes in a transmitter with  $J$  co-located antennas. The following  $J \times N$  SF codeword is formed in the transmitter

$$\mathbf{C}_{\text{SF}} = \begin{bmatrix} \mathbf{c}_1 \\ \mathbf{c}_2 \\ \vdots \\ \mathbf{c}_J \end{bmatrix} = \begin{bmatrix} c_1(0) & c_1(1) & \cdots & c_1(N-1) \\ c_2(0) & c_2(1) & \cdots & c_2(N-1) \\ \vdots & \vdots & \ddots & \vdots \\ c_J(0) & c_J(1) & \cdots & c_J(N-1) \end{bmatrix}, \quad (5)$$

where  $c_i(n)$  denotes the channel symbol transmitted over the  $n$ th subcarrier by transmit antenna  $i$ ,  $i = 1, \dots, J$ , and  $n = 0, \dots, N-1$ . The construction of space-frequency codes has been discussed in, for example, [8]–[10]. A detailed review of broadband MIMO-OFDM wireless communications is provided in [11].

The OFDM transformation of the source codeword applies an  $N$ -point IFFT to each row of the matrix  $\mathbf{C}_{\text{SF}}$ . To eliminate the effect of ISI, cyclic prefix is inserted in the beginning of the transformed matrix. The OFDM symbol corresponding to the  $i$ th row of  $\mathbf{C}_{\text{SF}}$  is then transmitted using the  $i$ th transmit antenna.

At the receiver, after matched filtering, removing the cyclic prefix, and applying FFT to recover the transmitted SF codeword, information can be decoded, for example, using maximum likelihood detection.

### B. Transmission of Distributed SF Codes

Now we extend the SF coded MIMO-OFDM approaches to the distributed wireless network. Similar to the protocol proposed in [3], [4] for constructing distributed ST codes,

distributed SF codes can be constructed at the source terminal during the first hop. This paper only considers the intermediate hops where information is relayed from multiple distributed transmit nodes to multiple distributed receive nodes, forming a distributed MIMO structure. In particular, a relay node transmits a specific row of a SF codeword. The proposed method can be used for both flat and dispersive channels. In the latter, path diversity can be achieved [9], [10].

The OFDM transformation at the source codeword applies an  $N$ -point IFFT to each row of the matrix  $\mathbf{C}_{\text{SF}}$ , resulting in

$$\mathbf{X}_{\text{SF}} = \mathbf{C}_{\text{SF}}\mathbf{F}, \quad (6)$$

where  $\mathbf{F}$  is the  $N \times N$  inverse Fourier transform matrix. After appending a cyclic prefix of length  $C$ , the  $J \times (N+C)$  matrix is obtained as

$$\mathbf{X}'_{\text{SF}} = [\mathbf{x}_{\text{SF}}(N-C), \dots, \mathbf{x}_{\text{SF}}(N-1), \mathbf{X}_{\text{SF}}], \quad (7)$$

where  $\mathbf{x}_{\text{SF}}(n)$  is the  $n$ th column of  $\mathbf{X}_{\text{SF}}$ . The frame length becomes  $N+C$  symbol durations after the cyclic prefix is inserted. The OFDM symbol corresponding to the  $i$ th ( $i = 1, \dots, J$ ) column of  $\mathbf{X}'_{\text{SF}}$  is transmitted by the source node through the  $i$ th time frame.

### C. Transmission Efficiency

In the single-carrier time-domain approaches, the minimum guard interval is  $L_C + L_D + L_W$ . In the proposed OFDM approach, the effective pulse tail  $L_W$  does not play a role. As a result, the guard interval is solely determined by the channel order,  $L_C$ , and the upper bound of the time delay between different transmit nodes,  $L_D$ . It could imply a significant reduction, particularly in flat or not heavily dispersive channels, since  $L_W$  may take a large value, depending on the synchronization strategy and the used pulse shaping function. By reducing the time delay effect from  $L_C + L_W + L_D$  to  $L_C + L_D$ , the guard interval can be designed to be smaller than that in the single-carrier counterparts, resulting in improved data rates.

### D. System Complexity

The proposed method requires transmitters and receivers to perform IFFT and FFT for OFDM modulations. It is usually a great reduction to the computational cost of an adaptive equalizer. It is noticed that the proposed approach requires IFFT or FFT operations in both transmit /receive sides, whereas the single-carrier time-domain approaches only require equalization performed at the receivers. Therefore, consider a multi-hop wireless network using the decode-and-forward algorithm, the proposed OFDM method provides significant advantages over the single-carrier approaches because the later demands high burden for equalizations in the decoding process. On the other hand, when we adopt the amplify-and-forward algorithm in the wireless network, single-carrier relaying

approaches accumulate the channel delays over multiple hops, whereas the proposed method can easily remove such delay accumulation by reconstructing the cyclic prefix. The reconstruction of the cyclic prefix is discussed in the following section.

## IV. Cyclic Prefix Reconstruction in Multi-Hop Amplify-and-Forward Relaying

At the  $k$ th relaying node, the received signal corresponding to the  $n$ th subcarrier is given by

$$y_k(n) = \sum_{i=1}^J \sqrt{P_i} \hat{c}_i(n) H_{i,k}(n) + v_k(n), \quad (8)$$

where  $\hat{c}_i(n)$  is the scaled and noisy replica of  $c_i(n)$  contaminated in the previous hops,  $H_{i,k}(n)$  is the Fourier transform of  $h_{i,k}(t)$ , and  $v_k(n)$  denotes the additive complex Gaussian noise at the  $n$ th subcarrier.

In matrix format, for the  $n$ th subcarrier, the received signal at the  $J'$  relaying nodes, after the prefix is removed, is expressed as

$$\mathbf{y}(n) = \mathbf{H}(n)\mathbf{P}^{1/2}\hat{\mathbf{c}}(n) + \mathbf{v}(n), \quad (9)$$

where

$$\mathbf{y}(n) = [y_1(n), y_2(n), \dots, y_{J'}(n)]^T,$$

$$\hat{\mathbf{c}}(n) = [\hat{c}_1(n), \hat{c}_2(n), \dots, \hat{c}_J(n)]^T,$$

$$\mathbf{v}(n) = [v_1(n), v_2(n), \dots, v_{J'}(n)]^T,$$

$$\mathbf{H}(n) = \begin{bmatrix} h_{1,1}(n) & h_{2,1}(n) & \dots & h_{J,1}(n) \\ h_{1,2}(n) & h_{2,2}(n) & \dots & h_{J,2}(n) \\ \vdots & \vdots & \ddots & \vdots \\ h_{1,J'}(n) & h_{2,J'}(n) & \dots & h_{J,J'}(n) \end{bmatrix},$$

and

$$\mathbf{P} = \text{diag}[P_1, P_2, \dots, P_J].$$

The signal is retransmitted after the prefix is reconstructed from  $\mathbf{y}(n)$ . At the next hop, the received signal vector at the relaying nodes is expressed as

$$\begin{aligned} \mathbf{y}'(n) &= \mathbf{H}'(n) \left[ \mathbf{H}(n)\mathbf{P}^{1/2}\hat{\mathbf{c}}(n) + \mathbf{v}(n) \right] + \mathbf{v}'(n) \\ &= \mathbf{H}'(n)\mathbf{H}(n)\mathbf{P}^{1/2}\hat{\mathbf{c}}(n) + [\mathbf{H}'(n)\mathbf{v}(n) + \mathbf{v}'(n)], \end{aligned} \quad (10)$$

where  $\mathbf{H}'(n)$  is the channel matrix corresponding to the next hop. In general, when  $M$  hops are involved, the above process can be repeated. Denoting  $\mathbf{H}_m(n)$  as the channel matrix corresponding to the  $m$ th hop and the  $n$ th subcarrier, it is seen that the proposed amplify-and-forward scheme relays SF coded information with equivalent channel matrix  $\prod_{m=1}^M \mathbf{H}_m(n)$ , whereas the noise is accumulated. The information can be decoded using standard coherent detection methods if the combined channel state information (CSI) can be estimated at the destination, or using noncoherent detection methods that do not require CSI.

It is emphasized that, unlike the single-carrier amplify-and-forward process where the time delay over different hops will be accumulated, resulting in unrealistic implementations in multi-hop systems, the effect of the time delay in each hop is decoupled by the insertion and subtraction of the prefix, rendering to a manageable frame design and high spectral efficiency.

## V. Numerical Results

### A. Effect of Pulse Shaping Functions

To illustrate the effect of pulse shaping functions in an dispersive or imperfectly synchronized network using the single-carrier time-domain approaches, Fig. 1 depicts the impulse response of raised cosine filters with different sampling position errors of 0,  $T_s/4$ , and  $T_s/2$ , where the roll-off factor is 0.3. Denote  $p(l)$  as the impulse response of the raised cosine filter sampled at the symbol rate, and define

$$\epsilon(n) = \frac{\sum_{\text{all symbols}} |p(l)| - \sum_{n \text{ dominant symbols}} |p(l)|}{\sum_{\text{all symbols}} |p(l)|} \quad (11)$$

as the residual error of the impulse response after  $n$  dominant symbols are counted, the result is summarized in Table I. It is clear that, the impulse response spans roughly five to six symbols for both sampling position errors of  $T_s/4$  and  $T_s/2$ . The long impulse response results in high equalizer complicity and, in a TR-STBC structure, a long guard interval.

The effect of sampling position errors can be reduced by using a large value of roll-off factor. Fig. 2 depicts the impulse response of raised cosine filters using roll-off factor of 1.0. While it implies a reduction in equalizer complexity and guard interval, the use of large roll-off factor results in a significant reduction of the spectral efficiency.

Unlike the time-domain approaches, the effect of pulse shaping in an OFDM system is minimal. As a result, the effect of sampling position errors to the proposed MIMO-OFDM approach is negligible.

### B. Effect of Cyclic Prefix Reconstruction

We consider a portion of a multi-hop relaying system as shown in Fig. 3. Two relaying hops (hop  $i$  and hop  $i + 1$ ) are included in this diagram to illustrate the capability of eliminating delay accumulation in the proposed scheme. Each relaying hop includes two transmit antennas ( $M_t = 2$ ). Only one receiver is assumed at the receive end of the  $(i + 1)$ th hop, because the performance of each node should be evaluated separately. While the relay nodes do not decode the signal, the bit error rate (BER) at the node is computed to measure the quality of the received signal.

The channels are considered to be independent random processes. It is assumed that the channels are fixed over a frame period, but vary randomly and assume complex Gaussian distribution over different frames. The channels

TABLE I  
Residual Error of Impulse Response

$n$	1	2	3	4	5	6
$T_s/4$	0.440	0.261	0.163	0.101	0.0617	0.0364
$T_s/2$	0.658	0.315	0.219	0.123	0.0838	0.0442

are normalized such that they have a unit variance. In each hop, the total power is unity and is equally distributed over the two transmit antennas. The noise power is assumed to be the same over different relaying hops. The input signal-to-noise ratio (SNR) is defined as the reciprocal of the receiver noise power at each hop.

128 subcarriers are used for the OFDM signals, and the guard interval consists of six symbols, resulting in total 134 symbol periods over an OFDM frame. The distributed SF codewords are generated based on [10] where  $\Gamma = 4$  and  $P = 16$  are used. The source stream before SF coding is a binary sequence. Random permutation is applied in generating the SF codewords to achieve path diversity. 5000 frames are used to evaluate the BER performance.

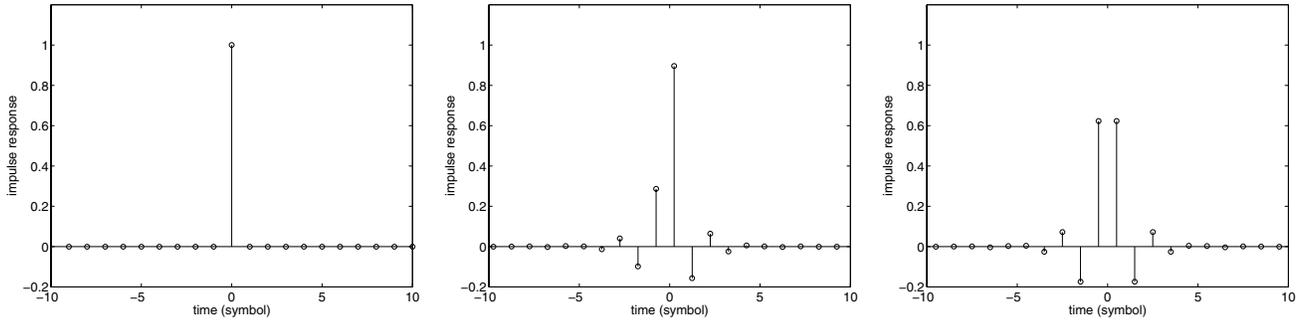
The order of each channel is assumed to be five symbols. Therefore, without prefix reconstruction, the time delay over the two hops will be accumulated, resulting a total channel order of ten symbols. As a result, the six-symbol prefix does not provide enough protection against the channel dispersion, resulting in a BER floor if no equalization is applied (the solid line in Fig. 4). However, when the prefix is removed and reconstructed in each relay, the time delay can be eliminated in each relay. It is evident in Fig. 4 (the dashed line) that the BER in such case does not have a floor and a high diversity gain is achieved in the relay process.

## VI. Conclusions

In a cooperative wireless network, channel dispersion exists as the result of several factors, e.g., channel length, pulse shaping function spreading, and spatial separations between the transmit nodes. We have considered the coded data transmission in a cooperative network with frequency-selective fading and proposed the distributed MIMO-OFDM schemes for effective multi-hop relaying. It is shown that, by reconstructing the cyclic prefix, the time delay effect in each relaying hop can be eliminated even using relatively simple amplify-and-forward algorithm. Therefore, the guard interval can be minimized and, thereby, high spectral efficiency is maintained.

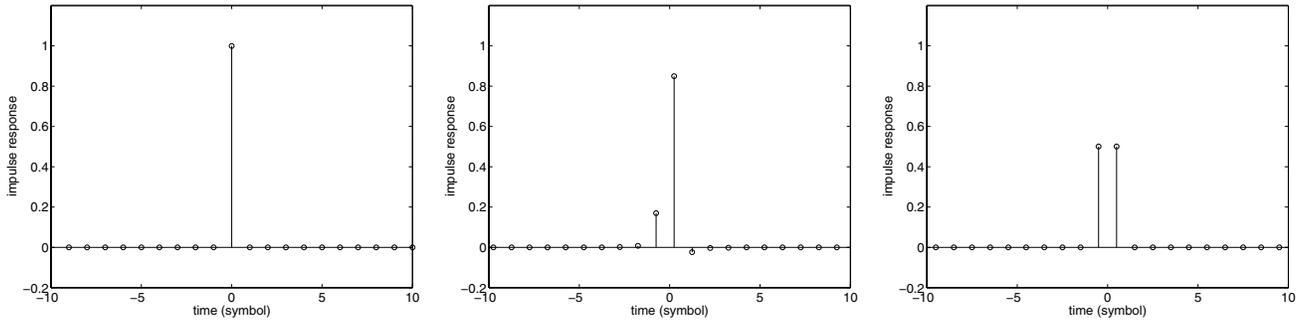
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(a) no sampling position error (b) sampling position error =  $T_s/4$  (c) sampling position error =  $T_s/2$

Fig. 1. Impulse response of raised cosine filter with different sampling position errors (roll-off factor is 0.3).



(a) no sampling position error (b) sampling position error =  $T_s/4$  (c) sampling position error =  $T_s/2$

Fig. 2. Impulse response of raised cosine filter with different sampling position errors (roll-off factor is 1.0).

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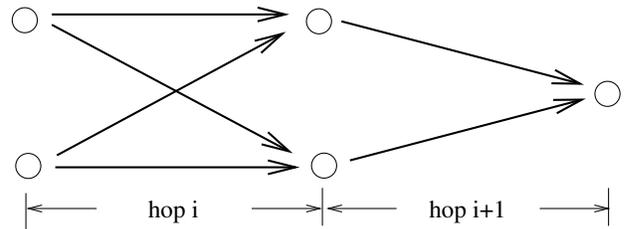


Fig. 3. Multi-hop relaying system under consideration.

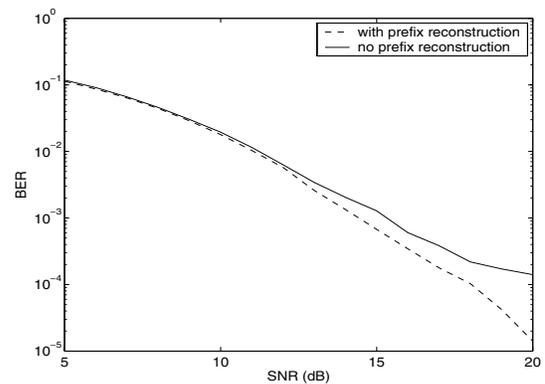


Fig. 4. BER with and without prefix reconstruction.