The Spatial Ambiguity Function and Its Applications

Moeness G. Amin, *Senior Member, IEEE*, Adel Belouchrani, *Member, IEEE*, and Yimin Zhang, *Member, IEEE*

***Abstract*—This letter introduces the spatial ambiguity functions (SAF's) and discusses their applications to direction finding and source separation problems. We emphasize two properties of SAF's that make them an attractive tool for array signal processing.**

***Index Terms*—Array signal processing, joint-variable MUSIC, nonstationary signals, spatial ambiguity function, time-frequency distribution.**

I. Introduction

THE EVALUATION of quadratic time-frequency distributions of the data snapshots across the array yields spatial time-frequency distributions (STFD's), which can be used to solve a large class of blind source separation and high resolution direction-of-arrival (DOA) estimation problems [1], [2]. STFD techniques are appropriate to handle sources of nonstationary waveforms that are highly localized in the time-frequency domain.

The concept of STFD can been extended to an arbitrary joint-variable domain [3], [4]. In this letter, the ambiguity functions are considered. Similar to STFD's, spatial ambiguity functions (SAF's) are discriminatory tools. The sources whose ambiguity domain signatures are used in constructing the SAF matrix are the only ones considered for signal separation and subspace estimation.

II. Analysis Model

The following linear data model

$$\mathbf{x}(t) = \mathbf{A}\mathbf{d}(t) + \mathbf{n}(t) \tag{1}$$

is commonly used in narrowband array processing, where $\mathbf{A}$ is the mixing matrix of dimension $m \times n$, $\mathbf{x}(t) = [x_1(t), \ldots, x_m(t)]^T$ is the sensor array output vector, and $\mathbf{d}(t) = [d_1(t), \ldots, d_n(t)]^T$ is the source signal vector. The superscript ^{T} denotes the transpose operator. $\mathbf{n}(t)$ is an additive noise vector. In direction-finding problems, we require $\mathbf{A}$ to have a known structure.

The SAF matrix of a signal vector $\mathbf{x}(t)$ is defined as

$$\mathbf{D}_{\mathbf{xx}}(\theta, \tau) = \int_{-\infty}^{\infty} \mathbf{x}(u + \tau/2)\mathbf{x}^H(u - \tau/2)e^{j\theta u}\, du \tag{2}$$

where θ and τ are the frequency lag and the time lag, respectively, and ^{H} denotes conjugate transpose. In a noise-free environment, $\mathbf{x}(t) = \mathbf{A}\mathbf{d}(t)$. In this case

$$\mathbf{D}_{\mathbf{xx}}(\theta, \tau) = \mathbf{A}\mathbf{D}_{\mathbf{dd}}(\theta, \tau)\mathbf{A}^H. \tag{3}$$

Equation (3) is similar to the formula that has been commonly used in blind source separation and DOA estimation problems, relating the data correlation matrix to the signal correlation matrix [5], [6]. Here, these matrices are replaced by the data spatial ambiguity function and signal ambiguity function matrices, respectively. The two subspaces spanned by the principle eigenvectors of $\mathbf{D}_{\mathbf{xx}}(\theta, \tau)$ and the columns of $\mathbf{A}$ are identical. This implies that array signal processing problems can be approached and solved based on the SAF.

III. Properties of Spatial Ambiguity Functions

The SAF's have the following two important offerings that distinguish them from other array spatial functions.

1) The crossterms in between source signals reside on the off-diagonal entries of matrix $\mathbf{D}_{\mathbf{dd}}(\theta, \tau)$, violating its diagonal structure, which is necessary to perform blind source separation. In the ambiguity domain, the signal autoterms are positioned near and at the origin, making it easier to leave out crossterms from matrix construction.
2) The autoterms of all narrowband signals, regardless of their frequencies and phases, fall on the time-lag axis ($\theta = 0$), while those of the wideband signals fall on a different (θ, τ) region or spread over the entire ambiguity domain. Therefore, the SAF is a natural choice for recovering and spatially localizing narrowband sources in broadband signal platforms.

IV. Ambiguity-Domain MUSIC

Similar to time-frequency MUSIC [2], the signal and noise subspaces $\mathbf{E} = [\mathbf{E_s}\ \mathbf{E_n}]$ of the SAF matrix $\mathbf{D}_{\mathbf{xx}}(\theta, \tau)$ can be obtained by the block joint-diagonalization of $\mathbf{D}_{\mathbf{xx}}(\theta, \tau)$ obtained at different (θ, τ) points. Once the noise subspace $\mathbf{E_n}$ is estimated, the ambiguity-domain MUSIC (AD-MUSIC) technique estimates the DOA's by finding the n_o largest peaks of the localization function $f(\phi) = |\hat{\mathbf{E}}_{\mathbf{n}}^H \mathbf{a}(\phi)|^{-2}$.

Consider the scenario of a four-element, equispaced linear array, where one chirp signal and two sinusoidal signals are received. The data record has 128 samples. All three signals have the same SNR of 20 dB. The DOA's of the chirp signal and the two sinusoidal signals are 15°, 10°, and 0°, respectively. While the ambiguity function of the chirp signal sweeps the ambiguity domain with contribution at the origin, the exact autoterm ambiguity function of the narrowband arrivals $s_1(t)$ and $s_2(t)$ is zero

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M. G. Amin and Y. Zhang are with the Department of Electrical and Computer Engineering, Villanova University, Villanova, PA 19085 USA (e-mail: moeness@ece.vill.edu).

A. Belouchrani is with the Department of Electrical Engineering, Ecole Nationale Polytechnique, Algiers, Algeria.

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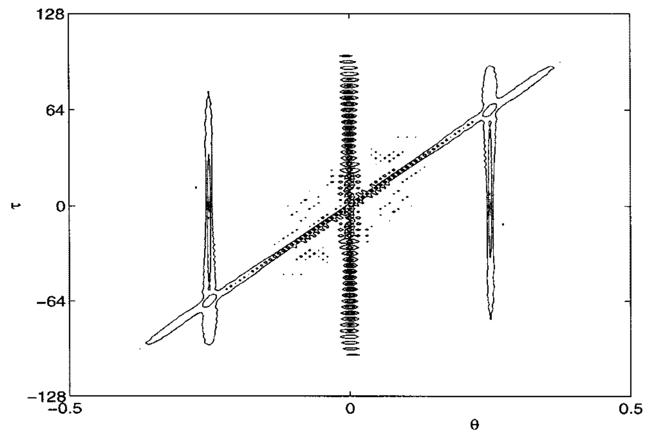

Fig. 1. Ambiguity functions of the chirp signal and two sinusoidal signals.

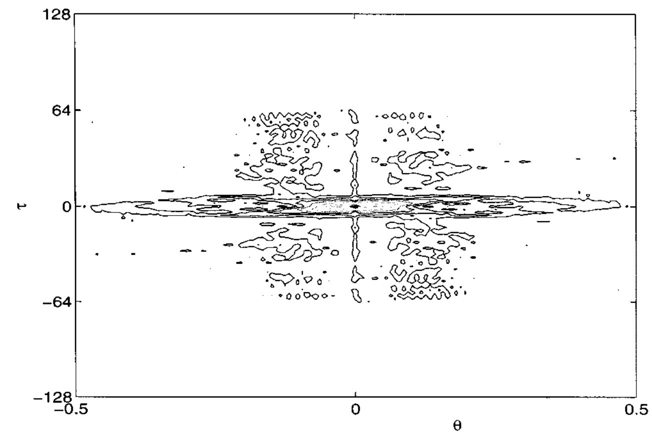

Fig. 3. Ambiguity functions of the mixed signal.

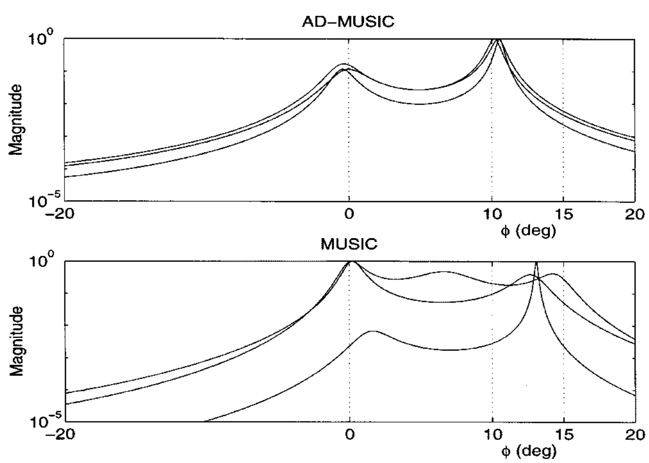

Fig. 2. Estimated spatial spectra of AD-MUSIC and conventional MUSIC.

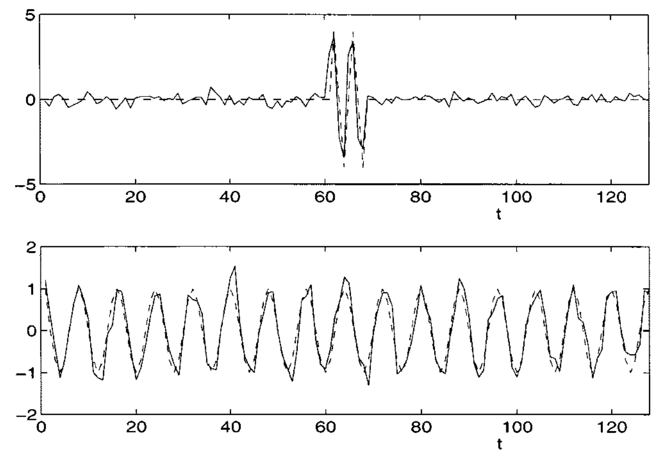

Fig. 4. Real part of the waveforms of the source signals (- - -) and the separated signals (——).

for nonzero frequency lags and may have nonzero values only along the vertical axis $\theta = 0$.

In this simulation example, we selected 24 points on the time-lag axis excluding the origin, and in so doing emphasized the narrowband components. Fig. 1 shows the ambiguity function where the two vertical lines represent the crossterms between the sinusoidal components. Fig. 2 shows the two estimated spatial spectra of three independent trials. One spectrum corresponds to the conventional method, and the other corresponds to the AD-MUSIC. There are two dominant eigenvalues for the case of the AD-MUSIC, since the chirp signal has been dropped out through our careful selection of the ambiguity-domain points. It is clear that the AD-MUSIC resolves the two sinusoidal signals, while the conventional MUSIC could not separate the three signals.

V. Ambiguity-Domain Source Separation

Analogous to blind source separation based on STFD [1], blind source separation based on SAF consists mainly of two steps. The first step is to whiten the array signal vector by an $m \times n$ matrix $\mathbf{W}$ such that $(\mathbf{WA})(\mathbf{WA})^H = \mathbf{UU}^H = \mathbf{I}$ (i.e., $\mathbf{WA}$ is a unitary matrix). The whitening matrix $\mathbf{W}$ can be obtained, for example, from the covariance matrix [1]. The second step is to perform joint diagonalization to obtain the unitary matrix $\mathbf{U}$ [1], which is then used to provide $\mathbf{A} = \mathbf{W}^{\#}\mathbf{U}$, where $^{\#}$ denotes pseudo-inverse, and the source signal vector is recovered as $\mathbf{s}(t) = \mathbf{U}^H\mathbf{W}\mathbf{x}(t)$. All of the above matrices are replaced by their estimates when dealing with one realization.

Assume that we have two sources and three equispaced sensors. One source is a sinusoid, whereas the other is a pulsed sinusoidal signal that extends over eight samples. The SNR of both signals, defined in the total power, is 10 dB. In this example, the mixing matrix did not have a presumed structure, and its columns were not complex exponential vectors.

The ambiguity function of the mixed signal at the first sensor is shown in Fig. 3. In this specific case, we select four points along the frequency-lag axis and the time-lag axis closest to the origin. Then, by using the spatial ambiguity functions, we are able to recover the original signals from only their observed mixture. Fig. 4 shows the waveforms of the original and the separated signals after multiplication by the proper complex scalar.

VI. Conclusion

The spatial ambiguity function and its application to direction finding and blind source separation have been discussed. Based on the spatial ambiguity functions, we have introduced the ambiguity-domain MUSIC and the ambiguity-domain blind source separation techniques.

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