Bilinear Signal Synthesis in Array Processing

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***Abstract*—Multiple source signals impinging on an antenna array can be separated by time–frequency synthesis techniques. Averaging of the time–frequency distributions (TFDs) of the data across the array permits the spatial signatures of sources to play a fundamental role in improving the synthesis performance. Array averaging introduces a weighting function in the time–frequency domain that decreases the noise levels, reduces the interactions of the source signals, and mitigates the crossterms. This is achieved independent of the temporal characteristics of the source signals and without causing any smearing of the signal terms. The weighting function may take noninteger values, which are determined by the communication channel, the source positions, and their angular separations. Unlike the recently devised blind source separation methods using spatial TFDs, the proposed method does not require whitening or retrieval of the source directional matrix. The paper evaluates the proposed method in terms of performance and computations relative to the existing source separation techniques based on quadratic TFDs.**

***Index Terms*—Array processing, signal synthesis, source separation, time–frequency distribution.**

I. Introduction

TIME–FREQUENCY distributions (TFDs) have been shown to be very useful for nonstationary signal analysis and synthesis [1]–[5]. While TFDs have been sought out and successfully used in the areas of speech, biomedicine, the automotive industry, and machine monitoring, their applications to sensor and spatial signal processing have not been sufficiently investigated. By properly incorporating the spatial dimension into time–frequency signal representations, the bilinear and higher order forms of TFDs can be a powerful tool for high-resolution angle-of-arrival estimation and recovery of the source waveforms impinging on a multisensor receiver, specifically those of nonstationary temporal characteristics.

Synthesizing the signal from bilinear distributions of the data at a single antenna receiver is often impeded by the presence of high levels of noise and crossterms. These undesired terms not only obscure the true signal power localization in the time—frequency (t-f) domain but also reduce the synthesized signal quality. Signal synthesis using TFDs can be improved using an antenna array receiver. The availability of the source signals at different array elements allows the implementation of t-f synthesis techniques that utilize the source spatial signatures for crossterm reduction and noise mitigation. In this paper, we introduce a new approach for signal synthesis in antenna arrays that utilizes the spatial separation of the sources as well as the sources' t-f characteristics. In effect, we perform source separation, or signal recovery, based on the difference in both the t-f and spatial signatures of the signal arrivals. The signals impinging on the multi-antenna receiver are assumed to be localizable in the t-f domain, e.g., FM and polynomial phase signals. The estimation of the signal waveforms and/or their respective spatial signatures are found important applications in, for example, jammer suppression in spread spectrum communication systems and global positioning system (GPS) receivers, and various radar and sonar systems (see, for example, [6] and [7]).

Unlike the proposed technique, the existing array signal processing techniques for nonstationary source separation using bilinear distributions require the construction of spatial TFD (STFD) matrices from the data snap shots. The elements of this matrix represent the auto- and cross-TFDs of the data across the array. It was shown in [14]–[16] that the formula relating the TFD matrix of the sensor data to that of the sources is identical to the relationship between the data covariance matrix and the source correlation matrix. Blind source separation (BSS) can therefore be performed using the source t-f signatures, instead of their correlation functions. The former is more suitable for nonstationary signal environments. The BSS based on TFDs method introduced in [14] first estimates the array, or the spatial signature, matrix from the STFD using joint diagonalization. Then, it proceeds to use this matrix estimate to undo the mixing at the array and recover the source signals.

The main difficulty of the above approach, however, is the need to construct the STFD matrices from auto-term points. Selections of cross-terms violate the diagonal structure of the source TFD matrix, which is a necessary condition for most blind source separation methods. Even if successfully selected, the autoterm region is often contaminated by high level of noise and intruded upon by the crossterms through the energy in their mainlobes and/or sidelobes. The key feature of the proposed technique is the utilization of the sources' spatial structures to enhance their t-f signatures in the t-f domain. Bilinear signal synthesis methods [8]–[10] can then be applied to the enhanced source t-f features to recover the signal waveform and its temporal characteristics. By averaging the TFDs of the data across the array, we permit the source spatial signatures (SS) to play a fundamental role in reducing noise and crossterm contamination of the true signal t-f power concentration, leading to improved synthesis performance. It is shown that the performance is determined by the inner product of the source array vectors

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and improves for weakly correlated and orthogonal source spatial signatures. In the case of Gaussian channel and omni-directional uniform linear arrays, spatial averaging produces a *sinc* weighting function whose maximum value, normalized to one, is assigned to all source autoterms, whereas its fractional values are assigned to the source crossterms, and thereby mitigating their effects. It is shown that the extent of crossterm reduction is a function of the spatial frequency separation of the sources and does not rely on the source specific locations or their t-f characteristics. However, sources positioned near the broadside will generally exhibit lower interactions in the t-f domain than those at the endfire.

Unlike source separation techniques based on STFD, the proposed approach does not require whitening or retrieval of the source directional matrix, thereby, simplifying the signal recovery process. Further, as a result of the averaging process in the proposed approach, a weighting function in the t-f domain is constructed, which decreases the noise levels, reduces the interactions of the source signals, and mitigates the cross-terms. This is achieved independent of the temporal characteristics of the source signals and without causing any smearing of the signal auto-terms.

The paper is organized as follows. The signal model is presented in Section II, and the proposed array averaging technique is also formulated. The effect of source angular separation on cross-term reduction is cast in Section II using the implicit beamforming properties of spatial averaging. Section II also addresses the equivalent t-f weighting introduced by the proposed technique. The complete synthesis procedure is devised in Section III, where the signals are synthesized from the array averaged extended Wigner-Ville distribution (WVD). The extended WVD [11] is used to avoid the need for extracting the odd-indexed and even-indexed vectors separately via eigenanalysis. Numerical simulations illustrating the performance of the proposed method are given in Section IV.

II. Problem Formulation

A. Signal Model

Assume L source signals incident on an M-sensor array. The propagation delay between antenna elements is assumed to be small relative to the inverse of the transmission bandwidth so that the received signals are identical to within a complex constant. The data received across the array is, in discrete-time expression, given by the narrowband model

$$\mathbf{x}(t) = \mathbf{y}(t) + \mathbf{n}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t),\ t = 1, \ldots, N \tag{1}$$

where $\mathbf{x}(t) = [x_1(t), \ldots, x_M(t)]^T$ and $\mathbf{s}(t) = [s_1(t), \ldots, s_L(t)]^T$ are the $M \times 1$ data snapshot vector and the $L \times 1$ source signal vector at time instant t, respectively. The superscript ^{T} denotes the vector/matrix transpose. It is assumed that $M \geq L$. The $M \times 1$ vector $\mathbf{n}(t)$ is the noise vector whose elements are modeled as stationary, spatially and temporally white complex Gaussian processes with zero mean and variance of σ^2, i.e.,

$$E\left[\mathbf{n}(t+\tau)\mathbf{n}^H(t)\right] = \sigma^2 \delta(\tau)\mathbf{I} \tag{2}$$

where the superscript ^{H} denotes transpose conjugation. Moreover, $\delta(\tau)$ is the Kronecker delta, $\mathbf{I}$ denotes the identity matrix, and $\mathbf{A}$ denotes the $M \times L$ mixing matrix

$$\mathbf{A} = [\mathbf{a}_1, \ldots, \mathbf{a}_L] \tag{3}$$

The columns of matrix $\mathbf{A}$ are the source spatial signatures (SSs) and are given by

$$\mathbf{a}_i = [a_{i1}, \ldots, a_{iM}]^T \tag{4}$$

where a_{ij} is the jth component of the ith SS $\mathbf{a}_i$. Matrix $\mathbf{A}$ serves as the transfer function between the source signals $\mathbf{s}(t)$ and the data $\mathbf{x}(t)$. Furthermore, we assume that matrix $\mathbf{A}$ is of full column rank, which implies that the SSs associated with the L sources are linearly independent. To simplify the discussion, we exchange any possible scalar factor embedded in $\mathbf{a}_i$ to the source signal and assume that $\|\mathbf{a}_i\|_2^2 = M$. It is clear that this exchange does not affect the data observed from the antenna array.

It is evident that when $L > 1$, (1) represents a multicomponent scenario due to the mixture of the signals at each sensor. Therefore, a quadratic TFD at the individual sensors would contain not only the autoterms of all source signals but the interactions of the source signals as well, causing undesirable crossterms.

For the purpose of subsequent derivation, we first expand (1) using definitions (3) and express the received noise-free data vector

$$\mathbf{y}(t) = \sum_{i=1}^{L} \mathbf{a}_i s_i(t). \tag{5}$$

Specifically, the data received at sensor k ($k = 1, 2, \ldots, M$) is given by

$$y_k(t) = \sum_{i=1}^{L} a_{ik} s_i(t). \tag{6}$$

B. Array-Averaged WVD

The discrete form of WVD of the signal $y(t)$ is given by [4][1]

$$W_{yy}(t,f) = \sum_{l=-\infty}^{\infty} y(t+l) y^*(t-l) e^{-j4\pi fl} \tag{7}$$

where * denotes complex conjugation, and t and f represent the time index and the frequency index, respectively. Equation (7) is often referred to as the auto WVD of the signal $y(t)$. Similarly, the cross WVD of any two signals $y_1(t)$ and $y_2(t)$ is defined as

$$W_{y_1 y_2}(t,f) = \sum_{l=-\infty}^{\infty} y_1(t+l) y_2^*(t-l) e^{-j4\pi fl}. \tag{8}$$

[1]There should be a factor of 2 in front of the summation. However, this factor is omitted in this and subsequent equations for simplicity.

Substituting (6) into (7), we can express the WVD of the signal at the kth sensor $y_k(t)$ as

$$\begin{aligned} W_{y_k y_k}(t,f) &= \sum_{i=1}^{L}\sum_{j=1}^{L} a_{ik}a_{jk}^{*} \sum_{l=-\infty}^{\infty} s_i(t+l)s_j^{*}(t-l)e^{-j4\pi fl} \\ &= \sum_{i=1}^{L}\sum_{j=1}^{L} a_{ik}a_{jk}^{*} W_{s_i s_j}(t,f) \end{aligned} \tag{9}$$

where $W_{y_k y_k}(t,f)$ will herein be referred to as the auto-sensor WVD of $y_k(t)$. $W_{s_i s_j}(t,f)$ corresponds to the auto-source or cross-source WVD, depending on whether $i=j$, or $i \neq j$. It is important to note that there are two types of crossterms in the underlying problem. The first type are the crossterms that are present in the auto-source WVD. These terms are the results of the interactions between the components of the same source signal, which is the case when the source signal itself is of multicomponents. We assume monocomponent sources for simplicity. The other type of crossterms are found in the cross-source WVD and generated from the interactions between two signal components belonging to two different sources. A variant of the two is the crossterms in the cross-sensor WVD, which results from the interactions of the signals from different array elements. It is generally the collection of the above crossterms and the source auto-terms. We note that the cross-sensor WVD does not play a role in the analysis presented in this paper.

Averaging the auto-sensor WVDs over the array yields

$$\begin{aligned} \bar{W}(t,f) &= \frac{1}{M}\sum_{k=1}^{M} W_{y_k y_k}(t,f) \\ &= \sum_{i=1}^{L}\sum_{j=1}^{L} \left(\frac{1}{M}\sum_{k=1}^{M} a_{ik}a_{jk}^{*}\right) W_{s_i s_j}(t,f) \\ &= \sum_{i=1}^{L}\sum_{j=1}^{L} \left(\frac{1}{M}\mathbf{a}_j^H \mathbf{a}_i\right) W_{s_i s_j}(t,f). \end{aligned} \tag{10}$$

In (10), $\mathbf{a}_j^H \mathbf{a}_i$ is the inner product of the SSs $\mathbf{a}_i$ and $\mathbf{a}_j$. For $i=j$, $\mathbf{a}_i^H \mathbf{a}_i = \|\mathbf{a}_i\|_2^2 = M$. Defining the spatial correlation coefficient

$$\beta_{ij} = \frac{1}{M}\mathbf{a}_j^H \mathbf{a}_i \tag{11}$$

(10) can be then rewritten as

$$\bar{W}(t,f) = \sum_{i=1}^{L}\sum_{j=1}^{L} \beta_{ij} W_{s_i s_j}(t,f). \tag{12}$$

The above equation shows that $\bar{W}(t,f)$ is a linear combination of the auto-source and cross-source WVDs of all signal arrivals. To obtain a general and compact form for $\bar{W}(t,f)$, we define the source WVD matrix that enters $W_{s_i s_j}(t,f)$ as its (i,j)th element

$$\mathbf{W_{ss}}(t,f) = \left[W_{s_i s_j}(t,f)\right],\ i,j = 1,\ldots,L \tag{13}$$

and

$$\mathbf{\Gamma} = \frac{1}{M}\mathbf{A}^H \mathbf{A} = [\beta_{ij}],\ i,j = 1,\ldots,L. \tag{14}$$

Accordingly, the averaged WVD $\bar{W}(t,f)$ could be simplified to

$$\bar{W}(t,f) = \mathbf{u}^H \left[\mathbf{W_{ss}}(t,f) \odot \mathbf{\Gamma}\right] \mathbf{u} \tag{15}$$

where $\odot$ denotes the Hadamard product, or the matrix element-by-element product, and $\mathbf{u}$ is an $L \times 1$ vector of unit values, i.e., $\mathbf{u} = [1,\ldots,1]^T$. Equation (15) is valid for every (t,f) point and elucidates the averaging of the WVD across the array. It includes all the signal autoterms and crossterms that naturally appear in a typical multicomponent WVD. However, in (15), these autoterms and crossterms are weighted by constant values, which are the spatial correlation coefficients that have resulted from the inner product between the sources' SSs, which are exhibited in the elements of matrix $\mathbf{\Gamma}$. It is important to note that by the virtue of the inner product, the source directional information carried by its respective SS is lost in $\bar{W}(t,f)$.

The diagonal elements of the matrix $\mathbf{W_{ss}}(t,f) \odot \mathbf{\Gamma}$ constitute all the autoterms of the L source signals, whereas the off-diagonal elements are their respective cross-terms. It is straightforward to show that for the ith and the jth sources

$$|\beta_{ij}| \leq 1, i \neq j \text{ and } \beta_{ij} = 1, i = j \tag{16}$$

indicating that the constant coefficients in (15) for the auto-source WVD' are always greater than, or at least equal to, those for the cross-source WVDs. For a large array or widely separated sources, $|\beta_{ij}| \ll 1$, leading to significant suppression of the crossterms. This property is utilized by the array averaging process and is shown to improve the signal synthesis performance.

An interesting case arises when all SSs are orthogonal, i.e., $\beta_{ij} = 0$ for any $i \neq j$. In this case, $\mathbf{\Gamma}$ becomes an identity matrix and yields

$$\bar{W}(t,f) = \sum_{k=1}^{L} W_{s_k s_k}(t,f). \tag{17}$$

It should be noted that $\bar{W}(t,f)$ in (17) is solely the summation of the source signal autoterms. The above equation highlights the fact that all crossterms between different source signals are entirely eliminated from $\bar{W}(t,f)$, and only the autoterms are maintained, which is most desirable from the synthesis perspective.

C. *Beamforming Effect*

In order to establish quantified analysis on the suppression effect of crossterms based on the proposed array averaging technique, we consider the special case of a Gaussian channel and a uniform linear array (ULA). With no signal scattering, the SS displays the structure

$$\mathbf{a}_i = \left[1, e^{j\omega_i}, \ldots, e^{j(M-1)\omega_i}\right]^T. \tag{18}$$

The spatial frequency of the ith source ω_i is given by

$$\omega_i = \frac{2\pi d}{\lambda}\sin(\theta_i) \tag{19}$$

where d is the interelement spacing, λ is the wavelength, and θ_i is the angle of arrival (AOA). From (11), we obtain

$$|\beta_{ij}| = \left|\frac{1-e^{jM\Delta\omega_{ij}}}{M\left(1-e^{j\Delta\omega_{ij}}\right)}\right| = \left|\frac{\sin\left(\frac{M\Delta\omega_{ij}}{2}\right)}{M\cdot\sin\left(\frac{\Delta\omega_{ij}}{2}\right)}\right| \qquad (20)$$

where $\Delta\omega_{ij} = \omega_j - \omega_i$ denotes the difference between the two spatial frequencies ω_j and ω_i. Equation (20) is the well-known *array factor* for an M-element ULA [17]. The spatial pattern represented by (20) reaches its maximum value within the mainlobe at $\Delta\omega_{ij} = 0$. The pattern has secondary maxima in the side lobes. The largest of those maxima occurs within the first sidelobe and is asymptotically (for large M) 13 dB down from the unit value (the highest normalized sidelobe level of a uniformly fed ULA is about -13 dB for large M). In this regard, if the difference in the spatial frequencies of adjacent sources are greater than $\Delta\Omega = 2\pi/M$, which is half of the mainlobe width, the suppression of crossterms could always be guaranteed by at least 13 dB for a large value of M. From (20), we have

$$\lim_{M\to\infty} |\beta_{ij}| = 0, \ \ i \neq j. \qquad (21)$$

Using this result, we could further rewrite (12) as

$$\lim_{M\to\infty} \bar{W}(t,f) = \sum_{k=1}^{L} W_{s_k s_k}(t,f) \qquad (22)$$

which is the asymptotical form of the orthogonal scenario described in (17). The importance of (22) lies in the fact that by utilizing the array averaging approach, crossterm could be suppressed to any extent if there are sufficient number of array elements. In other words, the orthogonality in SSs could be always approached by increasing the array manifold. As such, in the underlying problem, the array size is cast as an important parameter influencing the performance of crossterm suppression through array averaging.

Since M and d appear as a product in (20), then as the number of the sensor M increases, the sources could be more closely spaced without impeding crossterm suppression. It is important to note that because of the nonlinearity between $\Delta\omega$ and $\Delta\theta$, the condition on angular separation $\Delta\theta$ for the same level of cross-term suppression is more relaxed when the sources are near the broadside and more rigid when they are placed at the endfire.

The condition for the orthogonal structure of all SSs in a ULA, and subsequently full crossterm suppression, could be determined by simply setting (20) equal to 0. Consequently, we obtain

$$\Delta\omega_{ij} = \pm m\frac{2\pi}{M} = \pm m\Delta\Omega, \ m = 1, 2, \ldots \qquad (23)$$

and (17) follows. Conversely, the worst performance corresponds to the case in which the sources are closely spaced, i.e., $|\Delta\omega_{ij}| \ll 2\pi/M$. In this case, $|\beta_{ij}| \approx 1$, that means the received signal at different antenna sensors are highly correlated, and the crossterms would not encounter any significant changes as a result of array averaging. In general, if all the sources impinging on the array are closely spaced, the source signal crossterms cannot be substantially reduced by array averaging, and the averaged WVD has less difference to the WVD computed from the output of a single antenna receiver. It is noted that the degradation of source separation performance in the presence of closely spaced sources is typical and common for all spatial signal processing methods because of the close spatial signatures of the sources.

D. Equivalent Time-Frequency Weighting

The incorporation of the source SSs into WVD results in multiplying the original WVD with an appropriate weighting coefficient to an autoterm or a crossterm. When the autoterms and crossterms are separated in the t-f domain, a weighting coefficient is similar to a mask function in the t-f domain. However, unlike the conventional mask functions that typically, but not necessarily, assigns 1 for desired t-f regions and 0 elsewhere, the weights produced by the source spatial structure may assume any values, which are dependent on both the communication channels and source spatial locations. These weighting values are high over autoterm regions and small over crossterm regions, regardless of their specific locations in the t-f domain. It is emphasized that in our method, different weighting coefficients can be applied to an autoterm and a crossterm separately, even they are completely overlapping.

It is noteworthy that there are two types of crossterms in the underlying problem. The first type are the crossterms that are the results of the interaction between the components of the same source signals. Those crossterms are not harmful to signal synthesis and will not be suppressed in the process of spatial averaging. The other type of crossterms are those generated from the interactions between two signal components beloging to two different sources. The latter type of crossterms must be suppressed so that the source signals can be synthesized separately.

Since the suppression of crossterms between different source signals is controlled by the inner product of the source SSs and is not dependent on the source temporal characteristics and signal frequency contents, the evolved t-f mask in the underlying problem only reduces the crossterms that are produced from the interaction of the signals of different sources. That is, the array averaging process of the sensors' WVDs does not reduce the crossterms of the signal components belonging to the same source. These crossterms are, in essence, highlighted by the same coefficient that multiplies the respective source autoterms, and their retainment is important when synthesizing multicomponent signals. In other words, unlike reduced interference distributions (RIDs) [4], [12] in which appropriate kernels are applied for smoothing all crossterms, the proposed synthesis method using array processing applies selective mitigation of crossterms, as it identifies and eliminates the "undesired" crossterms over any t-f regions, even if they are overlapped by the source autoterms, which is the case shown in the simulation section. Another advantage of the proposed method over the RIDs lies in the fact that the array averaging technique does not produce any smearing effect. That is, the averaging process, apart from scalar multiplication, does not alter the shapes of the signal autoterms in the t-f domain.

The averaged WVD $\bar{W}(t,f)$ is not a valid WVD as there rarely exists a signal waveform that has the same WVD as $\bar{W}(t,f)$. In the sense of signal synthesis or blind source separation, the criterion of minimum square error (MSE) is often applied to obtain the signal $y_{\rm opt}(t)$ with WVD that best approximates the modified WVD, i.e.,

$$y_{\rm opt}(t) = \arg \min_y \sum_t \sum_f \left| W_y(t,f) - \bar{W}(t,f) \right|^2. \quad (24)$$

If desired, one may combine kernel smoothing and spatial smoothing. This is achieved by replacing the WVD in (13)–(17) with another member of Cohens class of TFDs. It is evident from above equations, however, that the extent to which the crossterms are mitigated via spatial averaging is kernel blind. It depends exclusively on the expression of $\mathbf{\Gamma}$, which is determined by the spatial information of the source signals and is independent of $\mathbf{W_{ss}}$, which is the specific t-f expression used. The integration of both spatial and t-f averaging can result in significant crossterm suppression that cannot be achieved by each type of averaging applied alone.

The above discussion is based on the noise-free assumption. Because the noise is spatially and temporally white, the averaging process in the presence of noise could also provide a reduction of the noise variance in the WVD in $\bar{W}(t,f)$ by a factor of M over the WVD of the single sensor. It is the randomness of the noise described by the noise variance, not the autoterm noise Wigner-Ville spectrum that remains unchanged in the array averaging, which contributes to the distortion of a synthesized signal. Therefore, the reduction of the noise variance amounts to increasing the synthesis robustness with respect to noise, which becomes important in the environment where the desired signals are submerged within the noise. Therefore, the benefits of the proposed method is two-fold: reduction of the signal crossterms and the additive noise level.

III. Signal Synthesis

A. Analogy of the Array WVD Signal Synthesis to "Weighted Model"

The proposed array averaging technique structurely resembles the "weighted model" introduced in [9] and is given by

$$\hat{W}(t,f) = \sum_{i=1}^{L} \sum_{j=1}^{L} \alpha_{ij} W_{s_i s_j}(t,f) \quad (25)$$

where α_{ij} is the weighting factor for the signal sources' auto- or cross-terms $W_{s_k s_l}(t,f)$. $\hat{W}(t,f)$ denotes the weighted t-f distribution and is similar in structure to the array averaged WVD $\bar{W}(t,f)$ defined in (12). The key difference between (25) and (12) is that in the underlying problem, we do not intentionally select the weight factors β_{ij} in (12). They evolve naturally from the inner products of the associated SSs embedded in the matrix of $\mathbf{\Gamma}$ and are generated without any "human intervention" through the process of array averaging.

B. Signal Synthesis From Averaged WVD

WVD-based signal synthesis using the MSE criterion (24) could be found in [8]–[11]. In the following, we derive the signal synthesis result for (12). Denote

$$\hat{p}\left(t, \frac{l}{2}\right) = \sum_f \bar{W}(t,f) e^{j2\pi f l} \quad (26)$$

where $\bar{W}(t,f)$ is defined in (12). Combining (8), we obtain

$$\hat{p}\left(t, \frac{l}{2}\right) = \sum_{i=1}^{L} \sum_{j=1}^{L} \beta_{ij} s_i \left(t + \frac{l}{2}\right) s_j^* \left(t - \frac{l}{2}\right). \quad (27)$$

Construct a matrix $\mathbf{Q} = [\hat{q}_{mn}]$ with

$$\begin{aligned} \hat{q}_{mn} =& \hat{p}\left(\frac{m+n}{2}, m-n\right) \\ =& \sum_{i=1}^{L} \sum_{j=1}^{L} \beta_{ij} s_i(m) s_j^*(n) \\ =& \mathbf{s}^H(n) \mathbf{\Gamma} \mathbf{s}(m) \end{aligned} \quad (28)$$

where $\mathbf{\Gamma} = [\beta_{ij}] = (1/M)\mathbf{A}^H\mathbf{A}$ and $\mathbf{s}(t) = [s_1(t), \ldots, s_L(t)]^T$. We also denote $\mathbf{S} = [\mathbf{s}(1), \ldots, \mathbf{s}(N)]$. The solution to the synthesis problem is equivalent to performing an eigendecomposition on $\hat{q}(t_1, t_2)$ and determining the eigenvector corresponding to the largest eigenvalue [9], [10].

Since $\mathbf{\Gamma}$ is complex conjugate symmetric

$$\mathbf{\Gamma} = \mathbf{C} \Lambda \mathbf{C}^H = \sum_{k=1}^{L} \lambda_k \mathbf{c}_k \mathbf{c}_k^H. \quad (29)$$

where λ_k and $\mathbf{c}_k$ are the real-valued eigenvalues and the orthonormal eigenvectors of $\mathbf{\Gamma}$. Inserting (29) into (28) and defining $\mathbf{U} = [\mathbf{u}_1, \ldots, \mathbf{u}_L]^T = \mathbf{C}^H\mathbf{S}$, we obtain

$$\hat{\mathbf{Q}} = \mathbf{U}^H \Lambda \mathbf{U} = \sum_{k=1}^{L} \lambda_k \mathbf{u}_k \mathbf{u}_k^H. \quad (30)$$

Let λ_1 be the largest eigenvalue, and let $\mathbf{u}_1$ be the respective eigenvector. The solution to the synthesis problem could be easily expressed as [9], [10]

$$\hat{s} = e^{j\phi} \sqrt{\lambda_1} \mathbf{u}_1 \quad (31)$$

where ϕ is an unknown value representing the phase ambiguity.

From the definition of $\mathbf{u}_1$, it is evident that the synthesized signal is a linear combination of the original signal components $s_k(t)$, $k = 1, \ldots, L$. However, this is not the desired result since, in most cases, we need to synthesize each individual signal component from the multicomponent WVD without the interference of the other signal components. This can be achieved by placing a mask on the desired signal autoterm and perform the synthesis process. The purpose of applying a mask is to reduce the crossterms and to mitigate other signals' autoterms. This is necessary because it has been shown that the existence of crossterms and the autoterms of other signal may completely fail the synthesis strategy [9]. With the use of averaged WVD across the antenna array, the autoterms are substantially enhanced with respect to both noise and crossterms and, thus, can be easily identified without the *a* *priori* knowledge of the signal waveforms and signatures, even in low SNR situations. It is noted that the construction of a mask does not require the exact knowledge or estimation of the signal waveforms and their WVDs. Rough bounds separating different autoterm areas would suffice when the noise and crossterms are sufficiently suppressed.

In this paper, we utilize the method of *extended discrete-time Wigner distribution* (EDTWD), which was introduced in [11], to the output of array averaged WVD. The advantage of using the EDTWD lies in the fact that it does not require *a priori* knowledge of the source waveform and thereby avoids the problem of matching the two "uncoupled" vectors (even-indexed and odd-indexed vectors).

The overall synthesis procedure is summarized in the following steps.

1) Given the received data of the ith sensor $x_i(t)$, compute the EDTWD

$$W_{x_ix_i}(t,f) = \sum_{k:t+(k/2)\in Z} x_i\left(t+\frac{k}{2}\right) x_i^*\left(t-\frac{k}{2}\right) e^{-j2\pi kf}$$
$$t = 0, \pm 0.5, \pm 1, \ldots. \quad (32)$$

2) Apply the averaging process, that is, summing the EDTWD across the array

$$\bar{W}(t,f) = \frac{1}{M}\sum_{k=1}^{M} W_{x_kx_k}(t,f). \quad (33)$$

3) Place an appropriate t-f mask on $\bar{W}(t,f)$ such that only the desired signal autoterms are retained.
4) Take the inverse fast Fourier transform (IFFT) of the masked WVD $\bar{W}(t,f)$

$$p(t,\tau) = \sum_f \bar{W}(t,f)e^{j2\pi\tau f}. \quad (34)$$

5) Construct the matrix $\mathbf{Q} = [q_{mn}]$ with

$$q_{mn} = p\left(\frac{m+n}{2}, m-n\right). \quad (35)$$

6) Apply eigendecomposition to the matrix $[\mathbf{Q}+\mathbf{Q}^H]$, and obtain the maximum eigenvalue $\lambda_{\max}$ and the associated eigenvector $\mathbf{u}$. The desired signal is given by

$$\hat{s}_{\text{opt}} = e^{j\phi}\sqrt{2\lambda_{\max}}\ \mathbf{u} \quad (36)$$

where again, ϕ is an unknown value representing the phase ambiguity.
7) Repeat step 3 through 6 until all source signals $\hat{s}_1(t), \hat{s}_2(t), \ldots, \hat{s}_L(t)$ are retrieved.

C. Array Matrix Estimation

Upon synthesizing all the source signals, the mixing, or array, matrix $\mathbf{A}$ can be estimated through the least-square (LS) criterion, which minimizes

$$\begin{aligned}\varepsilon &= \sum_{t=1}^{N}\left\|\mathbf{x}(t) - \mathbf{A}\hat{\mathbf{s}}(t)\right\|^2 \\ &= \sum_{t=1}^{N}\left(\mathbf{x}(t) - \sum_{i=1}^{L}\mathbf{a}_i\hat{s}_i(t)\right)^H\left(\mathbf{x}(t) - \sum_{j=1}^{L}\mathbf{a}_j\hat{s}_j(t)\right).\end{aligned} \quad (37)$$

That is

$$\hat{\mathbf{A}} = \arg\min_{\mathbf{A}} \varepsilon. \quad (38)$$

The LS estimate of the array matrix is obtained as [18]

$$\hat{\mathbf{A}} = \hat{\mathbf{r}}\hat{\mathbf{R}}^{-1} \quad (39)$$

where

$$\hat{\mathbf{R}} = \sum_{t=1}^{N}\hat{\mathbf{s}}(t)\hat{\mathbf{s}}^H(t) \quad (40)$$

represents the estimated signal source covariance matrix, and $\hat{\mathbf{r}} = [\hat{\mathbf{r}}_1, \ldots, \hat{\mathbf{r}}_L]$, with

$$\hat{\mathbf{r}}_i = \sum_{t=1}^{N}\mathbf{x}(t)\hat{s}_i^*(t) \quad (41)$$

is the correlation vector between the data vector received across the array and the ith source signal $\hat{s}_i(t)$. In (37)–(41), the notation "^" signifies the fact that we deal with estimated variables.

It is evident from the implementation of the proposed algorithm that the spatial information needs to be sacrified in the first phase to reduce crossterms and noise. The mixing matrix $\mathbf{A}$ could be estimated only after the complete retrieval of the signal source waveforms. This is fundamentally different from other algorithms that combine array signal processing with conventional bilinear distributions, e.g., the STFD, in which the mixing matrix estimation precedes the estimation of the source signals and is provided using relationship

$$\mathbf{D}_{\mathbf{xx}}(t,f) = \mathbf{A}\mathbf{D}_{\mathbf{ss}}(t,f)\mathbf{A} \quad (42)$$

where $\mathbf{D}_{\mathbf{ss}}(t,f)$ is the signal TFD matrix whose entries are the auto- and cross-TFDs of the sources, and $\mathbf{D}_{\mathbf{xx}}(t,f)$ is the data STFD matrix. In the STFD-based source separation, the estimate of $\mathbf{A}$ is provided using whitening, followed by joint diagonalization of $\mathbf{D}_{\mathbf{xx}}(t,f)$ for (t, $f \in$ autoterm regions). This estimate is then included to obtain the source signals using the pseudo-inverse of $\mathbf{A}$.

A hybrid technique based on both array averaging and STFD can be adopted. The array averaging of WVDs is first performed to offer a good estimate of t-f signatures of source signals through cross-term suppression properties. Once the auto-source WVDs are determined, we could then construct the STFD matrices and recover the synthesized signal waveforms, as well as the mixing matrix based on (39).

D. Signal Synthesis With Overlapping T-F Signatures

The procedures we have discussed is appropriate to synthesize the signal waveform whose t-f signatures are distinct. In this case, the masked t-f region always contains the autoterm of the desired source signal with the influence from other sources often negligible. However, if the source t-f signatures overlap, the mask is deemed to capture undesired autoterms. This problem cannot be mitigated by spatial averaging of TFDs and a modification of the proposed method is in order.

Assume that upon implementing the synthesis process described in Section III-B, we obtain the estimate of the mixing matrix $\hat{\mathbf{A}}$. Since there are interfering signal autoterms from other sources, $\hat{\mathbf{A}}$ should be considered different from $\mathbf{A}$. We use $\hat{\mathbf{A}}$ to construct a beamformer applied to the data received across the array (assume the noise-free scenario). That is

$$\mathbf{z}(t) = \frac{1}{M}\hat{\mathbf{A}}^H\mathbf{x}(t) = \frac{1}{M}\hat{\mathbf{A}}^H\mathbf{A}\mathbf{s}(t) \tag{43}$$

where $\mathbf{z}(t) = [z_1(t), \ldots, z_L(t)]$ is a $L \times 1$ vector. Clearly

$$z_k(t) = \left(\frac{1}{M}\hat{\mathbf{a}}_k^H\mathbf{a}_k\right)s_k(t) + \sum_{l\neq k}^{L}\left(\frac{1}{M}\hat{\mathbf{a}}_k^H\mathbf{a}_l\right)s_l(t). \tag{44}$$

It is expected that $\hat{\mathbf{a}}_k$ is a perturbed version of $\mathbf{a}_k$. With the approximations

$$\left|\frac{1}{M}\hat{\mathbf{a}}_k^H\mathbf{a}_k\right| \approx \beta_{kk} = 1 \tag{45}$$

and

$$\left|\frac{1}{M}\hat{\mathbf{a}}_k^H\mathbf{a}_l\right| \approx \beta_{lk} \ll 1, l \neq k \tag{46}$$

then, the WVD of $z_k(t)$ is given by

$$W_{z_kz_k}(t,f) \approx W_{s_ks_k}(t,f) + \sum_{i=1}^{L}\sum_{(j=1,j\neq i)}^{L}\beta_{ik}\beta_{jk}^*W_{s_is_j}(t,f). \tag{47}$$

for $j \neq i$, $\beta_{ik}\beta_{jk}^* \ll 1$. This shows that in (47), except the kth auto-source term, all other terms, either auto- or cross-source terms, are significantly reduced from $W_{z_kz_k}(t,f)$. In the case of ULA, the suppression of those terms are at least 13 dB for large value of M. The suppression of the autoterms other than source k is $|\beta_{ik}|^2$, which is more than 26 dB down from the kth source. Therefore, the effect of the overlapping autoterms from other sources becomes negligible. If we apply the steps 3–8 of the synthesis procedure of Section III-B using the improved WVD in (47), the synthesized signal will be enhanced.

E. Computational Cost

To compare the computational cost of the proposed method and STFD, we use the number of complex multiplications as the evaluation criterion. For the array averaged WVD, the computational cost is shown to be (Appendix A)

$$N_{aa} \approx (5 + L + 4\log_2 N)\cdot O(N^2) \tag{48}$$

where the operand $O(\cdot)$ denotes the order of "$\cdot$."

The computational cost for a typical STFD process is given as (also see Appendix A)

$$N_{\mathrm{STFD}} \approx \log_2 N \cdot O(N^2). \tag{49}$$

Thus, the array averaging technique requires higher computations than the STFD based method.

IV. Simulation Results

In this section, we provide computer simulations to demonstrate the improvement gained by the proposed technique in the reduction or elimination of crossterms. Specifically, we examine the effect of array averaging on the retrieval and separation of the nonstationary signals impinging on the multisensor array. In all the simulations presented in what follows, we consider several signals incident on an eight-sensor ULA ($M = 8$) with interelement spacing of half-wavelength. The additive noise is zero mean, Gaussian distributed, and spatially and temporally white. The length of the signal sequence is set to $N = 128$.

Moreover, we use the same performance index applied in [14] and [16] to evaluate the performance of the proposed technique

$$I_{pq} = E\left|\left(\hat{\mathbf{A}}^{\#}\mathbf{A}\right)_{pq}\right|^2 \tag{50}$$

where the superscript $^{\#}$ denotes the pseudo-inverse. Equation (50) defines the interference-to-signal ratio (ISR). Thus, I_{pq} measures the ratio of the power of the interference of qth source signal to the power of the pth source signal. For large enough N, we have $I_{pq} \approx 0$ for $p \neq q$. We also apply the global rejection level to evaluate the overall performance of the proposed method

$$I_{perf} = \sum_{q\neq p} I_{pq}. \tag{51}$$

In the first example, three chirp signals $s_1(t)$, $s_2(t)$, and $s_3(t)$ arrive at the array with AOAs of $-20°$, $0°$, and $20°$, with the respective start and end frequencies given by $(0.9\pi, 0.5\pi)$, $(0.66\pi, 0.26\pi)$, and $(0.5\pi, 0.1\pi)$. In the t-f plane, the source signals have parallel signatures, emulating a multipath environment. The crossterm of $s_1(t)$ and $s_3(t)$ also forms a chirp-like crossterm structure whose frequency starts from 0.7π and ends with 0.3π and, therefore, lies closely to the t-f signature of $s_2(t)$. Fig. 1 depicts the WVD of the signals at the reference sensor (sensor #1) for the case of noise-free environment. It is clear that the t-f signature of all signal autoterms and crossterms are parallel in the t-f domain. The crossterms produced from the three source signals are even more dominant than the source autoterms. In the single sensor receiver, it becomes difficult to distinguish the source autoterms from the crossterms without any *a priori* knowledge of the sources. From the above AOAs, we obtain

$$\left|\frac{1}{8}\mathbf{A}^H\mathbf{A}\right| = \begin{bmatrix} 1 & 0.2236 & 0.1048 \\ 0.2236 & 1 & 0.2236 \\ 0.1048 & 0.2236 & 1 \end{bmatrix}. \tag{52}$$

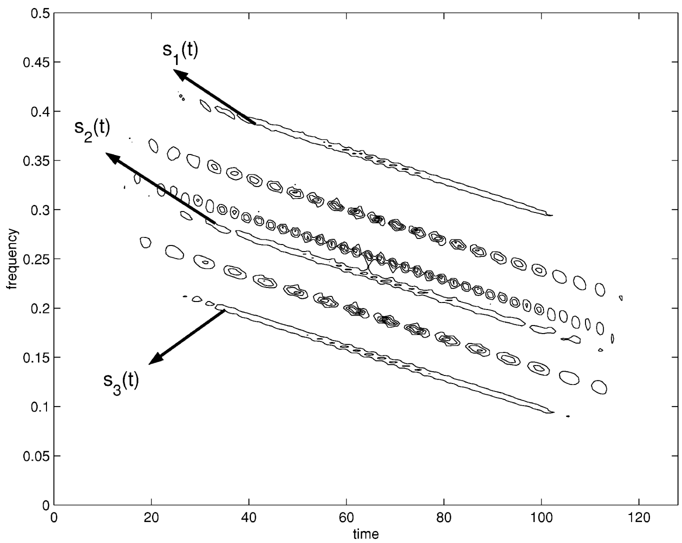

Fig. 1. WVD in noise-free case at reference sensor.

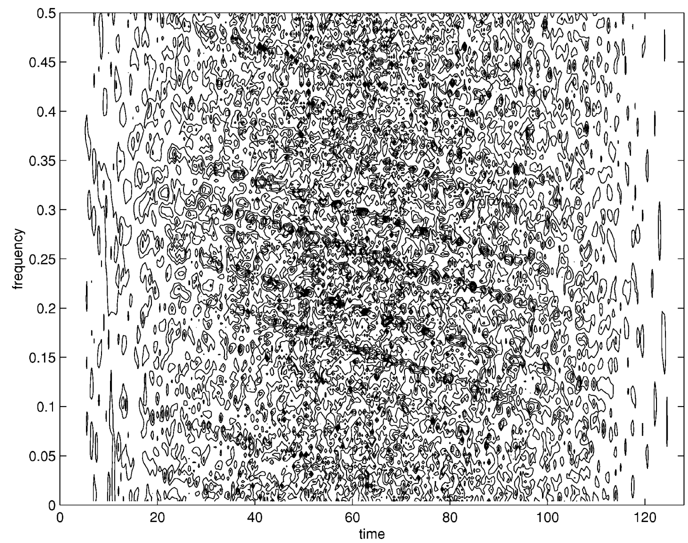

Fig. 3. WVD of the corrupted signals at reference sensor.

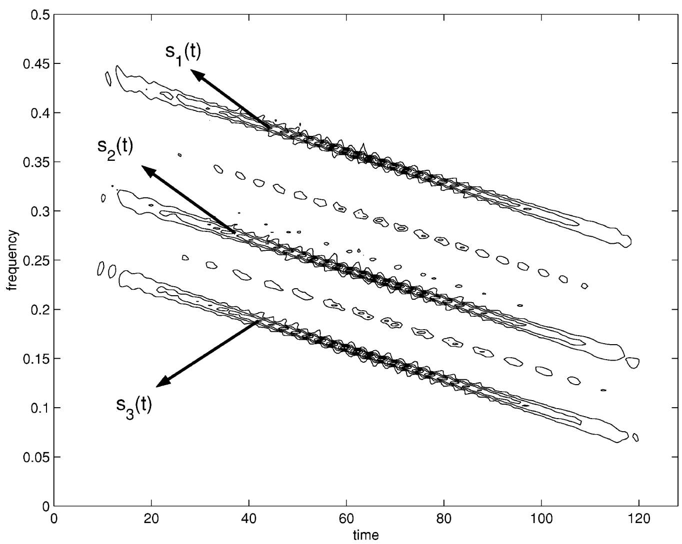

Fig. 2. Array-averaged WVD in noise-free environment.

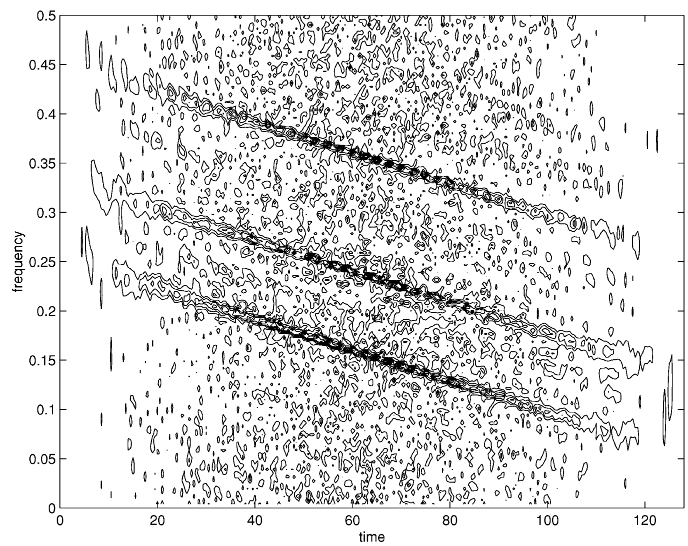

Fig. 4. Array-averaged WVD of the corrupted signals.

The off-diagonal elements are small (< -13 dB), compared with the matrix diagonal entries, indicating that the sources spatial signatures are weakly correlated, and the array averaging process could result in a substantial reduction in the crossterms. Fig. 2 shows the corresponding array-averaged WVD. Due to the reduction in cross-terms by more than 13 dB, the t-f signatures of the sources are distinctively exhibited in the plots. Explicitly, the crossterm from $s_1(t)$ and $s_3(t)$ ceased to become an interfering factor in identifying the adjacent signal source $s_2(t)$. In effect, averaging the WVDs across the array has significantly reduced the crossterms, whereas the three signals' autoterms have remained intact.

Next, we add 5 dB noise to the data at each array sensor so that the input SNR is -5 dB. Figs. 3 and 4 depict both the reference-sensor WVD and the array-averaged WVD. It is evident that the noise obscures both the signal autoterms and crossterms of the WVD at a single sensor. It is difficult, therefore, to retrieve the desired signal if we only synthesize from a single sensor.

Upon averaging, both noise and crossterms are sufficiently reduced to clearly manifest the individual source t-f signature, and the signals could be individually recovered if we place the appropriate masks in the t-f region. Figs. 5 and 6 shows the WVD of the synthesized signal $s_2(t)$ using the array averaging and STFD techniques, respectively. Fig. 7 displays the real parts of the original signal $s_2(t)$, the STFD-recovered $\hat{s}_2(t)$, and $\hat{s}_2(t)$ synthesized by the proposed method. It is clear that the result from the array averaging technique is closer to the original signal than the recovered signal from the STFD-based method. We also plot the global rejection level I_{perf} versus the input SNR in Fig. 8. The input SNR takes values from -10 dB to 20 dB. Both the STFD-based and the array averaging-based techniques are used to compute the empirical I_{perf} defined in (51). Increasing the SNR certainly improves the performance for both methods, and simulations show that the STFD-based method is outperformed by the array averaging technique, which is consistent with the results given in Fig. 7.

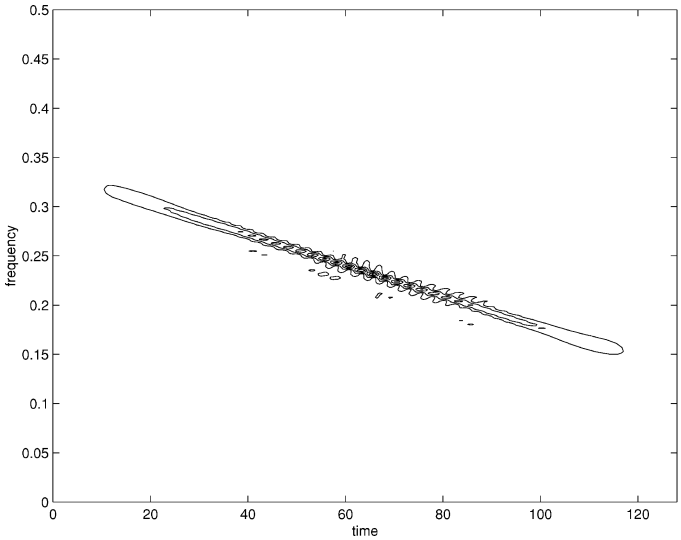

Fig. 5. WVD of synthesized $\hat{s}_2(t)$ using array averaging.

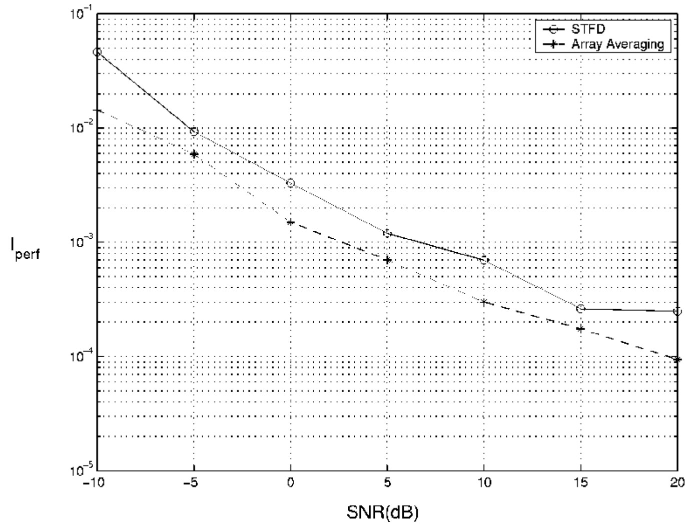

Fig. 8. Global rejection level versus input SNR ($\circ$: by STFD; $*$: by array averaging).

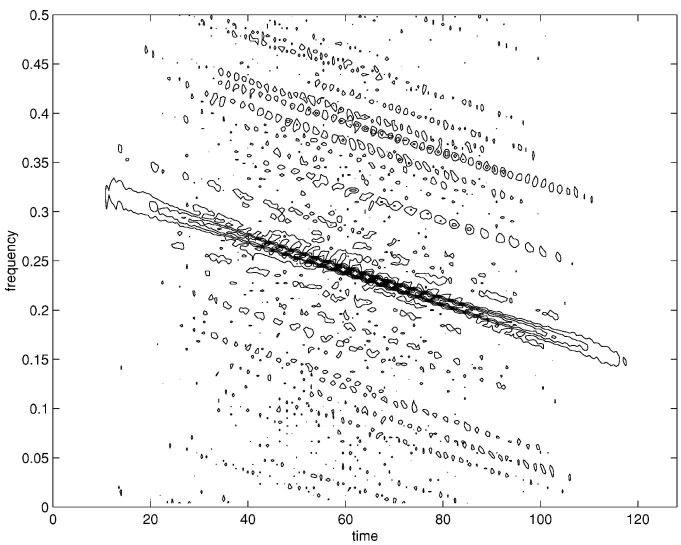

Fig. 6. WVD of synthesized $\hat{s}_2(t)$ using STFD.

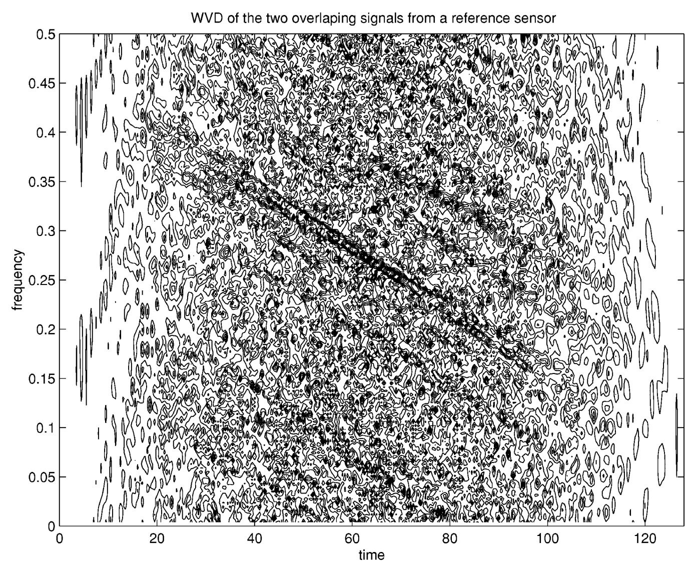

Fig. 9. WVD of the two overlapping signals from a reference sensor.

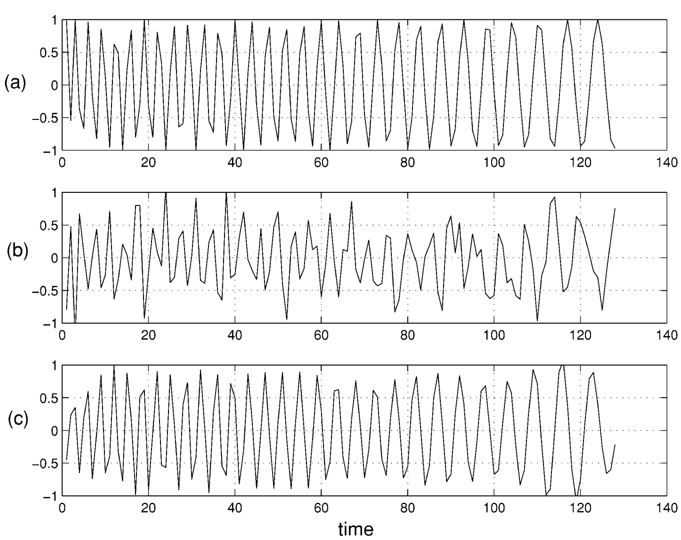

Fig. 7. (a) Real part of original $s_2(t)$. (b) Real part from the STFD-recovered $\hat{s}_2(t)$. (c) Real part from the array averaged $\hat{s}_2(t)$.

In the second example, we use two chirp signals with highly overlapping autoterms. The signals are from AOAs of -20° and 20° with start and end frequencies of $(0.90\pi, 0.96\pi)$ and $(0.16\pi, 0.10\pi)$, respectively. Fig. 9 shows the WVD of the data received at the reference sensor #1, where the input SNR is -5 dB. The two signal autoterms highly overlap. The respective crossterm is located in the middle which also overlaps with the autoterms. The array averaged WVD is plotted in Fig. 10. Using the conclusions derived in Section II, we expect that the cross term would be suppressed by about 19 dB after the array averaging process. This is supported by the plots in Fig. 10. To synthesize the signal, we place the mask along each t-f signature. Any reasonable selection of the mask inevitably includes components from the other source. Therefore, each signal synthesized following the procedures described in Section III-B is, in essence, corrupted by the other signal. By further implementing the beamformer and synthesis procedures from Section III-D, we could obtain an improved signal. The WVDs of the first synthesized signal before and after the beamforming process are shown in Fig. 11. The corresponding global rejection levels are

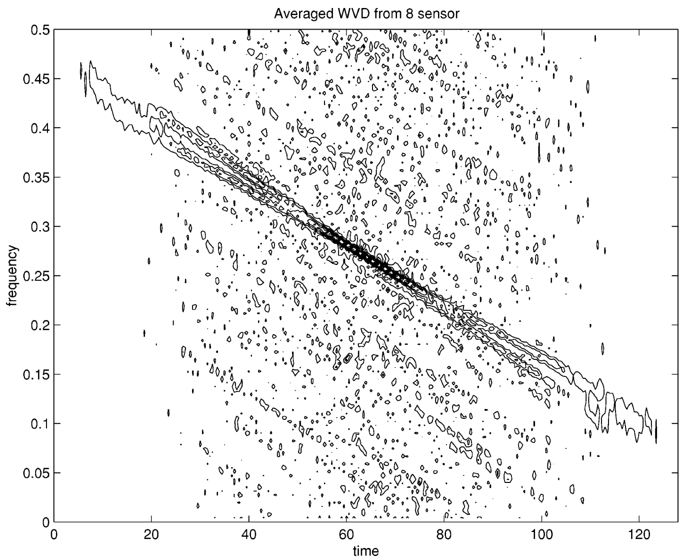

Fig. 10. Array averaged WVD.

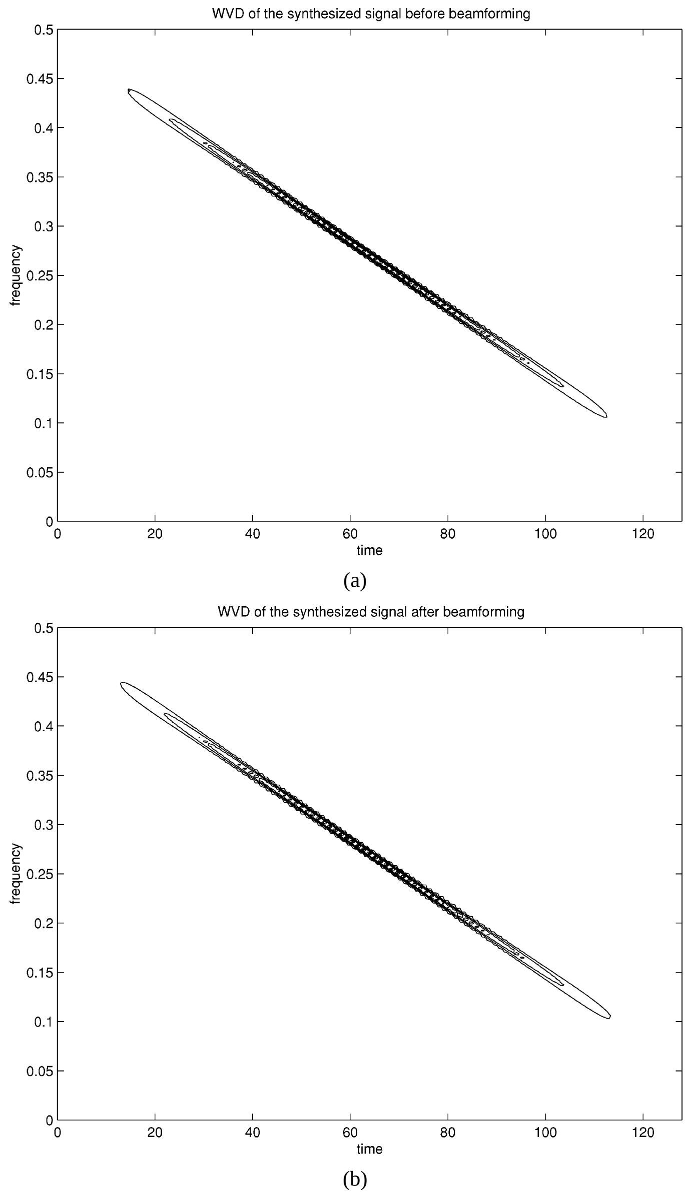

Fig. 11. WVD of the first synthesized signal (a) before beamforming and (b) after beamforming.

calculated and equal to -16.4 dB and -20.4 dB for the respective cases. Therefore, a 4-dB improvement is achieved using the beamforming process. It is evident from this example that the proposed method works well for highly overlapped signals.

V. CONCLUSION

A two-step synthesis technique using bilinear distributions was proposed for multisensor receivers. The first step is to average the Wigner–Ville distributions of the sensor data across the array. This averaging process allows the distinction in the spatial structures of the sources to play a key role in improving their t-f representations. This improvement is manifested in the reduction of the noise floor and mitigation of cross-terms in the t-f domain. The second step is to apply well-known bilinear synthesis methods to the averaged WVD. It was shown that the proposed synthesis approach is fundamentally different from the one recently devised using spatial TFDs. In the latter, the source spatial signatures need to be first estimated before the sources could be separated. The main attraction of the proposed approach is that it naturally extends bilinear signal synthesis to array processing. In doing so, it capitalizes on the spatial dimension to reduce the cross-terms without smearing the auto-terms, which cannot be achieved using the t-f smoothing operation via reduced interference distributions.

APPENDIX
COMPUTATION FOR STFD-BASED AND ARRAY AVERAGING-BASED TECHNIQUES

We first derive the computational requirement for a single EDTWD counting the complex multiplications. It is clear from (39) that for a sequence $y(t)$ $(t = 0, 1, \ldots, N-1)$, the EDTWD would generate a $(2N-1) \times (2N-1)$ two-dimensional TFD $\mathbf{W}_{yy}$. The computation cost is determined by constructing a $(2N-1) \times (2N-1)$ matrix whose elements are given by

$$W(t,k) = x_i\left(t+\frac{k}{2}\right) x_i^*\left(t-\frac{k}{2}\right) e^{-j2\pi kf}$$
$$t = 0, 0.5, 1, \ldots, N-1, k = 0, \pm 1, \ldots, \pm(N-1). \quad \text{(A.1)}$$

The number of complex multiplication involved in (A.1) is

$$n_1 = 1+2+\cdots+(N-1)+N+(N-1)+\cdots+1 = N^2. \quad \text{(A.2)}$$

The EDTWD could be obtained simply by calculating the fast Fourier transform (FFT) with respect to the k parameter in $W_1(t,k)$. Assuming that N_{FFT} is the computation cost for a sequence of length $(2N-1)$, we have $N_{FFT} \approx 2N \times \log_2 2N$. Therefore, the computation cost required in a single EDTWD is

$$n_2 = (2N-1) \times N_{FFT} \approx 4N^2 \log_2 2N. \quad \text{(A.3)}$$

Other costs are N_{FFT} for IFFT, $L \cdot O(N^2)$ (according to [19]) for eigendecomposition to recover source waveforms, and $L(L+1)/2N + LMN + O(L^2)$ for recovery of the mixing matrix. The total cost is computed by summing the above results. Since the number of samples N is often much greater than other parameters, we neglect all items but $O(N^2)$, yielding

$$\begin{aligned} N_{aa} =& n_1 + n_2 + N_{FFT} + L \cdot O(N^2) \\ \approx& (1 + L + 4\log_2 2N) \cdot O(N^2) \\ =& (5 + L + 4\log_2 N) \cdot O(N^2). \end{aligned} \quad \text{(A.4)}$$

The computation requirements of STFD could be calculated in two parts [20]. The first part is related to extractions of the t-f signature. This is achieved by computing the WVD of the data from a single reference sensor and selecting the t-f points that are associated with the different signal sources. Similar to EDTWD, the required cost for this process is $\log_2 N \cdot O(N^2)$. The second part involves the construction of the STFD matrices and recovery of the signal waveforms. It is straightforward to show that the respective computation cost is the following:

- estimation of the auto correlation matrix: $NM^2/2$;
- computation of the whitening matrix: $O(M^3)$;
- whitening of the data: NLM;
- estimation of the STFDs: $\alpha_T L(L-2)/2$;
- joint diagonalization: $O(KL^3)$;
- Separation: LMN.

The variables L, M, and N are the number of sources, sensors, and samples, respectively, whereas K is the number of the chosen t-f points. α_T is the cost of one classical TFD. In a typical scenario, $L, M, K, \alpha_T \ll N$, Therefore, the computation cost for STFD is

$$N_{\text{STFD}} \approx \log_2 N \cdot O(N^2). \quad \text{(A.5)}$$

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