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Joint Optimization of Source Power Allocation and Relay Beamforming in Multiuser Cooperative Wireless Networks

Xin Li · Yimin D. Zhang · Moeness G. Amin

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Abstract Relay beamforming techniques have been shown to significantly enhance the sum capacity of a multiuser cooperative wireless network through the optimization of the relay weights, where concurrent communications of multiple source-destination pairs are achieved via spatial multiplexing. Further optimization of the transmit power allocation over the source nodes is expected to improve the network throughput as well. In this paper, we maximize the sum capacity of a multiuser cooperative wireless network through the joint optimization of power allocation among source nodes and relay beamforming weights across the relay nodes. We consider a two-hop cooperative wireless network, consisting of single-antenna nodes, in which multiple concurrent links are relayed by a number of cooperative nodes. When a large number of relay nodes are available, the channels of different source-destination pairs can be orthogonalized, yielding enhanced sum network capacity. Such cooperative advantage is particularly significant in high signal-to-noise ratio (SNR) regime, in which the capacity follows a logarithm law with the SNR, whereas exploiting spatial multiplexing of multiple links yields capacity increment linear to the number of users. However, the capacity performance is compromised when the input SNR is low and/or when the number of relay nodes is limited. Joint optimization of source power allocation and relay beamforming is important when the input SNR and/or the number of relay nodes are moderate or the wireless channels experience different channel variances. In these cases, joint optimization of source power and distributed beamforming weights achieves significant capacity increment over both source selection and equal source power spatial multiplexing schemes. With consideration of the needs to deliver data from each source node, we further examine the optimization of global sum capacity in the presence of individual capacity requirements by maximizing sum capacity of the network subject to a minimum capacity constraint over each individual user.

Keywords distributed beamforming · cooperative network · optimization · resource allocation · spatial multiplexing · sum capacity

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X. Li · Y. D. Zhang (✉) · M. G. Amin
Center for Advanced Communications, Villanova University, Villanova, PA 19085, USA
e-mail: yimin.zhang@villanova.edu

X. Li
e-mail: xin.l@villanova.edu

M. G. Amin
e-mail: moeness.amin@villanova.edu

1 Introduction

Cooperative communications offer spatial processing capability in wireless networks beyond the limitation of physical number of antenna sensors to yield enhanced spectral and energy efficiency [1–4]. Among various approaches proposed in the literature, distributed beamforming techniques provide spatial diversity and array gains to yield improved link liability and enhanced channel capacity [5–10]. It was shown in [11] and [12] that, when the number of relay nodes is very large, the channels of the different users can be asymptotically orthogonalized (referred to as distributed orthogonalization). As such, concurrent communications between multiple source-destination pairs can be performed with negligible multiuser interference. As a result, in a cooperative wireless network with L pairs of source/destination nodes which are relayed by K relay nodes, the sum network capacity scales as $C = (L/2)\log(K) + O(1)$, that is, the sum capacity increases linearly with the number of source-destination pairs. Such advantage is particularly significant in wireless networks consisting of single-antenna nodes where spatial filtering is otherwise impossible and concurrent transmission within the same channel is infeasible.

Achieving the full advantages of distributed beamforming, needless to say, relies on the proper selection of the relay weights. In [13], the relay weights are optimized to minimize the overall mean square error (MSE) of all users. This scheme, while is convenient to provide controlled data error rates for each user, does not necessarily maximize the data transmission efficiency of the network, particularly when some users experience channel impairment. In [9], the transmit and relay power is minimized to meet the individual signal-to-interference-plus-noise ratio (SINR). Zhang et al. [1] optimizes the relay weights to maximize the sum capacity of all the users. It is seen that, in the presence of a large number of relay nodes, concurrent data transmission through spatial multiplexing yields higher capacity. Such multiplexing advantage is particularly significant in high signal-to-noise ratio (SNR) regime, where the capacity follows a logarithm law with the SNR, whereas exploiting spatial multiplexing of multiple links yields capacity increment linear to the number of users. The optimization in [1] and [9], however, is limited to the relay nodes, whereas the equal power allocation of all source nodes is assumed. As can be observed in [1], the scheme yields capacity degradation compared to the opportunistic source selection counterpart when the input SNR is low.

One objective of this paper is, through joint optimization of power allocation among source nodes and distributed beamforming weights across the relay nodes, to maximize the sum capacity of the wireless network. Optimized source power allocation is important in general, and its importance is particularly significant when the input SNR as well as the number of relay nodes are moderate or low, and when the channels corresponding to different users have unequal quality, for example, due to the presence of deep fading or shadowing in the wireless channels of some users. In such situations, the joint optimization of source power and distributed beamforming weights yields substantial capacity increment over both spatial multiplexing schemes with opportunistic source selection and equal source power allocation. It is pointed out in this paper that, in the low SNR regime, the optimized distributed beamforming degenerates to the opportunistic source selection scheme, where a single source-destination link with the highest capacity is selected.

In this paper, all the nodes considered in the multiuser cooperative wireless networks, including source, relay, and destination nodes, are assumed to use a single antenna, different from multi-antenna relay networks [14, 15]. It is further assumed that the optimization procedure is performed in a central station, which is usually resource-rich and is feasible to obtain the channel state information (CSI) of the entire network. The source power allocation and relay weights are respectively fed to the source and relay nodes. We consider a two-hop amplify-and-forward (AF) protocol. Both relay and source nodes are assumed to have their respective total power constraints. In addition to spatial multiplexing, the capacity performance of time-division multiple access (TDMA) and opportunistic source selection schemes is compared. We also examine the throughput when channels between source and relay nodes experience different variances.

Another important issue considered in this paper is the optimization of global sum capacity in the presence of individual capacity requirements. Depending on applications, the use of sum capacity as the optimization criterion in a multiuser cooperative network may raise fairness concerns in the sense that some users with poor channel quality may be designated only with low or no data rates and, therefore, their needs cannot be met. This may not be acceptable in certain applications. To solve this issue, a constrained optimization, in which each source-destination link is designated to have their respective predefined quality-of-service (QoS) requirement, such as individual capacity or output SINR, was considered in, for example, [9]. To examine the tradeoff between global sum capacity and individual capacity requirements, we also consider in this paper the sum capacity maximization of the network subject to a minimum capacity constraint over each individual user. This is a generalization of the optimization criterion in the sense that it degenerates to the sum rate optimization when the amount of individual capacity requirements is reduced to zero. As a general observation, a strict individual requirement yields a compromised performance from the point of view of the sum capacity. This is particularly evident when the individual constraints are difficult to meet due to high individual capacity requirements under worse channel conditions. In some situations, the individual capacity constraints may not be satisfied, yielding no solution in the joint optimization of the power allocation and distributed beamformer.

1.1 Notations

In this paper, lowercase (uppercase) bold characters are used to denote vectors (matrices). In particular, $\mathbf{I}$ denotes an identity matrix with an appropriate dimension. $(\cdot)^*$ denotes complex conjugate, $(\cdot)^T$ denotes transpose of a matrix or vector, and $(\cdot)^H$ is the conjugate transpose (Hermitian) operator. $\|\cdot\|$ denotes the Euclidian (Frobenius) norm of a vector or a matrix. Moreover, $E[\cdot]$ denotes statistical mean operator. In addition, $\mathrm{Diag}[\mathbf{a}]$ denotes a diagonal matrix with the elements of $\mathbf{a}$ as its diagonal elements, whereas $\mathrm{diag}[\mathbf{A}]$ denotes a vector consisting of the diagonal elements of square matrix $\mathbf{A}$. $\odot$ denotes Hadamard (element-wise) product of matrices or vectors.

This paper is organized as follows. The system model and a two-step communication protocol are briefly described in Section 2. In Section 3, three data transmission schemes including spatial multiplexing, TDMA and opportunistic source selection, are respectively presented and formulated. The optimization of relay weights and source power allocation is considered in Section 4, and the joint optimization with individual capacity constraints is provided in Section 5. In Section 6, simulation results are illustrated and compared. Finally, we conclude this paper in Section 7.

2 System model

Refer to Fig. 1, we consider a wireless network consisting of $K+2L$ single-antenna nodes with L designated pairs consisting of their respective source and destination nodes, denoted as S_l and D_l $(l=1,\cdots,L)$, respectively, and K single-antenna relay nodes, denoted as R_k $(k=1,\cdots,K)$. Similar to [1, 2] and [11], we assume that 1) Source node S_l intends to communicate solely with its corresponding destination node D_l; 2) No cooperation among source nodes, among relay nodes or among destination nodes is allowed; 3) No direct link between source and destination nodes exists; 4) The nodes work in half-duplex mode, i.e., they cannot transmit and receive simultaneously; 5) Communication takes place in two hops over two separate time slots; 6) All channels are independent, frequency-flat Rayleigh block-fading with independent realizations across blocks; and 7) Transmission/reception between the nodes is perfectly synchronized. We use a two-step AF protocol, described as follows.

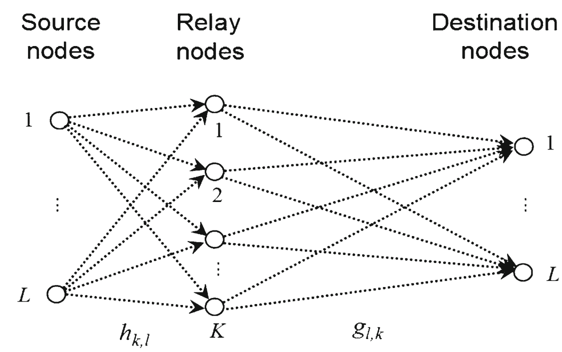

Fig. 1 The diagram of the two-hop relay system

1. During the first step (broadcast phase), the L source nodes transmit individual source information, individually (TDMA or source selection) or simultaneously (spatial multiplexing), to the K relay nodes. The lth source node uses power $P_l^{(S)}, l=1,\cdots,L$. It is assumed that the total power P^s transmitted from the L source nodes is fixed. Thus, $\sum_{l=1}^{L} P_l^{(S)} = P_s$ holds in the spatial multiplexing mode. The channel between the lth source node to the kth relay node is denoted as $h_{k,l}(t)$, where $l=1,\cdots,L$ and $k=1,\cdots,K$. The information symbol $x_l(t)$ is selected randomly from a complex codebook and satisfies $E(|x_l(t)|^2)=1$, $E(x_l^2(t))=0$, and $E(x_l^*(t)x_n(t))=0$ for $l\neq n$.
2. During the second step (relay phase), the K relay nodes amplify their respectively received signals and relay them to the L destination receivers. The noisy signal received at the kth relay node is scaled to unit amplitude and then amplified at the level of its relay power P_k and properly adjusted the phase according to the designed relay weight. We assume that the total relay power P_r used by the K relay nodes is subject to constraint $\sum_{k=1}^{K} P_k = P_r$. The channel between the kth relay node and the lth receiver is denoted as $g_{l,k}(t)$.

For notational simplicity, we will drop the time-dependence expression for the channel states because they are considered quasi-static and the optimization is performed over each fading block where the channel states remain unchanged.

3 Data transmission schemes

In this section, we describe the data transmission schemes in a general manner that the L source nodes simultaneously transmit individual signals. The TDMA and opportunistic source selection schemes are then described as the special cases where only one source is active in each time slot, i.e., all the other nodes are assigned zero transmission power.

3.1 Spatial multiplexing scheme

When the L source nodes simultaneously transmit signals, the lth source nodes uses transmit power of $P_l^{(\mathrm{S})}$ with $\sum_{l=1}^{L} P_l^{(\mathrm{S})} = P_s$. The signal received at the K relay nodes can be written as

$$\mathbf{r}(t) = \mathbf{H}[\mathbf{P}^{(\mathrm{S})}]^{1/2}\mathbf{x}(t) + \mathbf{v}(t), \tag{1}$$

where $\mathbf{H} = [\mathbf{h}_1, \cdots, \mathbf{h}_L] \in \mathbb{C}^{K\times L}$ represents the channel coefficients between the L source nodes and the K relay nodes with $\mathbf{h}_l = [h_{1,l}, \cdots, h_{K,l}]^T \in \mathbb{C}^{K\times 1}$, $\mathbf{P}^{(\mathrm{S})} = \mathrm{Diag}[P_1^{(\mathrm{S})}, \cdots, P_L^{(\mathrm{S})}] \in \mathbb{R}^{L\times L}$ is a diagonal matrix which denotes the source power allocation strategy, $\mathbf{x}(t) = [x_1(t), \cdots, x_L(t)]^T \in \mathbb{C}^{L\times 1}$ is the source signal vector, and $\mathbf{v}(t) \in \mathbb{C}^{K\times 1}$ is the relay noise vector observed at the K relay nodes and is assumed to be an independent and identically distributed (i.i.d.) complex Gaussian random variable vector with zero mean and unit variance. The scaling factor vector, $\beta = [\beta_1, \cdots, \beta_K]^T \in \mathbb{R}^{K\times 1}$, is obtained as

$$\beta = \mathrm{diag}\left[E\left(\mathbf{r}(t)\mathbf{r}^H(t)\right)\right]^{-1/2} = \mathrm{diag}(\mathbf{H}\mathbf{P}^{(\mathrm{S})}\mathbf{H}^H + \mathbf{I})^{-1/2}. \tag{2}$$

Denote $\mathbf{w}^* = [w_1, \cdots, w_K]^H \in \mathbb{C}^{K\times 1}$ as the unit-norm relay weight vector shared by all parallel user links. The actual relay weight vector thus is $\sqrt{P_r}\mathbf{w}^*$. The signal received at the lth destination node is then expressed as

$$\begin{aligned} y_l(t) &= \sqrt{P_r}\mathbf{w}^H\left(\mathbf{g}_l\odot\beta\odot\mathbf{r}(t)\right) + n_l(t) \\ &= \underbrace{\sqrt{P_r P_l^{(\mathrm{S})}}\mathbf{w}^H\left(\mathbf{g}_l\odot\beta\odot\mathbf{h}_l\right)x_l(t)}_{s_l(t)} \\ &\quad + \underbrace{\sqrt{P_r}\mathbf{w}^H\left[\mathbf{g}_l\odot\beta\odot\left(\mathbf{H}_{\bar{l}}[\mathbf{P}_{\bar{l}}^{(\mathrm{S})}]^{1/2}\mathbf{x}_{\bar{l}}(t)\right)\right]}_{j_l(t)} \\ &\quad + \underbrace{\sqrt{P_r}\mathbf{w}^H\left(\mathbf{g}_l\odot\beta\odot\mathbf{v}(t)\right) + n_l(t)}_{u_l(t)}, \end{aligned} \tag{3}$$

where $\mathbf{g}_l = [g_{l,1}, \cdots, g_{l,K}]^T \in \mathbb{C}^{K\times 1}$ is the channel vector between the K relay nodes and the lth destination node, $n_l(t)$ is the noise at the lth destination receiver, which is assumed to be i.i.d. complex Gaussian random variable of zero mean and unit variance. In addition,

$$\mathbf{H}_{\bar{l}} = [\mathbf{h}_1, \cdots, \mathbf{h}_{l-1}, \mathbf{h}_{l+1}, \cdots, \mathbf{h}_L] \in \mathbb{C}^{K\times(L-1)},$$

$$\begin{aligned}\mathbf{P}_{\bar{l}}^{(\mathrm{S})} &= \mathrm{Diag}[P_1^{(\mathrm{S})}, \cdots, P_{l-1}^{(\mathrm{S})}, P_{l+1}^{(\mathrm{S})}, \cdots, P_L^{(\mathrm{S})}] \\ &\in \mathbb{C}^{(L-1)\times(L-1)},\end{aligned}$$

$$\begin{aligned}\mathbf{x}_{\bar{l}}(t) &= [x_1(t), \cdots, x_{l-1}(t), x_{l+1}(t), \cdots, x_L(t)]^T \\ &\in \mathbb{C}^{(L-1)\times 1}.\end{aligned}$$

In (3), $s_l(t)$ is the desired signal, $j_l(t)$ denotes the multi-user interference, and $u_l(t)$ is the overall noise (consisting of relay noise and destination receiver noise). As such, the output SINR of the lth user at D_l can be written as

$$\begin{aligned}\rho_l &= \frac{E_{\{x_l(t)\}}|s_l(t)|^2}{E_{\{\mathbf{x}_{\bar{l}}(t)\}}|j_l(t)|^2 + E_{\{\mathbf{v}(t),n_l(t)\}}|u_l(t)|^2} \\ &= \frac{\mathbf{w}^H\mathbf{A}_l\mathbf{w}}{\mathbf{w}^H\mathbf{B}_l\mathbf{w} + 1},\end{aligned} \tag{4}$$

where

$$\begin{aligned}\mathbf{A}_l &= P_l^{(\mathrm{S})}\mathbf{A}_l' = P_l^{(\mathrm{S})}P_r\,(\mathbf{g}_l\mathbf{g}_l^H)\odot(\beta\beta^T)\odot(\mathbf{h}_l\mathbf{h}_l^H), \\ \mathbf{B}_l &= P_r\,(\beta\beta^T)\odot(\mathbf{g}_l\mathbf{g}_l^H)\odot(\mathbf{H}_{\bar{l}}\mathbf{P}_{\bar{l}}^{(\mathrm{S})}\mathbf{H}_{\bar{l}}^H + \mathbf{I}).\end{aligned} \tag{5}$$

The capacity for the lth link in the underlying wireless network, conditioned to the channel states $\mathbf{H}$ and $\mathbf{g}_l$, is then obtained as

$$C_l = \frac{1}{2}\log_2(1 + \rho_l), \tag{6}$$

where the factor 1/2 is used to emphasize the half-duplex signaling. Consequently, the instantaneous sum capacity of the L links, conditioned to the channel states, is expressed as

$$\begin{aligned}C = \sum_{l=1}^{L} C_l &= \frac{1}{2}\sum_{l=1}^{L}\log_2\left[1 + \frac{\mathbf{w}^H\mathbf{A}_l\mathbf{w}}{\mathbf{w}^H\mathbf{B}_l\mathbf{w} + 1}\right] \\ &= \frac{1}{2}\sum_{l=1}^{L}\log_2\left[1 + \frac{\mathbf{w}^H\mathbf{A}_l\mathbf{w}}{\mathbf{w}^H\mathbf{D}_l\mathbf{w}}\right],\end{aligned} \tag{7}$$

where $\mathbf{D}_l = \mathbf{B}_l + \mathbf{I}$. The mean value of the network capacity, referred to as the average sum capacity, is obtained as

$$\overline{C} = E_{\{\mathbf{h}_l,\mathbf{g}_l,l=1,\cdots,L\}}C. \tag{8}$$

3.2 TDMA scheme

In this scheme, a block of time slots are divided into L pairs of slots. In each time slot, the resource is devoted to a single source-destination link. Thus, the problem in each time slot is the same as a single-user relay network (see [6–8] and references therein) or the multiple access system [10]. Mathematically, it is equivalent to sequentially set only one non-zero diagonal element in matrix $\mathbf{P}^{(S)}$ at a time.

Consider the time slot $t = 2l-1$ in which the lth source node transmits a signal stream with power $P_l^{(S)} = P_s$. The signal received at the K relay nodes is expressed as

$$\mathbf{r}^{\mathrm{t}}(t) = \sqrt{P_s}\mathbf{h}_l x_l(t) + \mathbf{v}(t). \tag{9}$$

where $(\cdot)^{\mathrm{t}}$ is used to signify the TDMA scheme. The corresponding scaling factor vector β_l becomes

$$\beta_l^{\mathrm{t}} = \mathrm{diag}\left[P_s\mathbf{h}_l\mathbf{h}_l^H + \mathbf{I}\right]^{-\frac{1}{2}}. \tag{10}$$

Then, the signal received at the lth destination node now only contains $s_l(t)$ and $u_l(t)$, because there is no multiuser interference in the TDMA scheme. The output SNR, ρ_l, is expressed as

$$\rho_l^{\mathrm{t}} = \frac{E_{\{x_l(t)\}}|s_l(t)|^2}{E_{\{\mathbf{v}(t),n_l(t)\}}|u_l(t)|^2} = \frac{(\mathbf{w}_l^{\mathrm{t}})^H\mathbf{A}_l^{\mathrm{t}}\mathbf{w}_l^{\mathrm{t}}}{(\mathbf{w}_l^{\mathrm{t}})^H\mathbf{B}_l^{\mathrm{t}}\mathbf{w}_l^{\mathrm{t}} + 1}, \tag{11}$$

where

$$\begin{aligned}\mathbf{A}_l^{\mathrm{t}} &= P_s P_r(\mathbf{g}_l\mathbf{g}_l^H)\odot(\beta_l\beta_l^T)\odot(\mathbf{h}_l\mathbf{h}_l^H),\\ \mathbf{B}_l^{\mathrm{t}} &= P_r(\mathbf{g}_l\mathbf{g}_l^H)\odot(\beta_l\beta_l^H)\odot\mathbf{I}.\end{aligned} \tag{12}$$

Note that, $\mathbf{B}_l^{\mathrm{t}}$ is a diagonal matrix. The average capacity of the L users, conditioned with the channel states, is then expressed as

$$C^{\mathrm{t}} = \frac{1}{L}\sum_{l=1}^{L} C_l^{\mathrm{t}} = \frac{1}{2L}\sum_{l=1}^{L}\log_2\left(1+\rho_l^{\mathrm{t}}\right). \tag{13}$$

The mean network capacity is obtained in a similar manner to (8).

3.3 Opportunistic source selection

Among the L users, the one with the highest SNR $\rho_l^{\mathrm{t}}(l = 1, \ldots, L)$ achieves maximum instantaneous capacity under the same transmit power and bandwidth conditions. Thus, by selecting such a best source user at each time slot, opportunistic information transmission can be exploited to achieve user diversity [16, 17]. Thus, in the opportunistic source selection scheme, it remains true that the source power allocation matrix $\mathbf{P}^{(S)}$ given in (1) has only a single non-zero diagonal element, but the order is opportunistically selected, rather than sequentially determined as in the TDMA scheme. At a pair of time slots, the network capacity of the opportunistic source selection scheme is obtained as

$$C^{\mathrm{o}} = \max_{l=1,\cdots,L} C_l^{\mathrm{t}} = \max_{l=1,\cdots,L}\frac{1}{2}\log_2\left(1+\rho_l^{\mathrm{t}}\right), \tag{14}$$

where $(\cdot)^{\mathrm{o}}$ denotes opportunistic source selection. The mean network capacity can also be formulated similar to (8).

3.4 Remarks

We consider two special cases of the spatial multiplexing scheme, namely, that with a low SNR and that with a large number of relay nodes, K. When the input SNR is low, we can show that

$$\begin{aligned}C &= \sum_{l=1}^{L} C_l = \frac{1}{2}\log_2\prod_{l=1}^{L}(1+\rho_l)\\ &\approx \frac{1}{2}\log_2\left(1+\sum_{l=1}^{L}\rho_l\right) \le \frac{1}{2}\log_2\left(1+\sum_{l=1}^{L}\frac{P_l^{(S)}}{P_s}\rho_l^{\mathrm{t}}\right)\\ &\le \max_{l=1,\cdots,L}\frac{1}{2}\log_2\left(1+\rho_l^{\mathrm{t}}\right) = C^{\mathrm{o}}.\end{aligned} \tag{15}$$

That is, the opportunistic source selection scheme is optimum in the sense that the network capacity is maximized.

On the other hand, when K is sufficiently large, the equivalent channels corresponding to different users can be considered orthogonal [11]. In this case, $\mathbf{B}_l$ in (5) reduces to $\mathbf{B}_l^{\mathrm{t}}$ in (12), and $\mathbf{D}_l$ to $\mathbf{D}_l^{\mathrm{t}} = \mathbf{B}_l^{\mathrm{t}} + \mathbf{I}$ as well. Thus, the conditional capacity C becomes

$$C = \frac{1}{2}\sum_{l=1}^{L}\log_2\left[1+\frac{P_l^{(S)}}{P_s}\frac{\mathbf{w}^H\mathbf{A}_l^{\mathrm{t}}\mathbf{w}}{\mathbf{w}^H\mathbf{D}_l^{\mathrm{t}}\mathbf{w}}\right]. \tag{16}$$

4 Optimization of relay weights and power allocation

The instantaneous sum capacity C, given in (7), can be reformulated as

$$C = \frac{1}{2}\log_2\prod_{l=1}^{L}\frac{\mathbf{w}^H\mathbf{C}_l\mathbf{w}}{\mathbf{w}^H\mathbf{D}_l\mathbf{w}}, \tag{17}$$

where $\mathbf{C}_l = \mathbf{A}_l + \mathbf{D}_l$. Note that matrices $\mathbf{A}_l$, $\mathbf{B}_l$, $\mathbf{C}_l$, and $\mathbf{D}_l$, $l = 1, \cdots, L$, are all Hermitian. To Maximize the sum capacity above, optimization of source power allocation and relay weights are required. Such a multiplicative optimization, in a general case, becomes intractable due to non-convexity. In the following, we discuss the optimum problem from two aspects: optimization of the relay weights under fixed source power allocation and joint optimization of relay weights and source power allocation.

4.1 Optimization of relay weights

Under the assumption of a fixed source power allocation $P_l^{(S)} \geq 0$ and $\sum_{l=1}^{L} P_l^{(S)} = P_s$, the instantaneous sum capacity, C, described in (17), can be represented as

$$\max_{\mathbf{w}} \prod_{l=1}^{L} \frac{\mathbf{w}^H \mathbf{C}_l \mathbf{w}}{\mathbf{w}^H \mathbf{D}_l \mathbf{w}}, \quad \text{s.t. } \|\mathbf{w}\| = 1. \tag{18}$$

(a) *TDMA scheme*

In the TDMA scheme, there is only one active source node transmitting at the full source power P_s in each time slot and no interference exists. Therefore, the multiplicative optimization problem (19) reduces to a simple case where, at each time slot, only one term is involved. That is, at the lth time slot,

$$\max_{\mathbf{w}} \frac{\mathbf{w}^H \mathbf{C}_l^{\mathrm{t}} \mathbf{w}}{\mathbf{w}^H \mathbf{D}_l^{\mathrm{t}} \mathbf{w}} \quad \text{s.t. } \|\mathbf{w}\| = 1, \tag{19}$$

and the optimal relay weight vector can be obtained in a closed-form as [18]

$$\mathbf{w}_l^{\mathrm{t}} = \mathcal{P}\left((\mathbf{D}_l^{\mathrm{t}})^{-1} \mathbf{A}_l^{\mathrm{t}}\right) = \frac{(\mathbf{D}_l^{\mathrm{t}})^{-1} \tilde{\mathbf{h}}_l}{\|(\mathbf{D}_l^{\mathrm{t}})^{-1} \tilde{\mathbf{h}}_l\|}, \tag{20}$$

where $\mathcal{P}(\cdot)$ denotes the primary eigenvector operation and $\tilde{\mathbf{h}}_l = \sqrt{P_s P_r} \mathbf{g}_l \odot \beta_l \odot \mathbf{h}_l \in \mathbb{C}^{K \times 1}$ is the equivalent channel vector of the lth source-destination pair. Substituting the above result into (11), we obtain the maximum output SINR as

$$\rho_l^{\mathrm{t}} = \tilde{\mathbf{h}}_l^H (\mathbf{D}_l^{\mathrm{t}})^{-1} \tilde{\mathbf{h}}_l, \; l = 1, \ldots, L. \tag{21}$$

The sum capacity in the L pairs of time slots is then given by

$$C^{\mathrm{t}} = \frac{1}{L} \sum_{l=1}^{L} C_l^{\mathrm{t}} = \frac{1}{2L} \sum_{l=1}^{L} \log_2 \left(1 + \tilde{\mathbf{h}}_l^H (\mathbf{D}_l^{\mathrm{t}})^{-1} \tilde{\mathbf{h}}_l\right). \tag{22}$$

(b) *Opportunistic source selection scheme*

The optimization of relay weights is similar to the TDMA case except that the capacity of the best user is used as the instantaneous sum capacity, instead of that averaged over all the users.

(c) *Spatial multiplexing scheme*

Different from the TDMA scheme, multiple source nodes are simultaneously active and share the same wireless bandwidth. Consequently, for any user, the other user's signals become interference and thus the multiplicative optimization problem (19) cannot be simplified as in the TDMA scheme. As a result, the analytical solution to this optimization problem becomes difficult. A possible numerical solution is to use projection gradient method [21, 22], but the convergence to the global maxima needs careful examination. Alternatively, we developed a suboptimal solution to approximate the optimal solution or can be used as the initial value for further fine nonlinear search [1]. We separately consider the averaging of the matrices corresponding to the desired signals as well as the multiuser interference and noise components, i.e., we introduce

$$\bar{\mathbf{A}} = \frac{1}{L} \sum_{l=1}^{L} \mathbf{A}_l, \; \bar{\mathbf{D}} = \frac{1}{L} \sum_{l=1}^{L} \mathbf{D}_l, \; \bar{\mathbf{C}} = \frac{1}{L} \sum_{l=1}^{L} \mathbf{C}_l = \bar{\mathbf{A}} + \bar{\mathbf{D}}, \tag{23}$$

and the suboptimal relay weights are obtained from

$$\hat{\mathbf{w}} = \mathcal{P}\left(\bar{\mathbf{D}}^{-1} \bar{\mathbf{C}}\right) = \mathcal{P}\left(\bar{\mathbf{D}}^{-1} \bar{\mathbf{A}}\right). \tag{24}$$

Note that, while $\mathbf{A}_l$ is a rank-1 matrix, $\bar{\mathbf{A}}$ becomes full rank due to the averaging operation.

This suboptimal solution yields a sum capacity close to that corresponding to the optimum solution. The multiuser relay network, when K is much larger than L so as to facilitate the distributed orthogonalization, offers enhanced network capacity compared to TDMA or single-user transmission with opportunistic source selection. This solution can also serve as the initial value for standard nonlinear optimization methods, such as the MATLAB "fmincon" function and SOLNP [19, 20], to find out the optimal solution to the relay weights.

Note that, the individual channel coefficients, $\mathbf{h}_l$ and $\mathbf{g}_l$, $l = 1, \cdots, L$, are not required in performing the distributed beamforming optimization in (24). That is, there is no need for the relay nodes to estimate the CSI. Rather, the CSI required for the relay weight optimization is $\tilde{\mathbf{h}}_l$, $l = 1, \cdots, L$, which can be estimated at the destination nodes.

4.2 Joint optimization of relay weights and source power allocation

For the time-varying wireless channels, a fixed source power allocation scheme may not achieve the highest network capacity. Intuitively, source power allocation should adapt to channel variations so as to achieve efficient use of power and thereby to enhance the network capacity.

For the TDMA and opportunistic source selection schemes, the active node uses the full power P_s and thus it does not need to consider the source power allocation.

For the spatial multiplexing scheme, source power $P_l^{(S)}, l = 1, \cdots, L$, can be adaptively allocated to further improve the sum capacity. In this case, the maximization of the instantaneous capacity (17) can be written as

$$\max_{\mathbf{w}, P_l^{(s)}, l=1,\cdots,L} \prod_{l=1}^{L} \frac{\mathbf{w}^H \mathbf{C}_l \mathbf{w}}{\mathbf{w}^H \mathbf{D}_l \mathbf{w}},$$

$$\text{s.t. } \|\mathbf{w}\| = 1,\ P_l^{(S)} \geq 0, \sum_{l=1}^{L} P_l^{(S)} = P_s. \tag{25}$$

Similarly, an analytical solution to this optimization problem is intractable due to non-convexity. Instead, numerical solutions [21, 22] can be used again, although the convergence to the global maxima depends on the initial guess. Fortunately, it is a good choice to use equal source power allocation and the suboptimal solution given in (24) as the initial value of the search procedure.

To numerically solve the optimization problem (25), some nonlinear optimization tools can be used. In the following, we consider the multidimensional constrained nonlinear minimization procedure solver "`fmincon`" provided by the standard optimization toolbox of MATLAB [19]. It is a gradient-based constrained optimizer using sequential quadratic programming, where the gradients are calculated using an adaptive finite-difference method and thus analytical gradient functions of objective and constraint functions are not compulsively required. It is noted that, due to the non-convexity and highly nonlinear nature of the problem, the optimization may not always yield global optima. As mentioned above, the initial condition of the suboptimal solution given in (24) and equal source power allocation provides a good start of the constrained nonlinear search.

We provide two search procedures for the optimization problem. The first is the explicit iterative search procedure, where relay weights and source power allocation are individually optimized in an iterative manner. The search procedure is briefly described in Table 1. The second is the implicit iterative search procedure in which relay weights and source power allocation are jointly optimized at each iteration. The brief search procedure is given in Table 2. By contrast, the second procedure is less dependent on the initial guess of the search but more search iterations could be required.

The convergence curves of the explicit iterative search procedure is demonstrated in Fig. 2, where different numbers of source-destination pairs, relay nodes and SNR levels are considered. In each case, 5

Table 1 Explicit iterative search procedure

1) Define the capacity tolerance δ_C and the maximum number of iterations $N_{\max}$. Initialize $P_l^{(S)} := P_s/L$, $l = 1, \cdots, L$, and $i := 0$.
2) Compute the suboptimal relay weights.
3) Set $i := i + 1$. Optimize relay weights $\mathbf{w}$ to maximize the sum capacity, C, subject to constraint $\|\mathbf{w}\| = 1$.
4) Optimize source power allocation $P_l^{(S)}, l = 1, \cdots, L$, to maximize the sum capacity, C, subject to constraints $P_l^{(S)} \geq 0$ and $\sum_l^L P_l^{(S)} = P_s$. Denote the capacity obtained in the ith iteration as $C^{(i)}$.
5) If $(C^{(i)} - C^{(i-1)}) < \delta_C$ or $i \geq N_{\max}$, stop; Otherwise, go to step 3.

Table 2 Implicit iterative search procedure

1) Define the capacity tolerance δ_C and the maximum number of iterations $N_{\max}$. Initialize $P_l^{(S)} := P_s/L$, $l = 1, \cdots, L$, and $i := 0$.
2) Compute the suboptimal relay weights.
3) Set $i := i + 1$. Jointly optimize relay weights $\mathbf{w}_i$ and source power allocation $P_l^{(S)}, l = 1, \cdots, L$ to maximize the sum capacity, C, subject to constraints $\|\mathbf{w}\| = 1$, $P_l^{(S)} \geq 0$ and $\sum_l^L P_l^{(S)} = P_s$. Denote the capacity obtained in the ith iteration as $C^{(i)}$.
4) If $(C^{(i)} - C^{(i-1)}) < \delta_C$ or $i \geq N_{\max}$, stop; Otherwise, go to step 3.

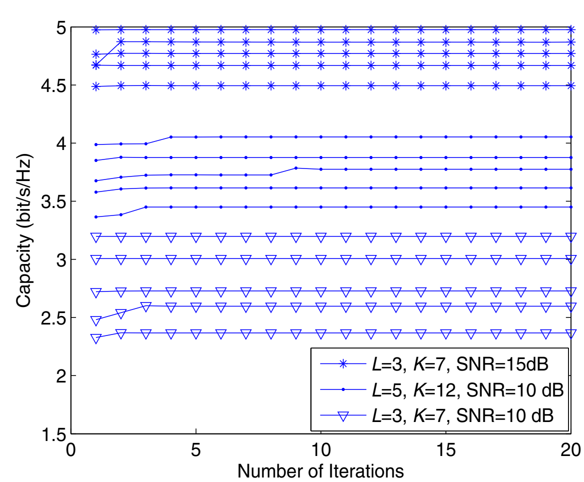

Fig. 2 Convergence behavior of the explicit iterative search procedure

realizations of the random channels are studied. While the joint optimization problem may suffer from the non-convexity, the convergence curves show that, due to the use of initial condition of the suboptimal solution given in (24) and equal source power allocation, a very rapid convergence is generally achieved. As thus, the computational complexity of the proposed iterative search procedure is considered to be affordable.

5 Joint optimization with individual capacity constraints

In the previous section, the optimization problems maximize the sum capacity without consideration of individual capacity requirement. However, the use of sum capacity as the optimization criterion in a multiuser cooperative network may raise fairness concerns in the sense that some users with poor channel quality may be designated only with low data rates that do not meet their needs. On the one hand, when users have similar long-term channel statistics and thereby the instantaneous channel quality is determined primarily by short-term fading, the unfairness issue may not be severe if the network can afford buffered data transmission and is tolerable to small delays in the network. In this case, the maximization of sum capacity over each time slot facilitates user diversity for efficient use of the available resources. On the other hand, when users experience significantly different long-term channel statistics due to, for example, unfavorable source node locations, the unfairness issue may exist a long period of time. In the worst case, some source nodes may have no chance to transmit at all. This issue may not be acceptable in certain applications. To solve this issue, the previous optimization problem should be revised as a constrained optimization, in which each source-destination link is designated to have their respective requirements of predefined QoS or individual capacity [9].

In this section, we consider the sum capacity maximization of the network subject to a minimum capacity constraint over each individual user. This is a generalization of the optimization criterion in the sense that it eventually degenerates to the sum rate optimization as considered in the previous section, when the individual capacity constraint is reduced to zero. The consideration of individual capacity constraints allows us to examine how the fairness affects the network efficiency. As a general observation, a strict individual requirement yields a compromised performance from the point of view of the sum capacity. This is particularly evident when the individual capacity requirement is difficult to meet due to low SNR or worse channel conditions. In some situations, the individual capacity constraints may not be satisfied, yielding no solution to the joint optimization of the power allocation and distributed beamformer.

The sum capacity optimization problem, constrained with individual capacity requirement, is expressed as

$$\max_{\mathbf{w}, P_l^{(s)}, l=1,\cdots,L} \prod_{l=1}^{L} \frac{\mathbf{w}^H \mathbf{C}_l \mathbf{w}}{\mathbf{w}^H \mathbf{D}_l \mathbf{w}}, \tag{26}$$

$$\text{s.t. } \|\mathbf{w}\| = 1,$$

$$C_l \geq \mathrm{C}_{0l},\ P_l^{(S)} \geq 0, l = 1, \cdots, L,$$

$$\sum_{l=1}^{L} P_l^{(S)} = P_s,$$

where C_{0l} is the minimum user capacity requirement of the lth user.

For this optimization problem, a gradient-based constrained optimizer can also be used. The initial condition of the suboptimal solution given in (24) and equal source power allocation provides a reasonable initialization for the nonlinear optimization procedure to yield the global optima. For demonstration, we consider the case $\mathrm{C}_{0l} = \mathrm{C}_0$, i.e., the minimum capacity of each user is not lower than C_0. An implicit iterative search procedure is designed to jointly optimize relay weights and source power allocation under such individual capacity constraints. The brief search procedure is described in Table 3. It is noted that, in the presence of individual capacity constraints, optimized solutions that satisfy all these constraints may not exist when the required capacity constraints are higher than the network can provide, given the available SNR and channel conditions.

Table 3 Search procedure with individual capacity constraints

1) Define the capacity tolerance δ_C and the maximum number of iterations $N_{\max}$. Initialize $P_l^{(S)} := P_s/L$, $l = 1, \cdots, L$, and $i := 0$.
2) Compute the suboptimal relay weights.
3) Set $i := i + 1$. Jointly optimize relay weights $\mathbf{w}$ and source power allocation $P_l^{(S)}, l = 1, \cdots, L$ to maximize the sum capacity, C, subject to constraints $\|\mathbf{w}\| = 1$, $C_l \geq \mathrm{C}_0$, $P_l^{(S)} > 0$ and $\sum_l^L P_l^{(S)} = P_s$. Denote the capacity obtained in the ith iteration as $C^{(i)}$.
4) If $(C^{(i)} - C^{(i-1)}) < \delta_C$, succeed and stop; If $(C^{(i)} - C^{(i-1)}) > \delta_C$ and $i > N_{\max}$, fail and stop; Otherwise, go to step 3.

6 Simulation results

In this section, we provide simulation results to demonstrate the performance of data transmission schemes and optimization methods discussed in Sections 3–5. Capacity comparison is performed among the joint optimization of source power allocation and relay beamformer for spatial multiplexing ("MP-Jopt"), optimization of relay beamformer for spatial multiplexing under equal source power allocation ("MP-opt"), opportunistic source selection ("SS") and TDMA ("TD") schemes. For all the schemes, due to the exploitation of relay beamforming, the allocation of relay power P_r across relay nodes depends on the designed beamforming weights. Without loss of generality, we assume $P_r = P_s$ and define the input SNR as P_s because unit noise power is assumed at the relay and destination nodes.

6.1 Instantaneous sum capacity

In the first demonstration, we show how the source power allocation and relay weights affect instantaneous user capacity and sum capacity. For clarity, we consider a case where $L = 2$ source nodes transmit and $K = 3$ relay nodes cooperate in distributed beamforming transmission. In a random channel realization, the channel matrices $\mathbf{H}$ and $\mathbf{G} = [\mathbf{g}_1, \mathbf{g}_2]$ described in (1) and (3), respectively, are given as

$$\mathbf{H} = \begin{bmatrix} 0.7079 - 1.1627i & 0.2619 + 0.1060i \\ 0.2643 - 0.0307i & -0.4235 + 0.3109i \\ 1.0511 - 0.3913i & 0.3613 - 0.6779i \end{bmatrix}, \tag{27}$$

$$\mathbf{G} = \begin{bmatrix} -0.5382 + 0.4078i & -0.6301 - 0.6864i \\ 0.2716 + 0.6887i & 0.7952 + 0.6798i \\ -0.2243 - 0.1854i & 1.4124 - 0.6072i \end{bmatrix}. \tag{28}$$

Following the data transmission schemes described in Section 3, both the user capacity C_1 and C_2 and the sum capacity C in each scheme can be obtained. For input SNR=5 dB, the instantaneous user capacity and sum capacity is compared in Table 4. In the TDMA scheme, two users transmit in turn and the sum capacity $C = 0.7152$ bit/s/Hz is achieved, while a higher capacity $C = 0.8135$ bit/s/Hz is reached in the

Table 4 Comparison of instantaneous capacity (SNR=5 dB)

Schemes	C_1	C_2	C
TD	0.6169	0.8135	0.7152
SS	0	0.8135	0.8135
MP-opt	0.2973	0.3623	0.6596
MP-Jopt	0	0.8135	0.8135

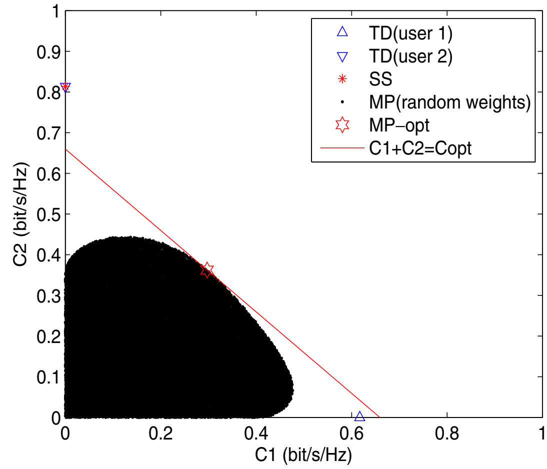

Fig. 3 Instantaneous capacity under various relay weights at a random channel realization (SNR = 5 dB, $L = 2$ and $K = 3$). MP-opt results are obtained under equal source power constraint

opportunistic source selection scheme. In the "MP-opt" scheme, although relay beamforming is performed across the relay nodes, the sum capacity $C_{\text{opt}} = 0.6596$ bit/s/Hz is lower than that of the "SS" scheme, implying that equal source power allocation is not most effective in this case. The optimal relay weight vector in this case is $\mathbf{w}^H = [0.6695-0.1988i, 0.1230-0.4434i, 0.5466+0.0416i]$.

Instead, in the "MP-Jopt" scheme, source power allocation is considered and thus the joint optimization of source power allocation and relay beamformer is performed. As such, the capacity as high as the "SS" scheme is obtained. In this example, due to low input SNR, the joint optimization converges to the "SS"

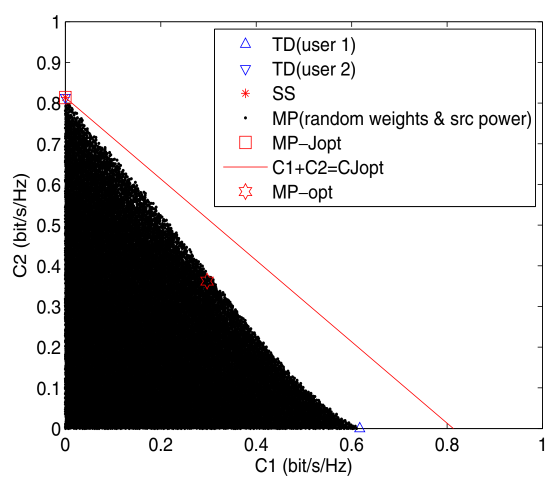

Fig. 4 Instantaneous capacity under various relay weights and source power allocations at a random channel realization (SNR = 5 dB, $L = 2$ and $K = 3$)

Table 5 Comparison of instantaneous capacity (SNR = 15 dB)

Schemes	C_1	C_2	C
TD	2.0331	2.3169	2.1750
SS	0	2.3169	2.3169
MP-opt	1.0027	1.5860	2.5887
MP-Jopt	0.8228	1.8334	2.6562

scheme. Indeed, the optimal source power allocation is obtained as $P_1^{(S)} = 0$ and $P_2^{(S)} = P_s$, and the corresponding relay beamforming weight vector is $\mathbf{w}^H = [0.2834 - 0.0791i, 0.0899 - 0.5560i, 0.7611 + 0.1305i]$.

The results are also illustrated in Figs. 3 and 4. The dots in Fig. 3 depict the instantaneous user capacity results by using random relay beamforming weights, conditioned with equal source power whereas in Fig. 4 no such constraint was applied. The plots reflect the results from 10^6 independent trials. It is seen that the majority of the capacity results are concentrated in low capacity region, demonstrating the importance of optimization of relay beamforming weights.

Next, we use the same channel coefficients as described above, but change the input SNR up to 15 dB and compare the capacity again. The instantaneous user capacity and sum capacity is summarized in Table 5. The average capacity $C = 2.175$ bit/s/Hz and $C = 2.3168$ is obtained respectively in the TDMA and opportunistic source selection schemes. In the "MP-opt" and "MP-Jopt" schemes, $C_{\text{opt}} = 2.5887$ bit/s/Hz and $C_{\text{Jopt}} = 2.6562$ are respectively achieved. In this relatively high SNR case, the "MP-opt" scheme achieves higher capacity than the "SS" scheme. Figures 5 and 6 illustrate the instantaneous user and the sum capacity is depicted using

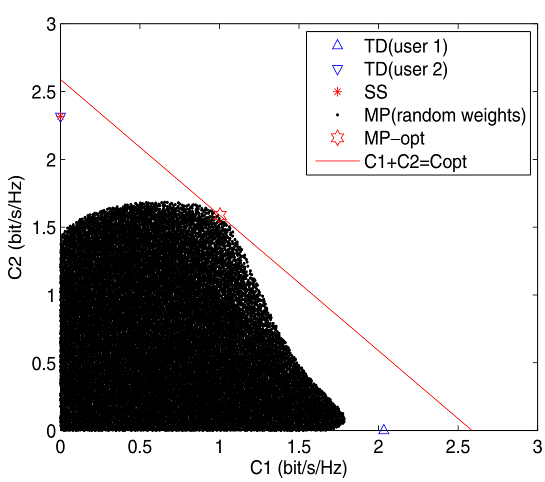

Fig. 5 Instantaneous capacity under various relay weights at a random channel realization (SNR = 15 dB, $L = 2$ and $K = 3$). MP-opt results are obtained under equal source power constraint

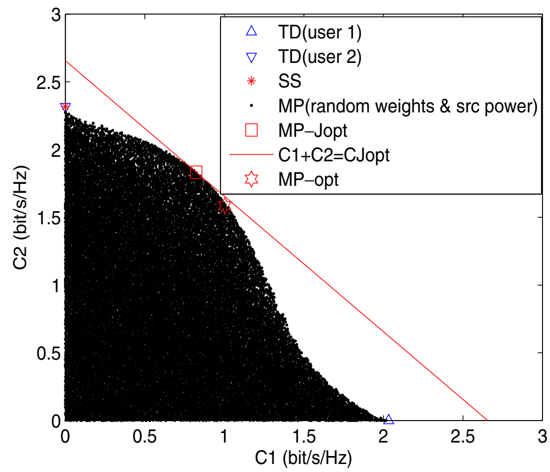

Fig. 6 Instantaneous capacity under various relay weights and source power allocations at a random channel realization (SNR = 15 dB, $L = 2$ and $K = 3$)

10^6 trials with random relay beamforming weights. The optimal relay beamformer, using weight vector $\mathbf{w}^H = [0.7392, -0.0991 - 0.2800i, 0.5519 - 0.2464i]$, outperforms other relay weights. Moreover, joint optimization provides highest capacity C_{Jopt} when the source power is allocated as $P_1^{(S)} = 0.3021P_s$ and $P_2^{(S)} = 0.6979P_s$, and the optimal relay weight vector is $\mathbf{w}^H = [0.6870 + 0.0065i, -0.1689 - 0.3106i, 0.5812 - 0.2553i]$.

6.2 Average sum capacity without individual user capacity constraints

Next, we evaluate the average sum capacity, rather than instantaneous capacity results. For each simulated point, 1,000 realizations of channel states are randomly and independently generated to compute the average sum capacity of the network. The variance of each

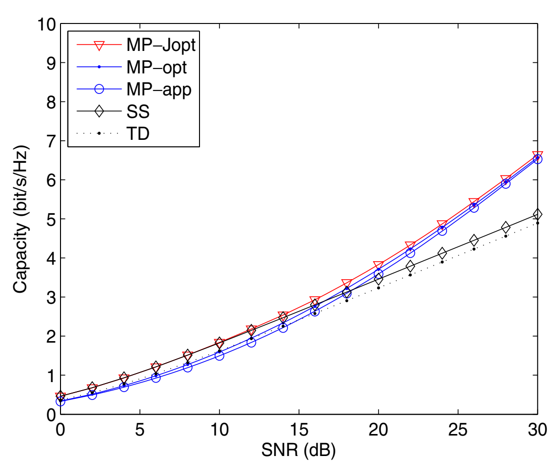

Fig. 7 Average capacity comparison ($L = 2$, $K = 3$)

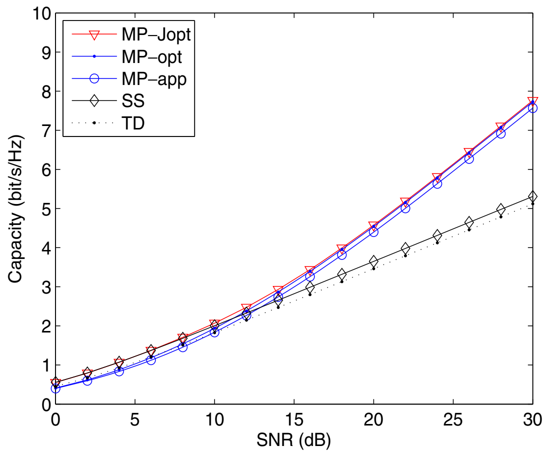

Fig. 8 Average capacity comparison ($L = 2$, $K = 4$)

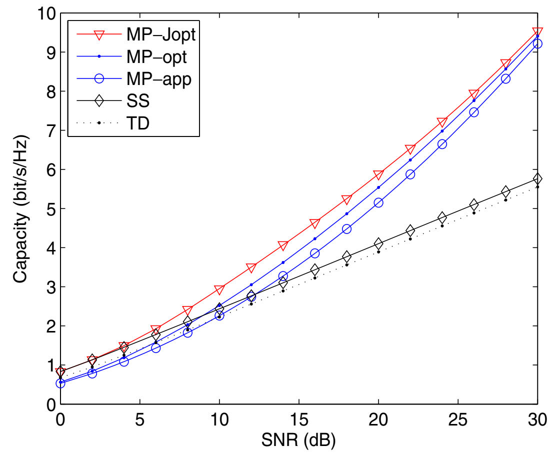

Fig. 10 Average capacity comparison ($L = 3$, $K = 7$)

channel segment is assumed to be unit, unless otherwise specified. Capacity performance of various schemes is compared.

In Figs. 7, 8, 9 and 10, the average sum capacity is plotted versus the input SNR for different schemes and varying values of L and K. It is observed that, regardless of the SNR levels, joint optimization of the source power allocation and relay beamformer achieves the highest capacity among the four schemes. It is also confirmed that, in the low SNR regime, the capacity obtained from the joint optimization scheme coincides that of the opportunistic source selection scheme. In addition, the suboptimal solution, denoted as "MP-app", provides satisfactory approximation of the optimal capacity when SNR is sufficiently high.

Comparing Figs. 7, 8, 9 and 10, one can see that, due to the spatial diversity from the cooperation among relay nodes, all schemes benefit from the increase of the number of relays nodes. However, as the input SNR increases, the two schemes exploiting spatial multiplexing (MP-Jopt and MP-opt) outperform the other two schemes ("SS" and "TD"). The difference becomes more significant when the SNR is high and/or when the number of relay nodes is large.

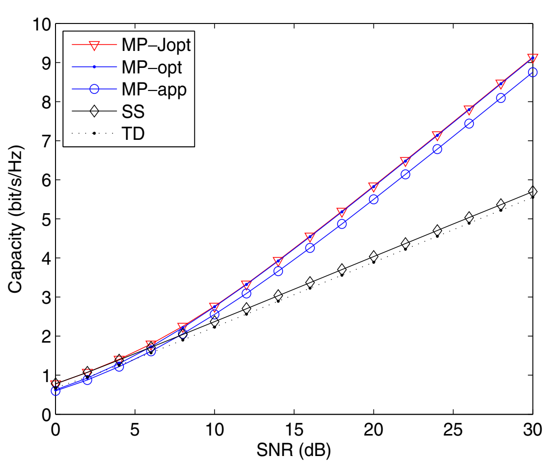

Fig. 9 Average capacity comparison ($L = 2$, $K = 7$)

Figures 9 and 10 compare the average network capacity with the same number of relays nodes but different number of source nodes. While in Fig. 9 the capacity obtained from the joint optimization is very close to that obtained from the relay beamforming scheme, implying that equal source power allocation is close to the optimum solution. In Fig. 10, however, as the number of active sources increases, the equal source power allocation scheme results in noticeable capacity reduction. Joint optimization of the source power allocation and the relay weights, therefore, becomes more important in such situations.

Figure 11 depicts the average sum capacity versus K for different values of L, where the input SNR is fixed at 15 dB. It is observed that, for a given value of L, the average capacity of each scheme is, asymptotically, a log-like increasing function of K. On the other hand, for a given K, the capacity of the TDMA scheme does not change with L, when each user has the same channel statistics. The sum capacity of the opportunistic source selection scheme slightly increases with L due to user diversity. For the "MP-opt" scheme, the capacity increases with L only when K is sufficiently large. However, for the "MP-Jopt" scheme, where joint optimization of the source power and relay weights are performed, the capacity increases with L, regardless of the value of K.

While the joint optimization scheme performs similarly to the relay beamforming scheme for a high value of K, the former outperforms the latter when K is moderate or low. Empirically, at this SNR level, the

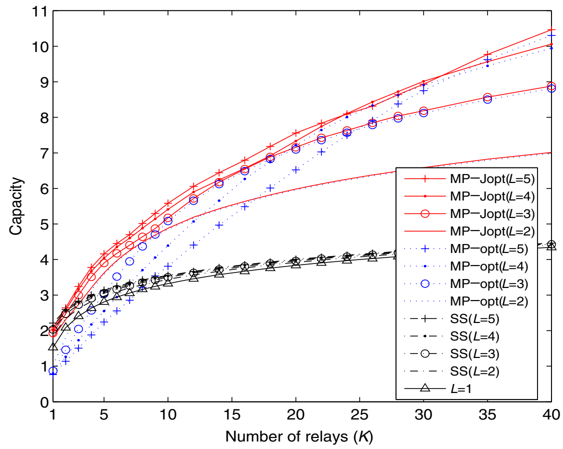

Fig. 11 Effect of the number of relays on the average capacity (SNR = 15 dB)

condition is observed as $K \leq L(L+1)$, in which source power allocation is important to the wireless network exploiting the relay beamforming techniques.

Furthermore, we examine the capacity performance when the channels between source and relay nodes experience different variances. Figure 12 illustrates the capacity for various schemes, where the input SNR remains at 15 dB. The channel variance vector is respectively $[1, 0.5]^T$ and $[1, 0.5, 0.3]^T$ for $L = 2$ and $L = 3$. Compared with the results depicted in Fig. 11, where a uniform channel variance is assumed, it is seen in Fig. 12 that the importance of proper source power allocation becomes more significant for the capacity enhancement when channel variances differ. This observation remains true even when K is sufficiently large. As a result, compared to equal source power allocation, jointly optimized solution of the source power allocation and distributed relay beamformer provides higher network capacity and more effective power utilization. It is worth noting that the capacity of the four schemes behaves very differently in the two examined channel cases. When another worse channel with a small variance 0.3 is added into existing channels with variance vector $[1\ 0.5]^T$, the capacity is improved when the joint optimization scheme is used, especially in the case of large K. The capacity of the "MP-opt" scheme, on the other hand, is enhanced only when K is sufficient high (e.g., $K > 20$ in Fig. 12), whereas the capacity of the TDMA scheme is degenerated and that of the opportunistic source selection scheme does not change.

6.3 Average sum capacity with individual user capacity constraints

In the final demonstration, we examine the average capacity with individual capacity constraints. Due to fading and noise in wireless channels, it is possible for the optimizer to find no solution to the optimization problem with individual capacity constraints. In this case, no capacity contributes to the instantaneous capacity. We consider $L = 3$ source nodes and $K = 7$ cooperative nodes in the network. It is assumed that each individual capacity should be higher than a common minimum requirement $C_0 = 1.0$ bit/s/Hz. Numerical results are illustrated in Fig. 13. It shows that the constrained capacity approaches zeros when the input SNR is lower than about 8 dB, since there is nearly no solution to the problem. However, the constrained capacity increases fast as the input SNR increases, and it eventually approaches that without individual capac-

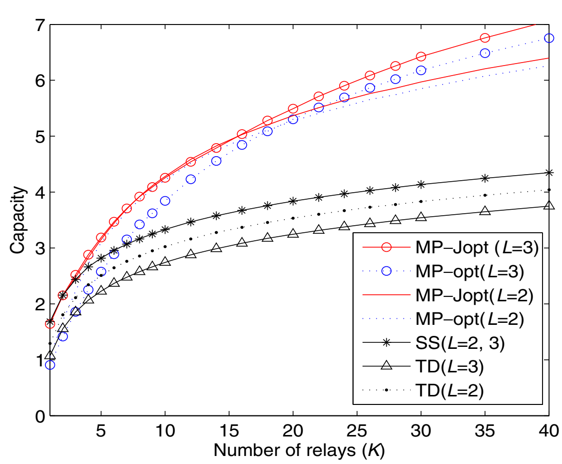

Fig. 12 Effect of channel variances on the average capacity (SNR = 15 dB, the channel variance vector is [1 0.5] for $L = 2$ and [1 0.5 0.3] for $L = 3$)

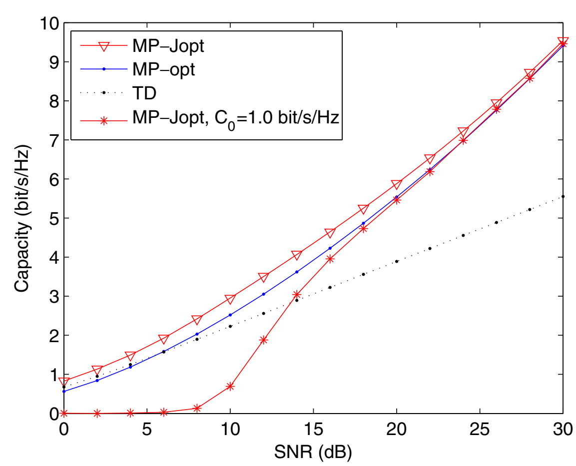

Fig. 13 Average capacity comparison with and without individual capacity constraints($L = 3$, $K = 7$)

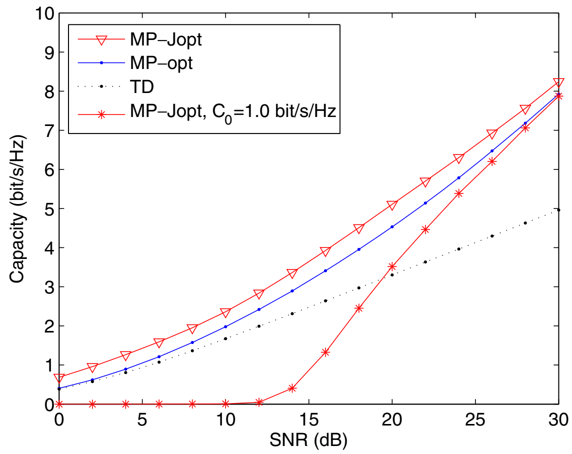

Fig. 14 Average capacity comparison with and without individual capacity constraints with unequal channel quality ($L = 3$, $K = 7$)

ity constraints when the input SNR is sufficiently high. By contrast, the constrained capacity increases slow as the input SNR increases, as demonstrated in Fig. 13, when the channels experience different variances (i.e., unequal channel attenuation). In Fig. 14, the channels between source and relay nodes have different variances of $[1\ 0.5\ 0.3]^T$, and the average capacity is even smaller in low input SNR regime, since in this case, the low-variance channels make the optimizer difficult to find optimal solutions.

7 Conclusions

In this paper, a joint source power allocation and distributed relay beamforming scheme has been developed for the maximization of the sum network capacity of a multiuser cooperative wireless network. The proposed analytical suboptimal solution provides satisfactory performance and can serve as the initial value for iterative search procedures. The proposed scheme outperforms all techniques compared herein, including TDMA, opportunistic source selection, and spatial multiplexing with equal source power allocation. Compared to single-user relay schemes (TDMA and opportunistic source selection), the advantages of spatial multiplexing schemes, with optimized or equal source power allocation, are obvious because they offer the capability of concurrently supporting multiple source streams and thus provide improved sum network capacity, particularly when the input SNR is high and/or when the number of relay nodes is high. In performing spatial multiplexing, the advantage of optimizing the source power allocation over equal source power allocation is significant where the input SNR and the number of relay nodes are not very high. Such an advantage becomes more significant when the user channels have different quality. Additionally, it is shown that, the opportunistic source selection scheme is the optimum solution in a low SNR regime. When each source-destination link has a minimum capacity requirement, the overall network performance, in terms of sum capacity, is reduced. No-solution scenarios arise when the individual user capacity requirement does not match the input SNR and channel conditions.

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