Wideband DOA Estimation for Uniform Linear Arrays Based on the Co-Array Concept

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***Abstract*—A novel design for wideband uniform linear arrays (ULAs) with the associated group-sparsity based direction-of-arrival (DOA) estimation method is proposed. This design allows the number of source signals to significantly exceed the number of sensors. Linear frequency modulated continuous wave (LFMCW) is used as the transmitted signal to ensure the required correlation property among different frequencies. The received echo signals from multiple targets are decomposed into different frequencies by discrete Fourier transform (DFT). Then these frequency bins are divided into several pairs to increase the degrees of freedom (DOFs) based on the co-array concept in the spatio-spectral domain. Group sparsity based signal reconstruction method is employed to jointly estimate the DOA results across multiple frequency pairs. Simulation results demonstrate a significantly improved performance achieved by the proposed method.**

I. Introduction

Direction-of-arrival (DOA) estimation is an important area in array signal processing [1]. Commonly used methods for DOA estimation include capon beamforming [2], MUSIC [3], and ESPRIT [4]. It is well known that, for a uniform linear array (ULA) with M sensors, $M-1$ source signals can be resolved effectively. To fully exploit the available degrees of freedom given a fixed number of sensors, various sparse array constructions have been proposed enabling the resolution of higher number of sources than physical sensors [1]. One representative example is the minimum redundant linear array (MRLA) [5], where the redundancy is minimum among all possible layouts of the array sensors. However, there is no systematic approach for designing an MRLA, especially when the number of array sensors is large.

Recently, two classes of sparse arrays have been proposed, namely nested arrays and co-prime arrays [6]–[8]. A two level passive nested array includes two ULAs to achieve high number DOFs based on the co-array concept. This property is exploited for narrowband DOA estimation [6], [9], [10], and then extended to the wideband case [11]. These methods apply the spatial smoothing approach to undo the coherence of the processed data. A typical co-prime array consists of two sub-arrays with significantly increased DOFs and subspace DOA estimation methods for narrowband co-prime arrays have been presented in [8], [12]–[14], whereas compressive sensing based methods have been proposed in [13], [14]. Generalization of co-prime arrays with compressed inter-element spacing and displaced sub-array design is considered in [15], and the achievable DOFs is provided. In [16], [17], the latter is extended to the wideband case based on the group sparsity concept. For both classes of sparse arrays, at least two ULAs are needed in their configurations.

More recently, the co-prime array concept is extended to the frequency domain in [13], where instead of employing two ULAs, a single ULA is used with two continuous-wave signals of co-prime frequencies. In this case, the ULA operating at two different frequencies acts as two equivalent sub-arrays in the co-prime array structure. The number of achievable degrees of freedom for such a ULA is derived in [18]. In this paper, this idea is further extended to the wideband case, and a method for wideband DOA estimation employing multiple frequency pairs is proposed. The key to the success of this method is to ensure the required correlation characteristic among different frequency components of the received echo signals, which can be achieved using linear frequency modulated continuous wave (LFMCW) signal as the transmitted waveform. A ULA structure can then be designed with adjacent inter-element spacing d according to the frequency band of interest. The received echo signals are first decomposed into different frequencies by a discrete Fourier transform (DFT), which are subsequently divided into several pairs to increase the number of DOFs based on the co-array concept in the spatio-spectral domain. The group sparsity based signal reconstruction method is employed to estimate the DOA results across multiple frequency pairs [19]. Simulation results show that a high number of DOFs can be provided by only a single ULA, with a significantly improved performance achieved.

This paper is organized as follows. The system model design for our proposed method is presented in Section II, and the proposed wideband DOA estimation method for multiple frequency pairs is proposed in Section III. Simulation results are provided in Section IV, and conclusions are drawn in Section V.

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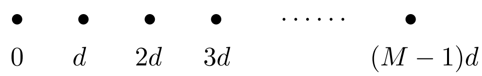


Fig. 1. Structure of a general uniform linear array.

II. System Model Design

A. Signal Model

For our wideband array model, the LFMCW signal is used as the wideband transmitted waveform. In addition to their desirable attributes in wideband high resolution applications, LFMCW signals are preferred in the underlying problem due to the correlation property among different frequencies. This property is required for generating the difference co-array needed in the spatio-spectral domain. The transmitted LFMCW signal with a bandwidth B and an initial frequency f_c can be expressed as

$$s(t) = Ae^{j(2\pi f_c t+\pi\alpha\cdot\mathrm{mod}(t+\tau,T)^2+\varphi)} , \quad (1)$$

where A is the signal amplitude, $\alpha = B/T$ is the chirp rate with T as the modulation period, τ is an initial time-offset, φ represents the initial phase, and $\mathrm{mod}(t+\tau, T)$ is a shorthand notation for ($t+\tau$ modulo T).

Consider an M-sensor ULA with an adjacent sensor spacing d, as shown in Fig. 1. The set of sensor positions can be expressed as S:

$$S = \{md,\ 0 \le m \le M-1, m \in \mathbf{Z}\} , \quad (2)$$

where $\mathbf{Z}$ is the complete integer set.

Assume that there are K targets distributed at incident angles $\theta_k,\ k = 1, 2, \ldots, K$, respectively, where θ_k is measured from the broadside of the array. Then, the echo signals observed at the m-th sensor can be expressed as:

$$x_m(t) = \sum_{k=1}^{K} \gamma_k(t) \cdot s\left[t - \tau_m(\theta_k)\right] + \overline{n}_m(t) , \quad (3)$$

where $\gamma_k(t)$ is the time-varying reflection coefficient owing to target motion or radar cross section (RCS) fluctuations. Since the target reflectivity may be different across the signal bandwidth and the phase delay varies with frequency, the reflection coefficient is in general frequency-dependent. Take the zeroth position of the ULA as the reference. Then $\tau_m(\theta_k)$ represents the time delay of the k-th echo signal with the incident angle θ_k arriving at the m-th sensor of the array. $\overline{n}_m(t)$ is the Gaussian white noise observed at the corresponding sensor.

Assume that the sampling frequency f_s is larger than twice the highest frequency of the signal. Then the discrete version of the received echo signals can be expressed as

$$\mathbf{x}[i] = \left[x_0[i], x_1[i], \ldots, x_{M-1}[i]\right]^T , \quad (4)$$

where $\{\cdot\}^T$ denotes the transpose operation.

Each received sensor signal is divided into non-overlapping groups with length L, and an L-point DFT is applied. The l-th frequency bin samples of the p-th group can be placed into one vector as follows

$$\mathbf{X}[l,p] = \left[X_0[l,p], X_1[l,p], \ldots, X_{M-1}[l,p]\right]^T , \quad (5)$$

where

$$X_m[l,p] = \sum_{i=0}^{L-1} x_m[L\cdot(p-1)+i] \cdot e^{-j\frac{2\pi}{L}il} , \quad (6)$$

with $p = 0, 1, \ldots, P-1$, and $l = 0, 1, \ldots, L-1$.

Define $S_k[l,p]$ and $\overline{N}_m[l,p]$ as the DFT of the p-th group discrete-time echo signals $\gamma_k[i]s[i]$ and discrete-time noises $\overline{n}_m[i]$ at the m-th sensor, respectively. $\mathbf{S}[l,p] = \left[S_1[l,p], \ldots, S_K[l,p]\right]^T$ is the column signal vector at the l-th frequency bin, while $\overline{\mathbf{N}}[l,p] = \left[\overline{N}_0[l,p], \ldots, \overline{N}_{M-1}[l,p]\right]^T$ is the corresponding column noise vector. Then, the output signal model in the DFT domain is given by

$$\mathbf{X}[l,p] = \mathbf{A}(l,\boldsymbol{\theta})\mathbf{S}[l,p] + \overline{\mathbf{N}}[l,p] , \quad (7)$$

where $\mathbf{A}(l,\boldsymbol{\theta}) = [\mathbf{a}(l,\theta_1), \ldots, \mathbf{a}(l,\theta_K)]$ is the steering matrix at frequency f_l corresponding to the l-th frequency bin. The frequency interval between adjacent frequency bins is $f_\Delta = f_s/L$. The steering vector $\mathbf{a}(l,\theta_k)$ at the l-th frequency bin and angle θ_k is expressed as

$$\mathbf{a}(l,\theta_k) = \left[1, e^{-j\frac{2\pi d}{\lambda_l}\sin(\theta_k)}, \ldots, e^{-j\frac{2\pi(M-1)d}{\lambda_l}\sin(\theta_k)}\right]^T , \quad (8)$$

where $\lambda_l = c/f_l$ and c is the propagation velocity of the signal.

We design the inter-element sensor spacing d to be

$$d = \frac{c}{2f_\Delta} \cdot \delta , \quad (9)$$

where δ is a variable used to adjust the spacing of the array. Based on the co-array concept, we will show that δ should be less than 1 to avoid spatial aliasing. The largest aperture corresponding to the best estimation performance is achieved at $\delta = 1$. Accordingly, (8) can be changed into

$$\mathbf{a}(l,\theta_k) = \left[1, e^{-j\pi l\delta\sin(\theta_k)}, \ldots, e^{-j\pi(M-1)l\delta\sin(\theta_k)}\right]^T . \quad (10)$$

III. Wideband DOA Estimation Based on Multiple Pairs of Frequency Bins Employing Group Sparsity

A. Virtual array generation for one frequency pair

Assume that the echo signal bandwidth covers Q frequency bins in the DFT domain. Φ_l represents the set of Q frequency bin indexes, and each frequency bin $l_q \in \Phi_l$, $0 \le q \le Q-1$.

We select N pairs of frequency bins, with the n-th pair consisting of the frequency bins l_{n_1} and l_{n_2}, where $l_{n_1} \in \Phi_l$, $l_{n_2} \in \Phi_l$, and $l_{n_1} \ne l_{n_2}$. Then, the auto-correlation matrix at the two frequencies can be obtained by

$$\begin{aligned}\mathbf{R}_{\mathbf{x}}[l_{n_1}, l_{n_1}] &= \mathrm{E}\left\{\mathbf{X}[l_{n_1},p]\cdot\mathbf{X}^H[l_{n_1},p]\right\}\\ &= \sum_{k=1}^{K}\sigma_k^2[l_{n_1},l_{n_1}]\mathbf{a}(l_{n_1},\theta_k)\mathbf{a}^H(l_{n_1},\theta_k) + \sigma_{n}^2[l_{n_1},l_{n_1}]\mathbf{I}_M ,\end{aligned} \quad (11)$$

$$\begin{aligned}\mathbf{R_x}[l_{n_2},l_{n_2}] &= \mathrm{E}\left\{\mathbf{X}[l_{n_2},p]\cdot\mathbf{X}^H[l_{n_2},p]\right\}\\ &= \sum_{k=1}^{K}\sigma_k^2[l_{n_2},l_{n_2}]\mathbf{a}(l_{n_2},\theta_k)\mathbf{a}^H(l_{n_2},\theta_k)+\sigma_{\bar{n}}^2[l_{n_2},l_{n_2}]\mathbf{I}_M\ ,\end{aligned} \tag{12}$$

where $\{\cdot\}^H$ denotes the Hermitian transpose, $\mathrm{E}\{\cdot\}$ is the statistical expectation operator, and $\mathbf{I}_M$ is the $M\times M$ identity matrix. The parameters $\sigma_k^2[l_{n_1},l_{n_1}]$ and $\sigma_k^2[l_{n_2},l_{n_2}]$ represent the powers of the k-th impinging signal at the corresponding frequency bins, whereas $\sigma_{\bar{n}}^2[l_{n_1},l_{n_1}]$ and $\sigma_{\bar{n}}^2[l_{n_2},l_{n_2}]$ define the corresponding noise powers.

The cross-correlation matrix across the two frequency bins is shown as

$$\begin{aligned}\mathbf{R_x}[l_{n_1},l_{n_2}] &= \mathrm{E}\left\{\mathbf{X}[l_{n_1},p]\cdot\mathbf{X}^H[l_{n_2},p]\right\}\\ &= \sum_{k=1}^{K}\sigma_k^2[l_{n_1},l_{n_2}]\mathbf{a}(l_{n_1},\theta_k)\mathbf{a}^H(l_{n_2},\theta_k)\ ,\end{aligned} \tag{13}$$

$$\begin{aligned}\mathbf{R_x}[l_{n_2},l_{n_1}] &= \mathrm{E}\left\{\mathbf{X}[l_{n_2},p]\cdot\mathbf{X}^H[l_{n_1},p]\right\}\\ &= \sum_{k=1}^{K}\sigma_k^2[l_{n_2},l_{n_1}]\mathbf{a}(l_{n_2},\theta_k)\mathbf{a}^H(l_{n_1},\theta_k)\ .\end{aligned} \tag{14}$$

In the above covariance matrices, $\sigma_k^2[l_{n_1},l_{n_1}]$, $\sigma_k^2[l_{n_2},l_{n_2}]$, $\sigma_{\bar{n}}^2[l_{n_1},l_{n_1}]$, and $\sigma_{\bar{n}}^2[l_{n_2},l_{n_2}]$ are all real and positive, while $\sigma_k^2[l_{n_1},l_{n_2}]$ and $\sigma_k^2[l_{n_2},l_{n_1}]$ are in general complex values due to the phase shift between different frequency bins caused by the LFMCW echo signal and the reflection coefficient.

Since $\mathbf{R_x}[l_{n_1},l_{n_2}]=\mathbf{R_x}^H[l_{n_2},l_{n_1}]$, we only use the former in the estimation process. In practice, $\mathbf{R_x}[l_{n_1},l_{n_1}]$, $\mathbf{R_x}[l_{n_2},l_{n_2}]$, and $\mathbf{R_x}[l_{n_1},l_{n_2}]$ can be replaced by their finite-sample estimates over P signal blocks under the assumption of wide-sense stationary.

By vectorizing the auto-correlation matrices, we obtain

$$\begin{aligned}\mathbf{z}[l_{n_1},l_{n_1}] &= \mathrm{vec}\left\{\mathbf{R_x}[l_{n_1},l_{n_1}]\right\}\\ &= \widetilde{\mathbf{A}}[l_{n_1},l_{n_1}]\widetilde{\mathbf{s}}[l_{n_1},l_{n_1}]+\sigma_{\bar{n}}^2[l_{n_1},l_{n_1}]\widetilde{\mathbf{I}}_M\ ,\end{aligned} \tag{15}$$

$$\begin{aligned}\mathbf{z}[l_{n_2},l_{n_2}] &= \mathrm{vec}\left\{\mathbf{R_x}[l_{n_2},l_{n_2}]\right\}\\ &= \widetilde{\mathbf{A}}[l_{n_2},l_{n_2}]\widetilde{\mathbf{s}}[l_{n_2},l_{n_2}]+\sigma_{\bar{n}}^2[l_{n_2},l_{n_2}]\widetilde{\mathbf{I}}_M\ ,\end{aligned} \tag{16}$$

with equivalent steering matrices of the two virtual arrays

$$\begin{aligned}\widetilde{\mathbf{A}}[l_{n_1},l_{n_1}] &= [\widetilde{\mathbf{a}}(l_{n_1},l_{n_1},\theta_1),\ldots,\widetilde{\mathbf{a}}(l_{n_1},l_{n_1},\theta_K)]\ ,\\ \widetilde{\mathbf{A}}[l_{n_2},l_{n_2}] &= [\widetilde{\mathbf{a}}(l_{n_2},l_{n_2},\theta_1),\ldots,\widetilde{\mathbf{a}}(l_{n_2},l_{n_2},\theta_K)]\ ,\end{aligned} \tag{17}$$

where the equivalent steering vectors $\widetilde{\mathbf{a}}(l_{n_1},l_{n_1},\theta_k)=\mathbf{a}^*(l_{n_1},\theta_k)\otimes\mathbf{a}(l_{n_1},\theta_k)$, and $\widetilde{\mathbf{a}}(l_{n_2},l_{n_2},\theta_k)=\mathbf{a}^*(l_{n_2},\theta_k)\otimes\mathbf{a}(l_{n_2},\theta_k)$, with $\otimes$ denoting the Kronecker product. The equivalent signal vectors $\widetilde{\mathbf{s}}[l_{n_1},l_{n_1}]=\left[\sigma_1^2[l_{n_1},l_{n_1}],\ldots,\sigma_K^2[l_{n_1},l_{n_1}]\right]^T$ and $\widetilde{\mathbf{s}}[l_{n_2},l_{n_2}]=\left[\sigma_1^2[l_{n_2},l_{n_2}],\ldots,\sigma_K^2[l_{n_2},l_{n_2}]\right]^T$. $\widetilde{\mathbf{I}}_M$ is a $(M)^2\times 1$ column vector obtained by vectorizing the identity matrix $\mathbf{I}_M$.

Vectorizing the cross-correlation matrices yields another virtual array, given by

$$\begin{aligned}\mathbf{z}[l_{n_1},l_{n_2}] &= \mathrm{vec}\left\{\mathbf{R_x}[l_{n_1},l_{n_2}]\right\}\\ &= \widetilde{\mathbf{A}}[l_{n_1},l_{n_2}]\widetilde{\mathbf{s}}[l_{n_1},l_{n_2}]\ ,\end{aligned} \tag{18}$$

with

$$\widetilde{\mathbf{A}}[l_{n_1},l_{n_2}]=[\widetilde{\mathbf{a}}(l_{n_1},l_{n_2},\theta_1),\ldots,\widetilde{\mathbf{a}}(l_{n_1},l_{n_2},\theta_K)]\ , \tag{19}$$

where its equivalent steering vectors $\widetilde{\mathbf{a}}(l_{n_1},l_{n_2},\theta_k)=\mathbf{a}^*(l_{n_2},\theta_k)\otimes\mathbf{a}(l_{n_1},\theta_k)$, and the equivalent signal vector $\widetilde{\mathbf{s}}[l_{n_1},l_{n_2}]=\left[\sigma_1^2[l_{n_1},l_{n_2}],\ldots,\sigma_K^2[l_{n_1},l_{n_2}]\right]^T$.

For different combinations of l_{n_1} and l_{n_2}, different co-arrays can be obtained. These co-arrays provided in (15), (16), and (18) can be combined together to characterise a large virtual array with positions distributed at the self-difference co-array sets

$$\{(l_{n_1}m_1-l_{n_1}m_2),0\le m_1,m_2\le M-1\}\ ,$$
$$\{(l_{n_2}m_1-l_{n_2}m_2),0\le m_1,m_2\le M-1\}$$

and the cross-difference set

$$\{\pm(l_{n_1}m_1-l_{n_2}m_2),0\le m_1,m_2\le M-1\}\ .$$

For a special case, l_{n_1} and l_{n_2} can be chosen to be co-prime. Then the signals at the l_{n_1}-th frequency bin and the l_{n_2}-th frequency bin can be considered as signals received by two sub-arrays of a co-prime array with $2M-1-\mathrm{floor}[L/\max(l_{n_1},l_{n_2})]$ equivalent physical sensors [13], where $\mathrm{floor}\{\cdot\}$ returns the largest integer not exceeding the argument and $\max\{\cdot\}$ returns the maximum value of the input vector. In so doing, an increased number of DOFs emerges by the equivalent virtual co-prime array [7], [13], [14].

In fact, no matter how l_{n_1} and l_{n_2} are selected, i.e., co-prime or nested, the number of co-array virtual sensors would be much larger than the number of physical sensors M. These increased degrees of freedom can be exploited for DOA estimation to handle a higher number of echo signals.

B. Group sparsity based DOA estimation for one frequency pair

Although $\widetilde{\mathbf{s}}[l_{n_1},l_{n_1}]$, $\widetilde{\mathbf{s}}[l_{n_2},l_{n_2}]$, and $\widetilde{\mathbf{s}}[l_{n_1},l_{n_2}]$ may be different from each other, these vectors share the same spatial support and we can estimate the DOA of targets based on the group sparsity concept.

For the n-th pair of frequency bins, denote $\mathbf{z}[n]=\left[\mathbf{z}^T[l_{n_1},l_{n_1}],\mathbf{z}^T[l_{n_2},l_{n_2}],\mathbf{z}^T[l_{n_1},l_{n_2}]\right]^T$. With a search grid of K_g potential incident angles $\theta_{g,1},\cdots,\theta_{g,K_g}$, we construct

$$\begin{aligned}\widetilde{\mathbf{A}}_{\mathbf{g}}[l_{n_1},l_{n_1}] &= \left[\widetilde{\mathbf{a}}(l_{n_1},l_{n_1},\theta_{g,1}),\ldots,\widetilde{\mathbf{a}}(l_{n_1},l_{n_1},\theta_{g,K_g})\right]\ ,\\ \widetilde{\mathbf{A}}_{\mathbf{g}}[l_{n_2},l_{n_2}] &= \left[\widetilde{\mathbf{a}}(l_{n_2},l_{n_2},\theta_{g,1}),\ldots,\widetilde{\mathbf{a}}(l_{n_2},l_{n_2},\theta_{g,K_g})\right]\ ,\\ \widetilde{\mathbf{A}}_{\mathbf{g}}[l_{n_1},l_{n_2}] &= \left[\widetilde{\mathbf{a}}(l_{n_1},l_{n_2},\theta_{g,1}),\ldots,\widetilde{\mathbf{a}}(l_{n_1},l_{n_2},\theta_{g,K_g})\right]\ ,\end{aligned}$$

and then a block diagonal matrix can be generated as

$$\widetilde{\mathbf{A}}_{\mathbf{g}}[n]=\mathrm{blkdiag}\left\{\widetilde{\mathbf{A}}_{\mathbf{g}}[l_{n_1},l_{n_1}],\widetilde{\mathbf{A}}_{\mathbf{g}}[l_{n_2},l_{n_2}],\widetilde{\mathbf{A}}_{\mathbf{g}}[l_{n_1},l_{n_2}]\right\}\ . \tag{20}$$

We also construct a $K_g\times 3$ matrix $\widetilde{\mathbf{S}}_{\mathbf{g}}[n]$ with $\widetilde{\mathbf{S}}_{\mathbf{g}}[n]=\left[\widetilde{\mathbf{s}}[l_{n_1},l_{n_1}],\widetilde{\mathbf{s}}[l_{n_2},l_{n_2}],\widetilde{\mathbf{s}}[l_{n_1},l_{n_2}]\right]$. By applying group sparsity concept, we can obtain the following virtual array model

$$\mathbf{z}[n]=\widetilde{\mathbf{A}}_{\mathbf{g}}[n]\widetilde{\mathbf{s}}_{\mathbf{g}}[n]+\widetilde{\mathbf{I}}\mathbf{w}[n]\ , \tag{21}$$

where the $3M^2 \times 2$ matrix $\widetilde{\mathbf{I}} = [\widetilde{\mathbf{I}}_1, \widetilde{\mathbf{I}}_2]$ with the first column vector $\widetilde{\mathbf{I}}_1 = [\widetilde{\mathbf{I}}_M^T, \mathbf{0}_M^T, \mathbf{0}_M^T]^T$ and the second column vector $\widetilde{\mathbf{I}}_2 = [\mathbf{0}_M^T, \widetilde{\mathbf{I}}_M^T, \mathbf{0}_M^T]^T$, $\mathbf{0}_M$ represents a column vector consisting of all zeros with the size of $M^2 \times 1$. In addition, $\widetilde{\mathbf{s}}_{\mathbf{g}}[n] = \text{vec}\left\{\widetilde{\mathbf{S}}_{\mathbf{g}}[n]\right\}$ is a $3K_g \times 1$ column vector by vectorizing $\widetilde{\mathbf{S}}_{\mathbf{g}}[n]$, and $\mathbf{w}[n] = \left[\sigma_{\overline{n}}^2[l_{n_1}, l_{n_1}], \sigma_{\overline{n}}^2[l_{n_2}, l_{n_2}]\right]^T$.

Equation (21) can be rewritten as

$$\mathbf{z}[n] = \widetilde{\mathbf{A}}_{\mathbf{g}}^{\circ}[n]\widetilde{\mathbf{s}}_{\mathbf{g}}^{\circ}[n] \ , \tag{22}$$

where $\widetilde{\mathbf{A}}_{\mathbf{g}}^{\circ}[n] = \left[\widetilde{\mathbf{A}}_{\mathbf{g}}[n], \widetilde{\mathbf{I}}\right]$ and $\widetilde{\mathbf{s}}_{\mathbf{g}}^{\circ}[n] = \left[\widetilde{\mathbf{s}}_{\mathbf{g}}^T[n], \mathbf{w}^T[n]\right]^T$.

We use the row vector $\mathbf{s}_{\mathbf{g},k_g}[n]$, $1 \leq k_g \leq K_g$, to represent k_g-th row of the matrix $\widetilde{\mathbf{S}}_{\mathbf{g}}[n]$. Then, we form a new $K_g \times 1$ column vector $\hat{\mathbf{s}}_{\mathbf{g}}[n]$ based on the l_2 norm of $\mathbf{s}_{\mathbf{g},k_g}[n]$, $1 \leq k_g \leq K_g$, as given below

$$\hat{\mathbf{s}}_{\mathbf{g}}[n] = \left[\left\|\mathbf{s}_{\mathbf{g},1}[n]\right\|_2, \left\|\mathbf{s}_{\mathbf{g},2}[n]\right\|_2, \ldots, \left\|\mathbf{s}_{\mathbf{g},K_g}[n]\right\|_2\right]^T \ , \tag{23}$$

where $\|\cdot\|_2$ denotes the l_2 norm.

Then, our group-sparsity based DOA estimation method is formulated as

$$\begin{aligned} &\min_{\widetilde{\mathbf{s}}_{\mathbf{g}}^{\circ}[n]} \quad \|\hat{\mathbf{s}}_{\mathbf{g}}[n]\|_1 \\ &\text{subject to} \quad \left\|\mathbf{z}[n] - \widetilde{\mathbf{A}}_{\mathbf{g}}^{\circ}[n]\widetilde{\mathbf{s}}_{\mathbf{g}}^{\circ}[n]\right\|_2 \leq \varepsilon \ , \end{aligned} \tag{24}$$

where ε is the allowable error bound and $\|\cdot\|_1$ is the l_1 norm. Here $\mathbf{w}[n]$ can also be considered as a variable due to the unknown noise powers, and the $\hat{\mathbf{s}}_{\mathbf{g}}[n]$ represents the DOA estimation results over the K_g search grids.

C. Wideband DOA estimation based on multiple frequency pairs

To estimate the DOA across the full frequency range of interest, we divide the frequency bins of interest into several pairs, and the group sparsity concept is expanded to all pairs for wideband DOA estimation due to the same spatial support.

Assume that all the frequency bins are divided into N pairs. Three matrices, a block diagonal matrix $\widetilde{\mathbf{B}}_{\mathbf{g}}^{\circ}$, a $K_g \times 3N$ matrix $\mathbf{R}_{\mathbf{g}}$, and a $(3K_g+2)N \times 1$ column vector $\mathbf{r}_{\mathbf{g}}^{\circ}$, are constructed using $\widetilde{\mathbf{A}}_{\mathbf{g}}^{\circ}[n]$, $\widetilde{\mathbf{S}}_{\mathbf{g}}[n]$, and $\widetilde{\mathbf{s}}_{\mathbf{g}}^{\circ}[n]$, expressed as

$$\begin{aligned} \widetilde{\mathbf{B}}_{\mathbf{g}}^{\circ} &= \text{blkdiag}\left\{\widetilde{\mathbf{A}}_{\mathbf{g}}^{\circ}[1], \widetilde{\mathbf{A}}_{\mathbf{g}}^{\circ}[2], \ldots, \widetilde{\mathbf{A}}_{\mathbf{g}}^{\circ}[N]\right\} \ , \\ \mathbf{R}_{\mathbf{g}} &= \left[\widetilde{\mathbf{S}}_{\mathbf{g}}[1], \widetilde{\mathbf{S}}_{\mathbf{g}}[2], \ldots, \widetilde{\mathbf{S}}_{\mathbf{g}}[N]\right] \ , \\ \mathbf{r}_{\mathbf{g}}^{\circ} &= \left[\widetilde{\mathbf{s}}_{\mathbf{g}}^{\circ T}[1], \widetilde{\mathbf{s}}_{\mathbf{g}}^{\circ T}[2], \ldots, \widetilde{\mathbf{s}}_{\mathbf{g}}^{\circ T}[N]\right]^T \ . \end{aligned} \tag{25}$$

Then the wideband DOA estimation can be formulated as

$$\begin{aligned} &\min_{\mathbf{r}_{\mathbf{g}}^{\circ}} \quad \|\hat{\mathbf{r}}_{\mathbf{g}}\|_1 \\ &\text{subject to} \quad \left\|\mathbf{z}_{\mathbf{g}} - \widetilde{\mathbf{B}}_{\mathbf{g}}^{\circ}\mathbf{r}_{\mathbf{g}}^{\circ}\right\|_2 \leq \varepsilon \ , \end{aligned} \tag{26}$$

where, $\mathbf{z}_{\mathbf{g}} = \left[\mathbf{z}^T[1], \mathbf{z}^T[2], \ldots, \mathbf{z}^T[N]\right]^T$ and

$$\hat{\mathbf{r}}_{\mathbf{g}} = \left[\left\|\mathbf{r}_{\mathbf{g},1}\right\|_2, \left\|\mathbf{r}_{\mathbf{g},2}\right\|_2, \ldots, \left\|\mathbf{r}_{\mathbf{g},K_g}\right\|_2\right]^T \ , \tag{27}$$

with the row vector $\mathbf{r}_{\mathbf{g},k_g}$, $1 \leq k_g \leq K_g$ representing the k_g-th row of the matrix $\mathbf{R}_{\mathbf{g}}$.

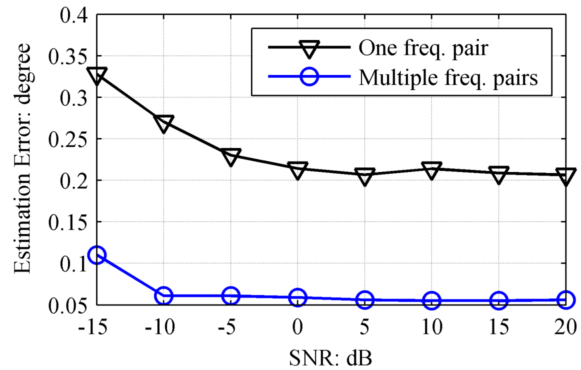


Fig. 2. RMSEs of estimated DOA versus input SNR.

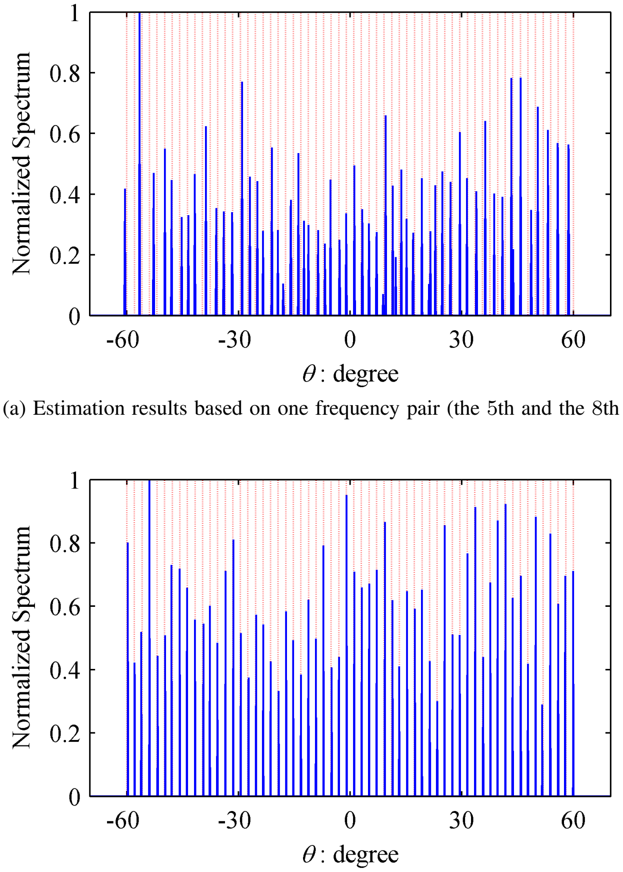
In (24) and (26), the K_g elements of the column vectors $\hat{\mathbf{s}}_{\mathbf{g}}[n]$ and $\hat{\mathbf{r}}_{\mathbf{g}}$ are the corresponding DOA estimation results over K_g search grids. These two optimization problems can be solved using CVX, a software package for specifying and solving convex programs [20], [21].

IV. Simulation Results

Consider an example of ULA with $M = 10$ sensors. To show the results clearly and also simplify the selection of frequency pairs, we choose $\frac{f_s}{2} = \frac{8}{3}B$ and $T = 64T_s$ with $T_s = 1/f_s$. The initial frequency, initial time-offset, and initial phase are set to be 0. Then the normalized frequencies of the echo signals cover the range from 0 to $\frac{3}{8}\pi$. The number of signal samples in the time domain at each sensor is 320000, and DFT of $L = 64$ points is applied. The number of data blocks used for estimating covariance matrices at each frequency bin is $P = 5000$, and there are $Q = 12$ frequency bins in total with the set of indexes $\Phi_l = [1, 2, \ldots, 12]$ in this example. All the $Q = 12$ frequency bins are divided into 6 frequency pairs with 1 and 12, 2 and 11, 3 and 10, 4 and 9, 5 and 8, as well as 6 and 7. d is set with $\delta = 1$. There are 40 far-field targets with incident angles uniformly distributed between $-60°$ and $60°$, and a search grid of $K_g = 3601$ incident angles is formed within the full angle range with a step size of $0.05°$.

In the first set of simulations, we focus on accuracy comparison between the DOA estimation results based only on one frequency pair and results based on multiple frequency pairs. The allowable error bound ε is chosen to give the best result for each method through trial-and-error in every experiment. The root mean square error (RMSE) results are shown in Fig. 2, where each point is based on an average of the results obtained by 500 simulation runs. In this simulation, the case with one frequency pair consists of the 5th and the 8th frequency bins. It is clear that the method based on multiple frequency pairs consistently outperforms the existing method exploiting only one co-prime pair of frequencies by a big margin.

For the second set of simulations, we give an example with the same setting as in the first set except that now 60 targets are uniformly distributed between $-60°$ and $60°$. The



(a) Estimation results based on one frequency pair (the 5th and the 8th bins).

(b) Estimation results based on multiple frequency pairs.

Fig. 3. DOA estimation results obtained by the estimation methods based on one frequency pair and multiple frequency pairs: the dotted lines represent the actual incident angles of the echo signals, while the solid lines represent the estimation results.

SNR is set to be 0 dB, and the results are shown in Fig. 3. Clearly the method based only on one pair of frequencies fails while accurate estimation results are obtained by our proposed wideband method exploiting multiple frequency pairs. This is because the same spatial distribution are shared by different frequencies, and the information provided by all frequencies of interest are fully utilized when jointly estimating the results across multiple frequency pairs.

V. Conclusion

A wideband uniform linear array with the associated group-sparsity based DOA estimation algorithm has been proposed which is capable of estimating more sources than the number of physical sensors. Towards this end, a linear frequency modulated continuous wave is used as the transmitted signal to ensure the required correlation property among different frequencies. The received echo signals from multiple targets are decomposed into different frequencies by DFT. These frequency bins are divided into several pairs to increase the number of DOFs based on the co-array concept in the spatio-spectral domain. Group sparsity based signal reconstruction method is employed to jointly estimate the DOA across multiple frequency pairs. It has been shown that the proposed design and the estimation method is much more accurate than the one based on a single frequency pair. In essence, the proposed method can still give an acceptable result, while the latter fails.

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