

# Subband CMA Adaptive Arrays in Multipath Fading Environment

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## SUMMARY

CMA (Constant Modulus Algorithm) adaptive arrays are considered a promising method in mobile communications to mitigate multipath fading and to suppress co-channel interference signals. However, since a CMA adaptive array assumes no *a priori* information of the desired signal, it is difficult to separate and combine the multipath rays of the desired signal. Therefore, when CMA is applied, the input signal power of the multipath rays cannot be efficiently utilized. Moreover, since suppressing the multipath rays requires many degrees of freedom (DOFs) of the array, the array system becomes complicated.

In this paper we propose the subband CMA adaptive array which enhances the signal correlation between the

multipath rays in each subband, as such the multipath fading effect can be greatly mitigated. By using this method, the multipath rays of the desired signal are equalized and combined, whereas an interference signal with multipath rays can be suppressed by using a single DOF. © 2000 Scripta Technica, Electron Comm Jpn Pt 1, 83(11): 43–54, 2000

**Key words:** CMA adaptive array; multipath fading; co-channel interference; filter banks; blind signal processing.

## 1. Introduction

In land mobile communications, a signal transmitted by a mobile station is reflected and scattered by surroundings before arriving at the base station. Therefore, the signal received at the base station is a multipath-faded version of the transmitted signal [1]. As the demand for multimedia communications increases, mobile communications are de-

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veloping toward high-speed digital wireless networks, for example, with a rate of several megabits per second. Therefore, in high-speed digital communications, the outdoor propagation delay at an urban area will exceed several symbol periods, as such yielding very low signal correlation between the different delayed paths. As a result, heavy intersymbol interference (ISI) problem occurs.

On the other hand, the co-channel interference (CCI) problem also exists due to the frequency band reuse at different cells, which is a common practice in order to effectively use the frequency resource. Therefore, both of the ISI and CCI problems exist simultaneously in high-speed digital mobile communications. Adaptive arrays, particularly under the scheme of space-time adaptive processing (STAP), have been proposed as a powerful means in mitigating both of the CCI and ISI problems [2–12]. Among these adaptive arrays, constant modulus algorithm (CMA) has been of special interest because it is a blind processing technique in the sense that it does not require *a priori* information of the desired signal [2–9]. A CMA adaptive array receives a single signal ray by suppressing not only the CCI signals, but also the delayed paths of the desired signal if the delays are long. However, suppressing the multipath rays of the desired signal wastes part of the available power and requires additional degrees of freedom (DOFs), that is, more array sensors.

In a fast-fading environment, the level of the received signals varies severely, whereas combining the multipath rays instead will stabilize the received signal level as well as improve the communication quality [1]. Furthermore, combining multipath rays instead of suppressing them improves the output performance with fewer number of DOFs [11].

When algorithms like LMS (Least Mean Square) [13] are used which assume that a training sequence is provided, the multipath rays of the desired signal can be separated based on the information of training signal, and the propagation channel can be estimated. Several methods have been proposed for adaptive arrays using LMS algorithms. For example, in Ref. 12 the multipath rays are received separately and then combined [12]; in Refs. 11 and 14 temporal equalization and array processing are simultaneously performed.

Separating direct path and delayed paths by using CMA adaptive arrays has also been investigated [e.g., 5, 8]. In Ref. 5 the beamspace approach was proposed, in which multiple beams are formed. When the direct path and the delayed paths fall in different beams, they can be separated by applying the CMA algorithm against different beams. However, this method is not valid when the direct path and delayed paths are closely spaced so that they cannot be separated by the prespecified beamforming. In Ref. 8, a relatively long known user code is assumed available and is used as the training signal.

This paper proposes the subband CMA adaptive array scheme for high-speed digital mobile communications. The proposed scheme not only suppresses interference signals, but also combines the multipath rays of the desired signal so that a diversity effect is achieved.

When interference signals also have constant modulus property, a CMA adaptive array may mistakenly catch the interference signal and suppress the desired signal [9, 15]. To avoid such a result, various information, such as the unique user codes [15], and the different cyclic correlation property of the users [16], can be used to identify the users and to ensure that the CMA adaptive array catches the desired signal. Such information is usually contained to maintain normal communications and is not necessary to be embodied specially for adaptive control purpose.

On the other hand, the subband processing based on filter banks does not require any *a priori* information, whereas it can increase the signal correlation between direct path and delayed paths in each subband, as such the faded signal is equalized [17, 18]. Note that such enhancement of the signal correlation does not depend on whether the signal is a desired signal or an interference signal. Applying this property to the CMA adaptive array yields the subband CMA adaptive array, which can suppress interference signals with fewer sensors, and at the same time, achieve diversity effect from the multiple paths of the desired signal.

Such a subband array processing, in essence, is a space-frequency signal processing scheme, which is substantially equivalent to the space-time signal processing scheme [17, 20]. However, compared with the space-time signal processing scheme, the proposed space-frequency signal processing scheme is much easier to compose. The merit is especially significant in performing the blind algorithm CMA.

So far, subband adaptive arrays have been proposed to improve the convergence speed [19] and to process wideband signals [7, 20]. The investigation of the space-frequency domain equalization by using subband adaptive arrays was first performed by the authors in Refs. 17 and 18.

This paper is organized as follows. In Section 2 the principles of CMA adaptive array are briefly reviewed, and the performance of CMA adaptive array in multipath environment is analyzed. In Section 3 the signal correlation enhancement by using subband decomposition is discussed, and the subband CMA algorithm is proposed. In Section 4 we give some simulation examples, which show that subband CMA arrays present improved performance in multipath environment compared with the conventional CMA adaptive arrays.

## 2. CMA Adaptive Arrays

In mobile communications, it is often difficult to obtain *a priori* information about the arrival direction of a signal. The usage of training sequence not only complicates the system, but also reduces the communication capacity. Therefore, there is a strong demand to use blind algorithms without any *a priori* information. Although the CMA algorithm was first proposed to suppress delayed paths of a constant modulus signal in mobile communications, it also has the ability to suppress interference signals. The CMA algorithm has been considered as a promising algorithm in mobile communications.

### 2.1. Principle of CMA Adaptive Array

Most signals commonly used in digital mobile communications, such as QPSK modulated signals, have the constant modulus property. When such a signal passes through a fading channel and arrives at a receiver with different time delay, the signal may lose the constant modulus property. On the other hand, the presence of interference signals also contributes to changing the constant modulus property of the received signal. The principle of the CMA adaptive array is to control the weights such that the distortion of the envelope of the array output is minimized in the presence of multipath fading and interference signals [2]. Figure 1 shows the block diagram of an  $N$ -element CMA adaptive array.

It is noted that when a signal is band-limited, it may lose its constant modulus property. Therefore, if a CMA adaptive array performs on the signal envelope at multiple samples for each symbol, the array performance will degrade [4]. However, since the signal does not lose the constant modulus property at the center of each symbol, the

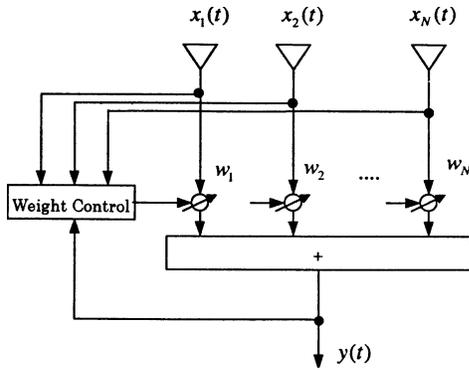


Fig. 1. A CMA adaptive array.

band-limit effect can be ignored if we update weights only once each symbol.

Without loss of generality, we assumed the amplitude of the desired output signal to be 1. The cost function  $Q(\mathbf{w})$  of the CMA adaptive array is expressed by

$$\begin{aligned} Q(\mathbf{w}) &= E \left[ \left| |y(m)|^p - 1 \right|^q \right] \\ &= E \left[ \left| |\mathbf{w}^H(m) \mathbf{x}(m)|^p - 1 \right|^q \right] \end{aligned} \quad (1)$$

where  $E[\cdot]$  is the statistical expectation operator, and  $\mathbf{w}(m)$ ,  $\mathbf{x}(m)$ , and  $y(m)$  are the weight vector, input signal vector, and the array output at time  $m$ , respectively.  $p$  and  $q$  are positive integers which often take a value of 1 or 2. In this paper, we use  $p = q = 2$ . Moreover, the superscript  $H$  denotes conjugate transpose.

The weights are controlled such that the cost function  $Q(\mathbf{w})$  in Eq. (1) is minimized. Since Eq. (1) is nonlinear in regard to the weights and has no closed-form expression of the optimum weights, asymptotic methods of optimization are often used.

When the steepest descent method is used, the weight vector  $\mathbf{w}$  is updated according to

$$\mathbf{w}(m+1) = \mathbf{w}(m) - \mu \nabla_{\mathbf{w}} Q(m) \quad (2)$$

where  $m$  is the number of iteration,  $\mu$  is the step size, and  $\nabla_{\mathbf{w}} Q(m)$  is the gradient vector of  $Q(m)$  with respect to  $\mathbf{w}$  in time  $m$ . In the case of  $p = q = 2$ ,  $\nabla_{\mathbf{w}} Q(m)$  is expressed by

$$\nabla_{\mathbf{w}} Q(m) = 4\mathbf{x}(m)\mathbf{y}^*(m) (|y(m)|^2 - 1) \quad (3)$$

where the superscript  $*$  denotes complex conjugate.

Since the statistical expectation in Eq. (1) cannot be obtained in practice, it is usually estimated from limited numbers of samples. The convergence speed can be improved by averaging  $K$  data samples such that [9]

$$Q(\mathbf{w}) = \frac{1}{K} \sum_{i=m-K+1}^m \left[ \left| |y_m(i)|^2 - 1 \right|^2 \right] \quad (4)$$

where  $K$  is the number of data samples used for the temporal average, and

$$y_m(i) = \mathbf{w}^H(m)\mathbf{x}(i)$$

is the array output obtained from weight vector at time  $m$ ,  $\mathbf{w}(m)$ , and input data vector at time  $i$ ,  $\mathbf{x}(i)$ .

Substituting Eqs. (3) and (4) into Eq. (2) yields

$$\begin{aligned} \mathbf{w}(m+1) &= \mathbf{w}(m) - \frac{4\mu}{K} \end{aligned}$$

$$\times \sum_{i=m-K+1}^m \left\{ \mathbf{x}(i) \mathbf{y}_m^*(i) \left[ |\mathbf{y}_m(i)|^2 - 1 \right] \right\} \quad (5)$$

To achieve fastest convergence, the step size is chosen according to [9]

$$\mu = 1 / \left[ \frac{1}{K} \sum_{i=m-K+1}^m 6\lambda_{\max}^2(i) \right] \quad (6)$$

where  $\lambda_{\max}(i)$  is the maximum eigenvalue of the matrix  $\mathbf{x}(i)\mathbf{x}^H(i)$ .

## 2.2. Behavior of CMA adaptive array in multipath environment

When the signal correlation between multipath rays of the desired signal is low, the CMA adaptive array accepts only one path while the others are suppressed [2, 3]. However, as the signal correlation increases, the multipath rays equivalently function as a single ray [21, 22]. Therefore, the multipath rays of the desired signal will be received by a CMA adaptive array.

Here we analyze the behavior of a CMA adaptive array in the presence of multiple paths. For simplicity, we consider the scenario in the presence of the desired signal and one interference signal, and each of them has a direct path and a delayed path. The desired signal and the interference signal are considered to be uncorrelated. Although we consider two-path scenario, the conclusion can be extended to other multiple-path scenarios.

The received signal vector  $\mathbf{x}(t)$  is denoted as

$$\mathbf{x}(t) = \mathbf{A}(\Theta_D) \mathbf{s}_D(t) + \mathbf{A}(\Theta_I) \mathbf{s}_I(t) + \mathbf{n}(t) \quad (7)$$

where  $\mathbf{A}(\Theta_D) = [\mathbf{a}(\theta_{D1}), \mathbf{a}(\theta_{D2})]$  is the mixing matrix whose columns are the steering vectors of the two paths of the desired signal, and  $\mathbf{s}_D(t) = [s_{D1}(t), s_{D2}(t)]^T$  is a vector with the complex signal waveforms of the two paths as its elements. Similarly,  $\mathbf{A}(\Theta_I) = [\mathbf{a}(\theta_{I1}), \mathbf{a}(\theta_{I2})]$  is the matrix with columns of steering vectors of the two paths of the interference signal, and  $\mathbf{s}_I(t) = [s_{I1}(t), s_{I2}(t)]^T$  is a vector with the complex signal waveforms of the two paths as its elements. Moreover,  $\mathbf{n}(t)$  is the additive noise vector.

We first consider the desired signal. Without loss of generality, we assume  $s_{D1}(t)$  as the desired path. The delayed path,  $s_{D2}(t)$ , can be decomposed into two components, where one is coherent with the direct path, whereas the other is orthogonal to the direct path [23], as

$$s_{D2}(t) = s_D^\perp(t) + \rho_D^* \xi_D s_{D1}(t) \quad (8)$$

and  $E[s_D^\perp(t)s_{D1}(t)] = 0$ . Denote  $\sigma_{D1}^2$  and  $\sigma_{D2}^2$  as the power of the direct path and delayed path, respectively, and  $\xi_D = \sigma_{D2}/\sigma_{D1}$  their power ratio. Also, we define  $\rho_D$  as the signal correlation between the two paths:

$$\rho_D = \frac{E[s_{D1}(t)s_{D2}^*(t)]}{\sigma_{D1}\sigma_{D2}} \quad (9)$$

Then, the covariance matrix of  $s_D(t)$  is obtained as

$$\begin{aligned} \mathbf{R}_{DD} &= E[\mathbf{s}_D(t)\mathbf{s}_D^H(t)] \\ &= \sigma_{D1}^2 \begin{bmatrix} 1 & \rho_D \xi_D \\ \rho_D^* \xi_D & |\rho_D \xi_D|^2 \end{bmatrix} \\ &\quad + \begin{bmatrix} 0 & 0 \\ 0 & (1 - |\rho_D|^2) \sigma_{D2}^2 \end{bmatrix} \end{aligned} \quad (10)$$

The first term of Eq. (10) is the desired component, which is expressed by an equivalent signal waveform  $\tilde{s}_D(t) = (1 + \rho_D^* \xi_D) s_{D1}(t)$  and an equivalent generalized steering vector

$$\tilde{\mathbf{a}}_D = \mathbf{a}(\theta_{D1}) + \rho_D^* \xi_D \mathbf{a}(\theta_2) \quad (11)$$

The second term of Eq. (10) is the interfering component of  $s_{D2}(t)$ .

It is clear from Eq. (10) that, as  $|\rho_D|$  approaches 1, the interfering component decreases, and the two paths are treated more likely as a single path. Therefore, the effect of intersymbol interference can be mitigated by increasing  $|\rho_D|$ . Increasing  $|\rho_D|$  also reduces the effect of the interference component on weight control [25].

For the interference signal, since both the direct path and the delayed path will be suppressed, we rather use eigen-decomposition to analyze their effect. The covariance matrix of the interfering component is

$$\begin{aligned} \mathbf{R}_{II} &= \mathbf{A}(\Theta_I) E[\mathbf{s}_I(t)\mathbf{s}_I^H(t)] \mathbf{A}^H(\Theta_I) \\ &= \sum_{i=1}^2 \lambda_{Ii} \mathbf{u}_{Ii} \mathbf{u}_{Ii}^H \end{aligned} \quad (12)$$

where  $\lambda_{Ii} (i = 1, 2)$  are the two nonzero eigenvalues of  $\mathbf{R}_{II}$ , and  $\mathbf{u}_{Ii}$  are their respective eigenvectors. The two eigenvalues are expressed as [21]

$$\begin{aligned} \lambda_{I1,2} &= \frac{N}{2} \left[ \sigma_{I1}^2 + \sigma_{I2}^2 + 2\sigma_{I1}\sigma_{I2}\text{Re}(\rho_I \beta_I^*) \right] \\ &\quad \times \left[ 1 \pm \sqrt{1 - \frac{4\sigma_{I1}^2\sigma_{I2}^2(1-|\beta_I|^2)(1-|\rho_I|^2)}{\sigma_{I1}^2 + \sigma_{I2}^2 + 2\sigma_{I1}\sigma_{I2}\text{Re}(\rho_I \beta_I^*)}} \right] \end{aligned} \quad (13)$$

where  $\sigma_{I1}^2$  and  $\sigma_{I2}^2$  are the power of the two paths of the interference signal, and  $\rho_I$  is the signal correlation coefficient

cient between the two paths. Moreover,  $\beta_I$  is the spatial correlation coefficient between the two paths, defined by

$$\beta_I = \frac{1}{N} \mathbf{a}^H(\theta_{I1}) \mathbf{a}(\theta_{I2}) \quad (14)$$

It is clear from Eq. (13) that, when either  $|\rho_I|$  or  $|\beta_I|$  becomes 1, that is, the two paths are either statistically or spatially coherent, the second eigenvalue becomes 0. Therefore, as either  $|\rho_I|$  or  $|\beta_I|$  becomes sufficiently high, the second eigenvalues will fall below the noise level, and the effect of the orthogonal component can be ignored. In this case, the two paths  $s_{I1}(t)$  and  $s_{I2}(t)$  can be equivalently treated as a single path, and a CMA adaptive array will suppress the two paths with a single DOF.

### 3. Subband CMA Adaptive Array

As discussed in the previous section, in principle, a CMA adaptive array suppresses the interference signal and the multipath rays of the desired signal, and receives only the direct path of the desired signal. However, if we can enhance the signal correlation between the multiple paths, the CMA algorithm will receive the multipath rays of the desired signal and combine them, whereas the interference signal will be suppressed by a single array DOF.

In this section, we introduce the concept of subband adaptive array. By using a filter bank to decompose a signal into different subbands, its multiple paths will become highly correlated in each subband. When the CMA algorithm is applied at each subband, diversity effect can be obtained for the desired signal, whereas the interference signals can be suppressed by fewer number of array DOFs.

#### 3.1. Correlation enhancement between multipath rays by subband decomposition

As shown in Fig. 2, subband decomposition is realized by using a set of analysis filters and a set of synthesis filters [24]. Moreover, since the subband signal can be processed at a lower rate, down-samplers and up-samplers are often used. To avoid loss of information, the following condition should be satisfied: decimation rate  $P \leq$  number of subbands  $M$ . In particular, when  $P = M$ , the filter bank is called a maximum decimated filter bank. When a filter bank satisfies the perfect-reconstruction condition, the output of the filter bank is the same as the input signal except for a time delay [24]. In this paper, we assume the filter bank used satisfies the perfect-reconstruction condition.

The subband decomposition divides an input data sequence  $x(n)$  into  $M$  subband sequences,  $x^{(1)}(n')$ ,  $\dots$ ,  $x^{(M)}(n')$ . The superscript ( $i$ ) denotes the signal component at the  $i$ -th subband. Compared to  $x(n)$ , each

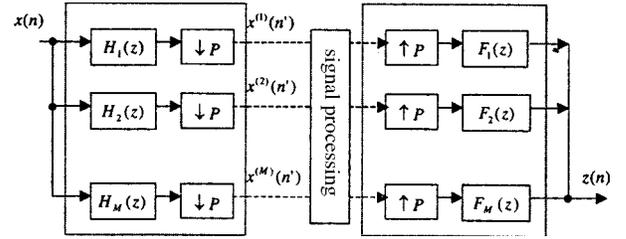


Fig. 2. Subband signal processing by filter banks.

subband signal  $x^{(i)}(n')$ ,  $i = 1, 2, \dots, M$ , has a bandwidth of as narrow as  $1/M$  of that of  $x(n)$ . When maximum decimation is done, the signal rate at a subband also becomes  $1/M$  of that at the full band.

Proper subband decomposition enhances the signal correlation between the direct path and the delayed path of a signal. Such property makes it possible to mitigate the effect of frequency-selective fading. For simplicity, we assume the source signal  $s(t)$  is band-limited and has a flat spectrum over  $[-B/2, B/2]$ , and is given by

$$p(f) = \begin{cases} 1, & -B/2 \leq f \leq B/2 \\ 0, & \text{elsewhere} \end{cases} \quad (15)$$

This corresponds to the case where the roll-off rate is 0. If the signal is QPSK-modulated,  $BT_s = 1$ , where  $T_s$  is the symbol period. Similar results can be obtained for other roll-off rate and modulation methods.

The autocorrelation function  $r(\tau)$  of signal  $s(t)$  is the Fourier transform of  $p(f)$ , and is given by

$$r(\tau) = E \{s(t)s^*(t - \tau)\} = \frac{\sin(\pi B\tau)}{\pi\tau} \quad (16)$$

We divide  $p(f)$  into  $M$  parts such that

$$p(f) = \sum_{k=1}^M p^{(k)}(f) \quad (17)$$

$$p^{(k)}(f) = \begin{cases} 1, & -\frac{B}{2} + \frac{k-1}{M}B \leq f < -\frac{B}{2} + \frac{k}{M}B \\ 0, & \text{elsewhere} \end{cases} \quad (18)$$

and decompose a signal  $s(t)$  into subband signals  $s^{(i)}(t)$  ( $i = 1, 2, \dots, M$ ). This is equivalent to the case where the analysis filters have nonoverlapping flat transfer functions. Similarly, the synthesis filters also have nonoverlapping flat transfer functions so that the analysis filters and

the synthesis filters form a perfect-reconstruction filter bank.

In the  $k$ -th subband, the autocorrelation function  $r^{(k)}(\tau)$  of  $s^{(k)}(t)$  is the Fourier transform of  $p^{(k)}(f)$ , and is given by

$$\begin{aligned} r^{(k)}(\tau) &= E \left\{ s^{(k)}(t) s^{(k)*}(t - \tau) \right\} \\ &= \frac{\sin(\pi B\tau/M)}{\pi\tau} e^{j(\pi B\tau/M)(2k-M-1)} \end{aligned} \quad (19)$$

The correlation coefficient between a signal  $s(t)$  and its delayed version  $s(t - \tau)$  is

$$\rho(\tau) = \alpha \frac{r(\tau)}{r(0)} = \alpha \frac{\sin(\pi B\tau)}{\pi B\tau} \quad (20)$$

where  $\alpha(|\alpha| = 1)$  expressed the phase shift in propagation. Similarly, at the  $k$ -th subband, the correlation coefficient between  $s^{(k)}(t)$  and  $s^{(k)}(t - \tau)$  becomes

$$\begin{aligned} \rho^{(k)}(\tau) &= \alpha \frac{r^{(k)}(\tau)}{r^{(k)}(0)} \\ &= \alpha \frac{\sin(\pi B\tau/M)}{\pi B\tau/M} e^{j(2k-M-1)\pi B\tau/M} \end{aligned} \quad (21)$$

Comparing Eqs. (20) and (21), it is evident that, for a given time-delay, the absolute value of the signal correlation function of signal  $s(t)$  becomes wider by a factor of  $M$ . For example, when  $B\tau = 1$ , in the full band we have  $\rho(1/B) = 0$ , whereas in a subband,  $|\rho^{(k)}(1/B)|$  becomes approximately 0.974 for  $M = 8$ , and 0.9936 for  $M = 16$ .

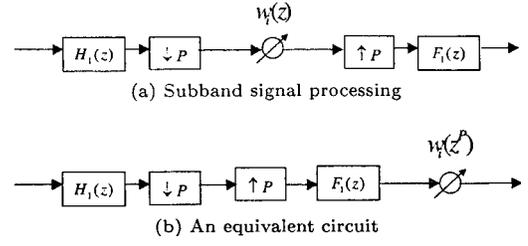


Fig. 3. Subband signal processing and its equivalent circuit.

Therefore, as  $M$  increases, the multipath rays of a signal can be treated as almost coherent signal components.

In Eq. (21),  $|\rho^{(k)}(\tau)|$  has a constant value with respect to different values of  $k$ . That is, the correlation coefficient is uniformly enhanced at different subbands.

From the above discussion, it is obvious that the multipath components in frequency-selective fading environment, regardless of the desired signal or the interference signal, become highly correlated based on the subband signal processing using filter bank. Applying this concept to array processing removes the necessity of temporal equalization and realizes the same results as a space-time adaptive processor.

It is noted that although the absolute value of the signal correlation between the direct path and the delayed path becomes higher with subband signal processing, the phase difference between the paths differs for different subbands. The phases of the two paths vary for different subbands, and in some subbands they may become oppo-

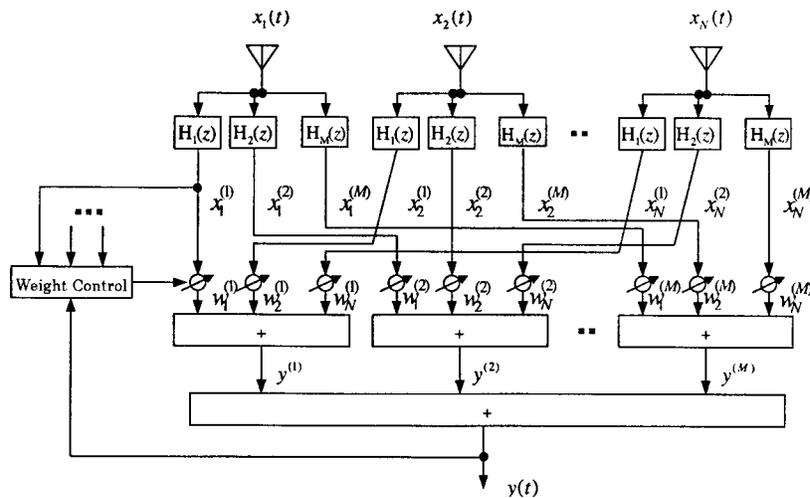


Fig. 4. A subband CMA adaptive array.

site. In this case, if array processing is not utilized, the two paths may cancel each other and yield a deep notch. However, when a CMA adaptive array is used, the effect of such cancellation is not significant unless the two paths have very high spatial correlation [25]. When the direct path and the delayed path arrive from the same direction and if they cancel each other in some frequency, the received signal itself has a physical notch at the frequency and such a notch cannot be removed by multipath suppression or equalization. In this case, the array performance may degrade.

The above discussion is based on ideal filter banks, where the analysis and synthesis filters have nonoverlapping flat transfer functions. Such filters require infinite number of taps and its time delay will be impracticably large. Instead, the modified-QMF filter bank can be used in practice which provides close performance to the ideal filter banks [18]. In this paper, we only consider ideal filter banks for simplicity.

### 3.2. Subband CMA adaptive arrays

In a subband CMA array the signals received at the array elements are decomposed into different subbands. After weighting in each subband, the output of each subband is combined which yields the array output.

When a signal is band-limited, as previously discussed, the constant modulus property is valid only at a single sampling point for each symbol. Therefore, for CMA adaptive array, the weights can be updated only once a symbol. When the subband signals are decimated by a rate  $P$ , the weights are usually updated every  $P$  symbols. To simplify the analysis, here we assume  $P = 1$ , and the weights are updated every symbol.

Figure 3(a) shows the signal flow at the  $k$ -th subband. The weight processing can be equivalently moved out from the filter bank, yielding Fig. 3(b). In particular, when  $P = 1$ , there are no down- and up-sampling operations, and the analysis and synthesis filters can be combined. Figure 4 shows an implementation of the subband CMA adaptive array.

Let  $x_j^{(k)}(m)$  denote the signal of  $j$ -th array element at  $k$ -th subband, and let its corresponding weights be  $w_j^{(k)}(m)$ , then the array output is expressed as

$$\begin{aligned} y(t) &= \sum_{k=1}^M y^{(k)}(m) \\ &= \sum_{k=1}^M \mathbf{W}^{(k)H} \mathbf{X}^{(k)} \\ &= \sum_{k=1}^M \sum_{j=1}^N w_j^{(k)}(m) x_j^{(k)}(m) \end{aligned} \quad (22)$$

where

$$\mathbf{x}^{(k)} = [x_1^{(k)}, x_2^{(k)}, \dots, x_N^{(k)}]^T \quad (23)$$

$$\mathbf{w}^{(k)} = [w_1^{(k)}, w_2^{(k)}, \dots, w_N^{(k)}]^T \quad (24)$$

Substituting Eq. (22) into Eqs. (1) and (3), the cost function and its gradient vector with respect to  $\mathbf{w}^{(k)}$  are obtained as

$$\begin{aligned} Q(\mathbf{w}^{(k)}) &= E \left[ \left| |y(t)|^2 - 1 \right|^2 \right] \\ &= E \left[ \left| \left| \sum_{k=1}^M \mathbf{w}^{(k)H} \mathbf{x}^{(k)} \right|^2 - 1 \right|^2 \right] \end{aligned} \quad (25)$$

and

$$\nabla_{\mathbf{w}^{(k)}} Q(m) = 4 \mathbf{x}^{(k)}(m) y^*(m) (|y(m)|^2 - 1) \quad (26)$$

respectively. Accordingly, the weights are updated according to

$$\mathbf{w}^{(k)}(m+1) = \mathbf{w}^{(k)}(m) - \mu \nabla_{\mathbf{w}^{(k)}} Q(m) \quad (27)$$

Similar to the discussion in Section 2, if we replace the statistical average by the temporal average, the weight update equation becomes

$$\begin{aligned} &\mathbf{w}^{(k)}(m+1) \\ &= \mathbf{w}^{(k)}(m) - \frac{4\mu^{(k)}}{K} \\ &\quad \times \sum_{i=m-(K+1)}^m \left\{ \mathbf{x}^{(k)}(i) y_m^*(i) [ |y_m(i)|^2 - 1 ] \right\} \end{aligned} \quad (28)$$

where

$$\mu^{(k)} = 1 / \left[ \frac{1}{K} \sum_{i=m-K+1}^m 6\lambda_{\max}^{(k)2}(i) \right] \quad (29)$$

is the step size of the weight update in  $k$ -th subband.

## 4. Performance Evaluation

In order to evaluate the effectiveness of the proposed method, in this section, we show simulation examples to examine the basic behavior of CMA adaptive arrays in multipath environment, the performance with closely

spaced direct path and delayed path, the performance in the presence of interference signals, as well as the diversity effect of the array. QPSK modulation is assumed for both desired signal and interference signals, and the roll-off rate is chosen as 0.5. Uniform linear array with half-wavelength interelement spacing is used, and the direction of arrival is measured from the broadside direction.

The array performance is evaluated in terms of the residual error and the bit error rate (BER). The residual error is defined as

$$\text{Residual error (dB)} = 10 \log \left[ \left| |y(m)|^2 - 1 \right| \right] \quad (30)$$

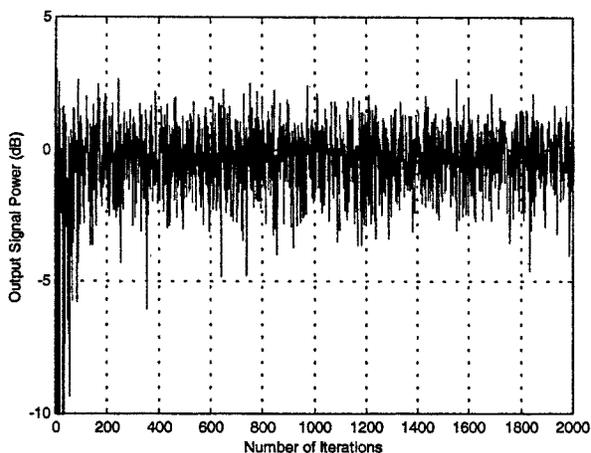
and is calculated after the array is converged.

#### 4.1. Array behavior in multipath environment

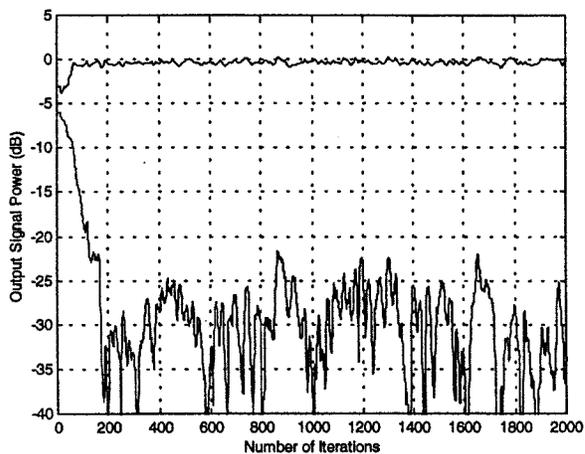
We first consider the scenario where only the desired signal is present. The signal arrives via one direct path and one delayed path, and a three-element array is used. In this case, even though we do not utilize the subband processing, the DOFs are enough to suppress the delayed path.

Figure 5 shows the output signal level. The direct path arrives from  $0^\circ$ , and its input power is 0 dB. The delayed path arrives from  $20^\circ$ , and the power is  $-3$  dB. The time delay between the two paths is  $T_S$ . The power of additive noise is  $-10$  dB, and the number of subbands is  $M = 8$ . The number of data samples used for weight update is  $K = 15$ .

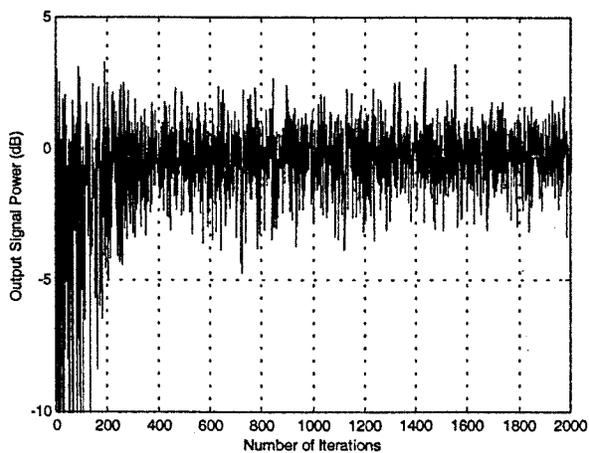
In the underlying scenario, since both the conventional CMA adaptive array and the subband CMA adaptive



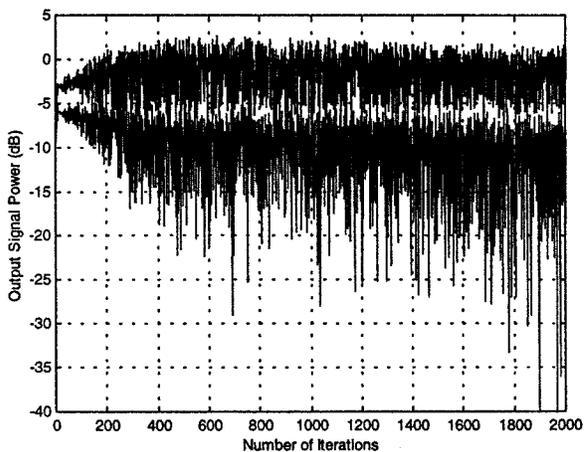
(a) Output signal power of the array (without subband processing)



(b) Output signal power of the two rays (without subband processing)



(c) Output signal power of the array (with subband processing :  $M=8$ )

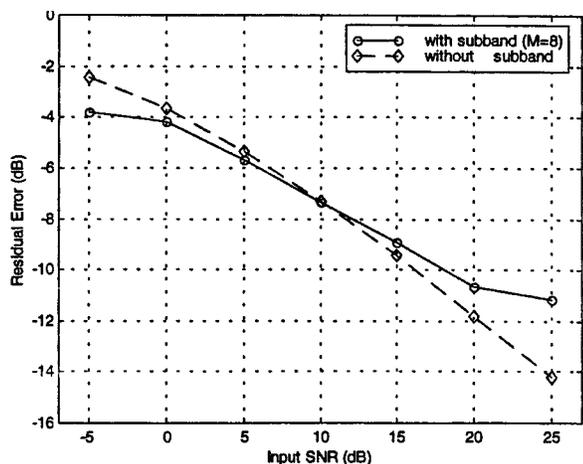


(d) Output signal power of the two rays (with subband processing :  $M=8$ )

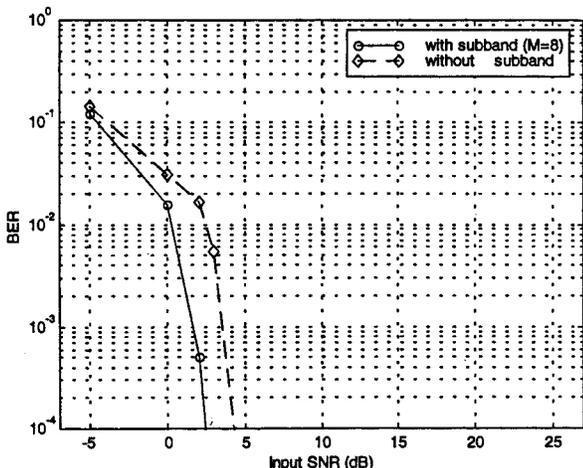
Fig. 5. The output signal power performance ( $N = 3$ ,  $\sigma_{D1}^2 = 0$  dB,  $\sigma_{D2}^2 = -3$  dB,  $\sigma_n^2 = -10$  dB,  $\phi_{D1} = 0^\circ$ ,  $\phi_{D2} = 20^\circ$ ,  $\tau = T_S$ ).

array have enough DOFs, both of them output good performance. However, their behaviors are different. In contrast to the conventional CMA adaptive array which receives the direct path and suppresses the delayed path [Figs. 5(a) and (b)], the subband CMA adaptive array combines both paths [Figs. 5(c) and (d)]. In the subband CMA adaptive array, since the direct and delayed paths are highly correlated in the subbands, the solution is not unique to satisfy the condition that the output signal has a constant modulus. Therefore, the contribution of the direct and delayed paths varies with time [Fig. 5(c)], whereas the array output is relatively stable.

In Fig. 6, the residual error and the output BER are shown with respect to the input SNR defined as the power ratio between the direct path and the additive noise. Both



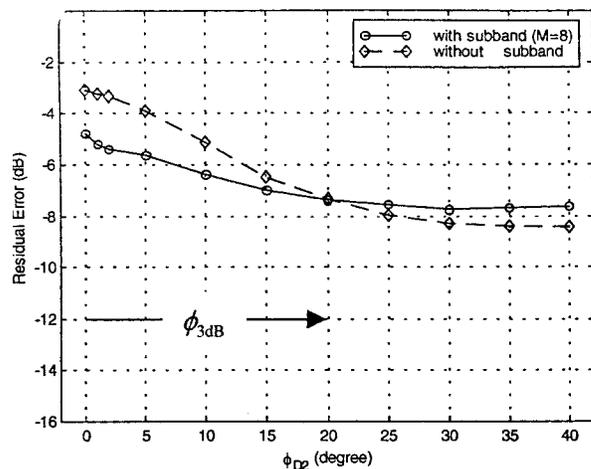
(a) Residual error



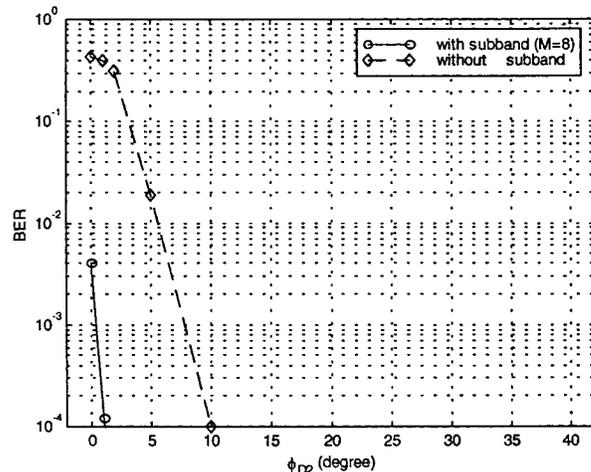
(b) Output BER

Fig. 6. The array performance versus input SNR ( $N = 3$ ,  $\sigma_{D1}^2 = 0$  dB,  $\sigma_{D2}^2 = -3$  dB,  $\phi_{D1} = 0^\circ$ ,  $\phi_{D2} = 20^\circ$ ,  $\tau = T_S$ ).

arrays show improved output performance as the input SNR increases. However, the subband CMA adaptive array shows lower residual error than the conventional CMA adaptive array when the input SNR is low. When the SNR exceeds 10 dB, on the other hand, the residual error of the subband CMA adaptive array is greater than that of the conventional CMA adaptive array. As a matter of fact, in the subband CMA adaptive array, there is a floor at the residual error due to the presence of orthogonal component of the delayed signal, which is more apparent in high-SNR cases. However, the effect of such component is usually not significant in practical applications. For example, in this figure, when the SNR exceeds 5 dB, for both arrays the BER



(a) Residual error



(b) Output BER

Fig. 7. The array performance versus the DOA of the delayed ray ( $N = 3$ ,  $\sigma_{D2}^2 = -3$  dB,  $\sigma_n^2 = -10$  dB,  $\phi_{D1} = 0^\circ$ ,  $\tau = T_S$ ).

is lower than  $10^{-4}$ , which is negligible in practical applications.

#### 4.2. Performance in the case of closely spaced direct and delayed paths

In practical mobile communication environment, it is realistically possible that the direct path and the delayed paths arrive from closely spaced directions. A conventional CMA adaptive array will try to form a null in the direction of the delayed paths. However, when the delayed paths are close to the direct path, the beamforming becomes difficult due to the limitation of the array resolution. In contrast, a

subband CMA adaptive array is not necessary to suppress the delayed paths, and therefore its performance is less affected by the array resolution.

Figure 7 shows the residual error and the output BER as the angle of arrival of the delayed path changes, where  $\phi_{3\text{dB}}$  denotes the 3-dB beamwidth when the array elements are identically fed. When the angle separation between the direct path and the delayed path is small, the conventional CMA adaptive array cannot effectively suppress the delayed path, as such yields a large residual error and the BER is as high as of the order of  $10^{-1}$ . In contrast, the subband CMA array provides much smaller residual error, and the BER is less than  $10^{-4}$  even when the angle separation between the two paths is very small.

#### 4.3. Performance in the presence of interference signals

Figure 8 shows the residual error and the output BER performance, where an interference signal is added to the scenario of Fig. 6. The interference signal also arrives via two paths. The direct path is from  $20^\circ$  and its input power is  $-3$  dB, whereas the delayed path with 1 symbol delay time is from  $-40^\circ$  and the power is  $-6$  dB.

In this case, the DOFs of the conventional CMA adaptive array become insufficient, yielding ineffective suppression of the interference signal and the delayed path of the desired signal. As a result, the array output performance degrades and the output BER shows a floor on the order of  $10^{-1}$ . On the other hand, the subband CMA adaptive array equivalently deals with the two paths of each signal as a single path, as such it has enough DOFs to suppress the interference signal. When the input SNR exceeds 10 dB, the output BER is as low as less than  $10^{-4}$ .

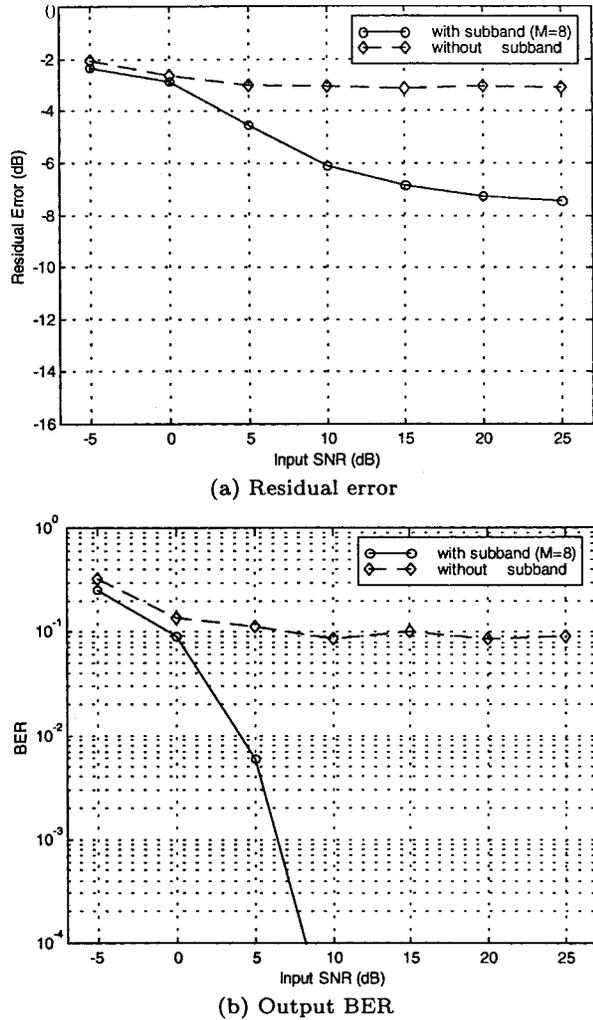


Fig. 8. The array performance when the number of rays exceeds array DOFs ( $N = 3$ ,  $\sigma_{D1}^2 = 0$  dB,  $\sigma_{D2}^2 = -3$  dB,  $\phi_{D1} = 0^\circ$ ,  $\phi_{D2} = 20^\circ$ ,  $\tau_D = T_S$ ,  $\sigma_{I1}^2 = -3$  dB,  $\sigma_{I2}^2 = -6$  dB,  $\phi_{I1} = -20^\circ$ ,  $\phi_{I2} = -60^\circ$ ,  $\tau_I = T_S$ ,  $\sigma_n^2 = -10$  dB).

## 5. Conclusions

Although its blind processing nature makes the CMA algorithm a promising algorithm in mobile communications, it has been considered difficult to separate multiple paths by using this algorithm. This paper proposes the subband CMA adaptive array, which enhances the signal correlation between multiple paths based on subband decomposition, and subsequently mitigates the effect of frequency-selective fading.

As a result, the paths of the desired signal are equalized and combined with achieved diversity effect. On the other hand, the paths of an interference signal become highly correlated and can be suppressed with a single DOF, as such the DOFs of the array can be effectively utilized. Accordingly, the required number of array elements is reduced, and the array performance is improved, particularly when the number of array elements is small, or/and

the direct path and the delayed paths arrive from close directions.

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## REFERENCES

1. Lee WC. Mobile communication engineering. McGraw-Hill; 1982.
2. Treichler JR, Agee BG. A new approach to multipath correction of constant modulus signals. *IEEE Trans Acoust Speech Signal Process* 1983;ASSP-31:459–472.
3. Ogane T. Characteristics of CMA adaptive array for selective fading compensation in digital land mobile radio communications. *IEICE Trans Commun* 1990;J73-B-II:489–497.
4. Fujimoto M, Kikuma N, Inagaki N. Behavior of the CMA adaptive array to filtered  $\pi/4$ -shifted QPSK signals. *IEICE Trans Commun* 1991;J74-B-II:497–500.
5. Chiba, Miura R, Karasawa Y. Separation of direct wave and delay wave using a beam space CMA adaptive array antenna. *Proc IEICE General Conf*, No. B-84, 1994.
6. Chiba, Chujo W, Fujise M. Beam space CMA adaptive array antennas. *IEICE Trans Commun* 1994;J77-B-II:130–138.
7. Sekiguchi T, Karasawa Y. CMA adaptive array antennas using analysis and synthesis filter banks. *IEICE Trans Fundam* 1998;E81-A:1570–1577.
8. Furukawa H, Kamio Y, Sasaoka H. A study on interference reduction and delayed signal combining scheme using a CMA adaptive array antenna. *Tech Rep IEICE* 1998;AP97-213, RCS97-251:89–94.
9. Yuan H, Hirasawa K, Zhang Y. The mutual coupling and diffraction effects on the performance of a CMA adaptive array. *IEEE Trans Veh Technol* 1998;VT-47:728–736.
10. Ogawa Y, Ohmiya M, Itoh K. An LMS adaptive array for multipath fading reduction. *IEEE Trans Aerosp Electron Syst* 1987;AES-23:17–23.
11. Doi Y, Ohgane T. The performance of the high gain interference canceller combining adaptive array and MLSE when number of antenna elements is smaller than that of signals. *Tech Rep IEICE* 1996;RCS96-56:27–32.
12. Ogawa Y, Nagashima Y, Itoh K. An adaptive antenna system for high-speed digital mobile communications. *IEICE Trans Commun* 1992;E75-B:413–421.
13. Widrow B, Mantev PE, Griffiths LJ, Goode BB. Adaptive antenna systems. *Proc IEEE* 1967;55:2143–2159.
14. Fujii M. A study on joint processing of adaptive array antenna and MLSE for multipath countermeasure. *Tech Rep IEICE* 1995;RCS95-97:1–6.
15. Furukawa H, Kamio Y, Sasaoka H. Co-channel interference reduction method using CMA adaptive array antenna. *IEICE Trans Commun* 1997;J80-B-II:292–295.
16. Kukuma N, Kihira K, Inagaki N. Correlation-constrained CMA adaptive arrays using cyclostationary signal properties. *Proc IEEE AP-S Int Symp*, Atlanta, p 376–379, 1998.
17. Zhang Y, Yang K, Amin MG. Adaptive subband arrays for multipath fading mitigation. *Proc IEEE AP-S Int Symp*, Atlanta, p 380–383, 1998.
18. Zhang Y, Yang K, Amin MG. Performance analysis of subband adaptive arrays in multipath propagation environment. *Proc 9th IEEE Signal Processing Workshop on Statistical Signal and Array Signal Processing*, Portland, 1998.
19. Khalab JM, Ibrahim MK. Novel multirate adaptive beamforming technique. *Electron Lett* 1994;30:1194–1195.
20. Compton RT, Jr. The relationship between tapped delay-line and FFT processing in adaptive arrays. *IEEE Trans Antennas Propagat* 1988;AP-36:15–26.
21. Hudson JE. Adaptive array principles. Peter Peregrinus; 1989.
22. Zhang Y, Karasawa Y. Performance analysis of an adaptive array in the presence of interference signals with angular spread. *Tech Rep IEICE* 1997;AP97-108, RCS97-123:71–78.
23. Yang J, Swindlehurst A. Maximum SINR beamforming for correlated sources. *Proc ICASSP'95*, Detroit, p 1916–1919.
24. Fliege NJ. Multirate digital signal processing. John Wiley; 1994.
25. Zhang Y, Yang K. CMA adaptive array performance in the presence of correlated signals. *Proc 3rd Int Symp on Multi-Dimensional Mobile Communications*, Menlo Park, 1998.

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