Local CSI Based Selection Beamforming for AF MIMO Relay System with Direct Link

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***Abstract*—We propose a novel transmission scheme for a relay system that consists of a source, an amplify-and-forward (AF) relay, and a destination, which are all multi-antenna nodes and employ beamforming. The proposed scheme is based merely on local channel state information. The source transmits to the relay only when the source finds that the source-destination link will be in outage. In so doing, the proposed scheme lends itself to providing better average spectral efficiency. We derive an exact closed-form expression of the outage probability for Rayleigh fading channels and determine the system diversity gain. Numerical results validate the analytical results and show that, for the scenarios considered and for given spectral efficiency, the proposed scheme provides improved outage probability compared to the recently developed technique.**

***Index Terms*—MIMO relays, outage probability, selection beamforming, degrees-of-freedom.**

I. Introduction

In recent years, the application of multiple-input multiple-output (MIMO) relays for cooperative communications has garnered significant interest (e.g., [1]). The end-to-end error performance of a network consisting of multi-antenna threshold-based decode-and-forward relays, and single-antenna source and destination nodes, is analyzed in [2]. It is assumed that the relays use maximum ratio combining (MRC) and selection combining to decode the received signals. In [3], the source and destination beamformers, and the amplify-and-forward (AF) MIMO relay processing matrix are jointly optimized to maximize the signal-to-noise ratio (SNR) at the destination. We refer to this method as *max-SNR* scheme throughout the rest of this letter. If the direct link between the source and destination is neglected, the max-SNR scheme results into the application of MRC plus maximum ratio transmission for both the source-relay ($\mathcal{S}$-$\mathcal{R}$) and relay-destination ($\mathcal{R}$-$\mathcal{D}$) links. This leads to simple closed-form solutions for the beamformers and the AF MIMO relay. If the source-destination ($\mathcal{S}$-$\mathcal{D}$) link is taken into account, however, the max-SNR method requires the destination to combine the signals received at two time slots from the two-hop and $\mathcal{S}$-$\mathcal{D}$ links using the MRC or minimum mean-square error (MMSE) schemes. This approach yields optimum SNR at the destination. It requires, however, a) two time slots (channel uses) for each symbol transmission, b) solving a nonconvex source beamformer optimization problem (see Section IV in [3]) and c) non-local channel state information (CSI) depending on the node where the latter optimization problem is solved. Note that non-local channels for the source, relay and destination nodes are $\mathcal{R}$-$\mathcal{D}$, $\mathcal{S}$-$\mathcal{D}$ and $\mathcal{S}$-$\mathcal{R}$ channels, respectively. Although with some additional overhead a node can obtain its non-local CSI using the training and limited feedback methods [3]-[6], the computational cost for obtaining the global optimal solution of the abovementioned nonconvex problem can be significantly high.

In this letter, we propose a selective source beamforming technique for a network setup which is similar to that of [3], i.e., the multi-antenna cooperative MIMO relay system with a direct link. The source beamformer is selected depending on the outage condition of the instantaneous $\mathcal{S}$-$\mathcal{D}$ link. This link is considered to be in outage if it fails to guarantee the successful delivery of information above a certain target rate. In our approach, whenever the direct link is not in outage, the source can transmit two symbols in two time slots. In so doing, it efficiently utilizes the available degrees-of-freedom (DoF) to provide better average spectral efficiency than the max-SNR approach. For a given spectral efficiency, the proposed scheme can provide improved outage performance compared to the max-SNR scheme. This performance improvement is verified for different propagation conditions using simulation results. The proposed scheme achieves the maximum cooperative diversity gain, requires only local CSI, and avoids solving the nonconvex source beamformer optimization problem.

In making a *selection* between the $\mathcal{S}$-$\mathcal{D}$ and the two-hop relay link, our approach bears resemblance to the relay selection schemes in systems with single-antenna nodes (see [7] and [8] and references therein). However, the selection criterion, utilization of the available DoF, and the impacts on the outage probability are different from the schemes in [7], [8]. In [8], no direct link between the source and destination is assumed. The best relay is selected from a pool of relays using (mainly) the SNRs corresponding to all two-hop links. Unlike in [8], our approach makes a selection between the two-hop link and the direct link on the basis of its outage condition. In [8], irrespective of the selected relay, the end-to-end transmission from the source to the destination requires two time slots, since the relays operate in a half-duplex mode. The same is true for the cooperative system [7] in which the signals associated with the best relay and the $\mathcal{S}$-$\mathcal{D}$ link over two time slots are combined at the destination. This requirement is relaxed in our approach, which allows the direct link, when selected, to transmit two symbols in two time slots. This improves DoF utilization and reduces the outage probability of the cooperative system. In this letter, rather

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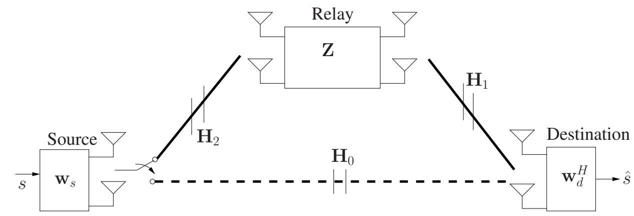

Fig. 1. Selection beamforming for MIMO relay channel.

than single-antenna systems, we deal with multiple antennas in which selection beamforming is performed at the source. We derive the exact outage probability expression and provide the diversity analysis involving the statistics of the largest eigenvalue of a Wishart distributed matrix [9].

Notations: Upper (lower) bold face letters will be used for matrices (vectors); $(\cdot)^H$, $f_{\alpha_\mu}(x)$, $\Pr\{\cdot\}$, and $\mathcal{C}^{n\times n}$ denote Hermitian transpose, probability density function (PDF) of α_μ, the probability operator and the space of $n\times n$ complex matrix.

II. System Model

Fig. 1 shows the cooperative MIMO relay system under consideration. The source, relay, and destination have, respectively, $n_{\rm s}$, $n_{\rm r}$ and n_d antennas. Spatially uncorrelated Rayleigh flat-fading MIMO channels are assumed. Let $\mathbf{H}_0 \in \mathcal{C}^{n_{\rm d}\times n_{\rm s}}$, $\mathbf{H}_1 \in \mathcal{C}^{n_{\rm d}\times n_{\rm r}}$, and $\mathbf{H}_2 \in \mathcal{C}^{n_{\rm r}\times n_{\rm s}}$ represent the fast fading parts of the $\mathcal{S}$-$\mathcal{D}$, $\mathcal{R}$-$\mathcal{D}$, and $\mathcal{S}$-$\mathcal{R}$ channels, respectively. The entries of $\mathbf{H}_0$, $\mathbf{H}_1$ and $\mathbf{H}_2$ are assumed to be independent, zero-mean, circularly symmetric complex Gaussian (ZMCSCG) random variables with unit variance. The path gains of the $\mathcal{S}$-$\mathcal{D}$, $\mathcal{R}$-$\mathcal{D}$ and $\mathcal{S}$-$\mathcal{R}$ links are denoted as $c_0 = \tilde{d}_0^{-\eta}$, $c_1 = \tilde{d}_1^{-\eta}$ and $c_2 = \tilde{d}_2^{-\eta}$, respectively, where η is the path loss exponent, and $\tilde{d}_0$, $\tilde{d}_1$ and $\tilde{d}_2$ are the respective distances corresponding to the $\mathcal{S}$-$\mathcal{D}$, $\mathcal{R}$-$\mathcal{D}$ and $\mathcal{S}$-$\mathcal{R}$ links. We assume that, using training [4] or feedback [3]-[6], each node obtains its local CSI perfectly, i.e., the source has the CSI of the $\mathcal{S}$-$\mathcal{R}$ and $\mathcal{S}$-$\mathcal{D}$ links, the relay has the CSI of the $\mathcal{S}$-$\mathcal{R}$ and $\mathcal{R}$-$\mathcal{D}$ channels, and the destination knows the CSI of the $\mathcal{S}$-$\mathcal{D}$ and $\mathcal{R}$-$\mathcal{D}$ channels. In the following, we present the proposed transmission protocol which is based on selection beamforming at the source.

Proposed Protocol: Each symbol transmission can take up to two time slots during which the channels are assumed to remain constant. The relay operates in a half-duplex mode. At the beginning of each transmission (first time-slot), selection beamforming is performed at the source as follows. If the source finds that the $\mathcal{S}$-$\mathcal{D}$ link is not in outage, the source transmits two symbols over two time slots by matching $\mathbf{w}_{\rm s}$ to the $\mathcal{S}$-$\mathcal{D}$ channel and also asks the relay to be silent. On the other hand, if the source finds that the $\mathcal{S}$-$\mathcal{D}$ link will be in outage, the source matches its beamformer $\mathbf{w}_{\rm s} \in \mathcal{C}^{n_{\rm s}\times 1}$ to the $\mathcal{S}$-$\mathcal{R}$ link and transmits one source symbol to the relay during the first time slot. In the latter case, the destination does not record what it receives from the source. The relay processes its received signal with the optimized linear operator $\mathbf{Z} \in \mathcal{C}^{n_{\rm r}\times n_{\rm r}}$ and transmits the resulting signal to the destination in the second time slot. The destination matches its receive beamformer $\mathbf{w}_{\rm d} \in \mathcal{C}^{n_{\rm d}\times 1}$ either to $\mathcal{S}$-$\mathcal{D}$ or $\mathcal{R}$-$\mathcal{D}$ channel depending on the route chosen by the source.

Let $B_{\rm R}$ denote the state of the relay, which is 1 if the relay is active and 0 if the relay is idle. On the basis of the proposed protocol, we can express $B_{\rm R}$ as

$$B_{\rm R} = \begin{cases} 1, & \text{if } \log_2(1+\gamma^{\rm D}) < r_{\rm th} \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

where $\gamma^{\rm D}$ is the instantaneous received SNR at the destination corresponding to the $\mathcal{S}$-$\mathcal{D}$ link and $r_{\rm th}$ is the target rate or the spectral efficiency attempted by the source. We will keep $r_{\rm th}$[1] same for all schemes so that their outage probabilities are comparable. For $B_{\rm R} = 1$, the optimal $\mathbf{w}_{\rm s}$, $\mathbf{w}_{\rm d}$ and $\mathbf{Z}$ happen to be the same as the solutions given in [3] for the case of two-hop relay channel without the direct link. Hence, we obtain the following SNR at the destination for $B_{\rm R} = 1$

$$\gamma^{\rm R} = \frac{ef\alpha_1\alpha_2}{e\alpha_1 + f\alpha_2 + 1} \tag{2}$$

where α_1 and α_2 are, respectively, the largest eigenvalues of $\mathbf{H}_1^H\mathbf{H}_1$ and $\mathbf{H}_2^H\mathbf{H}_2$, and $e = \frac{c_1 P_{\rm R}}{\sigma_{\rm d}^2}$ and $f = \frac{c_2 P_{\rm S}}{\sigma_{\rm r}^2}$. The source and relay transmit powers are denoted by $P_{\rm S}$ and $P_{\rm R}$, respectively, whereas the respective noise powers for the relay and destination are $\sigma_{\rm r}^2$ and $\sigma_{\rm d}^2$. When $B_{\rm R} = 0$, the destination SNR, obtained after using the optimal $\mathbf{w}_{\rm s}$ and $\mathbf{w}_{\rm d}$, is given by

$$\gamma^{\rm D} = \frac{c_0 P_{\rm S}\alpha_0}{\sigma_{\rm d}^2} \triangleq g\alpha_0 \tag{3}$$

where α_0 is the largest eigenvalue of $\mathbf{H}_0^H\mathbf{H}_0$, and $g \triangleq \frac{P_{\rm S}c_0}{\sigma_{\rm d}^2}$ is the average SNR of the $\mathcal{S}$-$\mathcal{D}$ link. The PDF of $\{\alpha_\mu\}_{\mu=0}^2$ is given by [9]

$$f_{\alpha_\mu}(x) = \sum_{\tilde{p}=1}^{p_{\rm u}} \sum_{\tilde{q}={\rm a}_\mu}^{{\rm b}_\mu} d_{\tilde{p},\tilde{q}}\tilde{p}^{\tilde{q}+1}\frac{x^{\tilde{q}}{\rm e}^{-\tilde{p}x}}{\tilde{q}!}, \forall\mu \in \{0,1,2\} \tag{4}$$

where $d_{\tilde{p},\tilde{q}}$ are obtained from [9]. For brevity, we use the following definitions for the indexes in (4). For α_0, α_1 and α_2, variables $\tilde{p}$ are replaced by k, j, and i, respectively, whereas the respective values of $\tilde{q}$ are replaced by l, n, and m. In (4), $p_{\rm u}$ takes values of $s_0 = \min(n_{\rm d}, n_{\rm s})$, $s_1 = \min(n_{\rm r}, n_{\rm d})$ and $s_2 = \min(n_{\rm s}, n_{\rm r})$ for α_0, α_1 and α_2, respectively. Moreover, we define ${\rm a}_0 = |n_{\rm d} - n_{\rm s}|$, ${\rm a}_1 = |n_{\rm d} - n_{\rm r}|$, ${\rm a}_2 = |n_{\rm s} - n_{\rm r}|$, ${\rm b}_0 = (n_{\rm d} + n_{\rm s})k - 2k^2$, ${\rm b}_1 = (n_{\rm d} + n_{\rm r})j - 2j^2$, and ${\rm b}_2 = (n_{\rm s} + n_{\rm r})i - 2i^2$.

III. Outage Probability Analysis

In the proposed protocol, when the direct link is selected, the symbol rate is 1 symbol per time slot (pts), whereas when the two-hop link is selected, the symbol rate is $1/2$ symbol pts. By using a larger constellation in the latter case, we can maintain the same spectral efficiency as in the 1 symbol pts case[2]. Hence, the outage probability at the destination can be expressed as the joint probability of the events $B_{\rm R} = 1$ and

[1] By changing $r_{\rm th}$ [7], diversity multiplexing tradeoff can be analyzed for our scheme. However, such analysis is difficult, although not necessarily impossible for the max-SNR scheme since the PDF of the corresponding optimum destination SNR is not known in closed-form.

[2] Thus, the outage of the two-hop relay link is also based on the same $r_{\rm th}$ which is then used to define the outage probability of the max-SNR scheme.

$\frac{1}{2}\log_2(1+\gamma^{\mathrm{R}}) < r_{\mathrm{th}}$. The end-to-end outage probability P_{out} can be expressed as

$$\begin{aligned} P_{\mathrm{out}} &= \Pr\left\{\frac{1}{2}\log_2(1+\gamma^{\mathrm{R}}) \le r_{\mathrm{th}}, B_{\mathrm{R}}=1\right\} \\ &= \Pr\left\{\gamma^{\mathrm{R}} \le 2^{2r_{\mathrm{th}}}-1\right\}\Pr\left\{\gamma^{\mathrm{D}} \le 2^{r_{\mathrm{th}}}-1\right\} \quad (5) \end{aligned}$$

where the second equality is due to the statistical independence of the events $B_{\mathrm{R}} = 1$ and $\frac{1}{2}\log_2(1+\gamma^{\mathrm{R}}) \le r_{\mathrm{th}}$, and $\Pr\{B_{\mathrm{R}}=1\} = \Pr\left\{\log_2(1+\gamma^{\mathrm{D}}) \le r_{\mathrm{th}}\right\}$ according to (1) and (3). For notational simplicity, we define $R_1 \triangleq 2^{r_{\mathrm{th}}}-1$ and $R_2 \triangleq 2^{2r_{\mathrm{th}}}-1$. Applying (4), the PDF of SNR (3) can be expressed as

$$f_{\gamma^{\mathrm{D}}}(x) = \sum_{k=1}^{s_0}\sum_{l=a_0}^{b_0}\frac{d_{k,l}k^{l+1}}{l!g^{l+1}}x^l \mathrm{e}^{-\frac{k}{g}x}. \quad (6)$$

Using (6) and eq. (3.381.1) of [10], we obtain

$$\tilde{P}_0 \triangleq \Pr\left\{\gamma^{\mathrm{D}} \le R_1\right\} = \sum_{k=1}^{s_0}\sum_{l=a_0}^{b_0}\frac{d_{k,l}}{l!}\gamma_{\mathrm{L}}\left(l+1, \frac{k}{g}R_1\right) \quad (7)$$

where $\gamma_{\mathrm{L}}(\cdot,\cdot)$ is the lower-incomplete Gamma function. The probability $\tilde{P}_1 \triangleq \Pr\left\{\gamma^{\mathrm{R}} \le R_2\right\}$ is given by

$$\begin{aligned} \tilde{P}_1 &= 1 - 2\sum_{i=1}^{s_2}\sum_{m=a_2}^{b_2}\sum_{j=1}^{s_1}\sum_{n=a_1}^{b_1}\frac{d_{i,m}d_{j,n}}{m!} \\ &\times i^{m+1}\mathrm{e}^{-\left(\frac{i}{f}+\frac{j}{e}\right)R_2}\sum_{p=0}^{n}\sum_{r=0}^{p}\sum_{s=0}^{m}\frac{j^p}{p!}\binom{p}{r}\binom{m}{s} \\ &\times \frac{R_2^{\frac{2m-s+p+r+1}{2}}(1+R_2)^{\frac{p-r+s+1}{2}}}{e^{\frac{p+r+s+1}{2}}f^{\frac{2m-s+p-r+1}{2}}}\left(\frac{j}{i}\right)^{\frac{r+s-p+1}{2}} \\ &\times K_{r+s-p+1}\left(2\sqrt{\frac{ijR_2(1+R_2)}{ef}}\right) \end{aligned} \quad (8)$$

where $K_n(\cdot)$ is the modified Bessel function of the second type with order n and $\binom{p}{r}$ stands for the binomial coefficient [10]. The derivations of (8) are shown in the Appendix.

Diversity analysis: For determining the diversity order of the proposed scheme, we use the asymptotic behavior of $f_{\alpha_\mu}(x)$ (as $x \to 0^+$) [11]

$$f_{\alpha_\mu}(x) \approx \frac{q_\mu\prod_{k=0}^{s_\mu-1}k!}{\prod_{k=0}^{s_\mu-1}(t_\mu+k)!}x^{q_\mu-1} \triangleq r_\mu x^{q_\mu-1} \quad (9)$$

where $q_0 = n_{\mathrm{d}}n_{\mathrm{s}}$, $q_1 = n_{\mathrm{d}}n_{\mathrm{r}}$, $q_2 = n_{\mathrm{s}}n_{\mathrm{r}}$, $t_0 = \max(n_{\mathrm{d}}, n_{\mathrm{s}})$, $t_1 = \max(n_{\mathrm{d}}, n_{\mathrm{r}})$, and $t_2 = \max(n_{\mathrm{s}}, n_{\mathrm{r}})$. For large e and f, (2) can be approximated as

$$\gamma^{\mathrm{R}} \approx \frac{ef\alpha_1\alpha_2}{e\alpha_1+f\alpha_2} \le \min(e\alpha_1, f\alpha_2). \quad (10)$$

Substituting (10), (2) and (3) into (5) and using the fact that $\min(e\alpha_1, f\alpha_2) \ge R_2$ is same as $(e\alpha_1 \ge R_2, f\alpha_2 \ge R_2)$, (5) can be written as

$$\begin{aligned} P_{\mathrm{out}} \approx \Pr\{g\alpha_0 \le R_1\}\Big\{&\Pr\{f\alpha_2 \le R_2\} + \Pr\{e\alpha_1 \le R_2\} \\ &-\Pr\{f\alpha_2 \le R_2\}\Pr\{e\alpha_1 \le R_2\}\Big\}. \end{aligned} \quad (11)$$

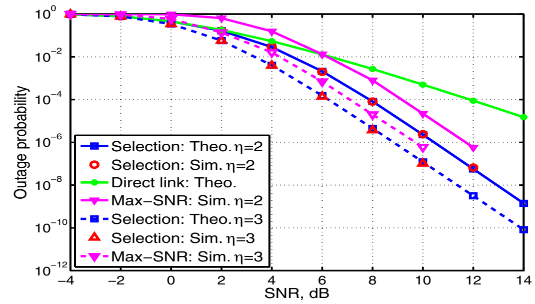

Fig. 2. Outage probabilities versus average SNR for $n_{\mathrm{r}} = 2$.

Now, applying (9) into (11), we get

$$\begin{aligned} P_{\mathrm{out}} \approx\, &\tilde{s}_0 e^{-n_{\mathrm{d}}n_{\mathrm{r}}}g^{-n_{\mathrm{d}}n_{\mathrm{s}}} + \tilde{s}_1 f^{-n_{\mathrm{s}}n_{\mathrm{r}}}g^{-n_{\mathrm{d}}n_{\mathrm{s}}} \\ &-\tilde{s}_2 e^{-n_{\mathrm{d}}n_{\mathrm{r}}}f^{-n_{\mathrm{s}}n_{\mathrm{r}}}g^{-n_{\mathrm{d}}n_{\mathrm{s}}} \end{aligned} \quad (12)$$

where $\tilde{s}_0 = \frac{r_0r_1}{q_0q_1}R_1^{n_{\mathrm{d}}n_{\mathrm{s}}}R_2^{n_{\mathrm{d}}n_{\mathrm{r}}}$, $\tilde{s}_1 = \frac{r_0r_2}{q_0q_2}R_1^{n_{\mathrm{d}}n_{\mathrm{s}}}R_2^{n_{\mathrm{s}}n_{\mathrm{r}}}$ and $\tilde{s}_2 = \frac{r_0r_1r_2}{q_0q_1q_2}R_1^{n_{\mathrm{d}}n_{\mathrm{s}}}R_2^{n_{\mathrm{s}}n_{\mathrm{r}}+n_{\mathrm{d}}n_{\mathrm{r}}}$. As $e, f, g \to \infty$, the first two terms in (12) dominate the third term. Without loss of generality, we assume $e = \bar{\gamma}$, $f = \nu_1\bar{\gamma}$ and $g = \nu_2\bar{\gamma}$ where ν_1 and ν_2 are some positive scalars. Then, the diversity order is given by $\min(n_{\mathrm{d}}n_{\mathrm{r}}, n_{\mathrm{s}}n_{\mathrm{r}}) + n_{\mathrm{d}}n_{\mathrm{s}}$, which shows that the maximum diversity order of cooperative system is achieved.

For comparison, the outage probability of the max-SNR scheme is given by $\tilde{P}_{\mathrm{out}} = \Pr\left\{\frac{1}{2}\log_2(1+\gamma^{\mathrm{Opt}}) \le r_{\mathrm{th}}\right\}$, where γ^{Opt} for our case, can be given as [3],

$$\gamma^{\mathrm{Opt}} = \max_{||\mathbf{w}_{\mathrm{s}}||^2 \le P_{\mathrm{S}}}\frac{\nu_1||\mathbf{H}_2\mathbf{w}_{\mathrm{s}}||^2}{\nu_2||\mathbf{H}_2\mathbf{w}_{\mathrm{s}}||^2+\nu_3} + \nu_4||\mathbf{H}_0\mathbf{w}_{\mathrm{s}}||^2 \quad (13)$$

where $\nu_1 = ec_2\alpha_1/\sigma_{\mathrm{r}}^2$, $\nu_2 = c_2/\sigma_{\mathrm{r}}^2$, $\nu_3 = e\alpha_1 + 1$ and $\nu_4 = c_0/\sigma_{\mathrm{d}}^2$.

IV. Numerical Results

In this section, Monte Carlo simulations are performed to assess the accuracy of the closed-form expression (5) incorporating (7) and (8). In all simulations, we take $n_{\mathrm{s}} = n_{\mathrm{d}} = 2$, $\sigma_{\mathrm{d}}^2 = \sigma_{\mathrm{r}}^2 = \sigma^2$, P_{S}=P_{R}=P, $r_{\mathrm{th}} = 2$ bits/sec/Hz, $\tilde{d}_0 = 1$, $\tilde{d}_1 = \tilde{d}_2 = 0.5\tilde{d}_0$. The entries of $\mathbf{H}_0, \mathbf{H}_1$, and $\mathbf{H}_2$ are taken to be ZMCSCG with unit variance. The coefficient values $d_{i,m}$, $d_{j,n}$ and $d_{k,l}$ for different antenna configurations are taken from [9]. We also compare our method with the max-SNR method, for which we numerically solve (13) [3]. Fig. 2 shows the outage probability, obtained analytically and via simulations, versus the average SNR (i.e., by varying P and taking $\sigma^2 = 1$) for $n_{\mathrm{r}} = 2$ and different η. With the same settings, but with $n_{\mathrm{r}} = 3$, the outage probability results are depicted in Fig. 3. It is evident from both figures that there exists a fine agreement between analytical and simulated results for the proposed scheme. Moreover, in both figures for the cases considered, our method performs better than both the direct transmission and the max-SNR scheme. The advantage of the proposed scheme in terms of performance over the max-SNR method, however, decreases as η and n_{r} increase. This is an expected outcome since the outage probability of the two-hop relay link dominates the end-to-end outage probability. The slopes of the outage probability curves at high SNR also verify the analytically derived diversity order $\min(n_{\mathrm{d}}n_{\mathrm{r}}, n_{\mathrm{s}}n_{\mathrm{r}}) + n_{\mathrm{d}}n_{\mathrm{s}}$

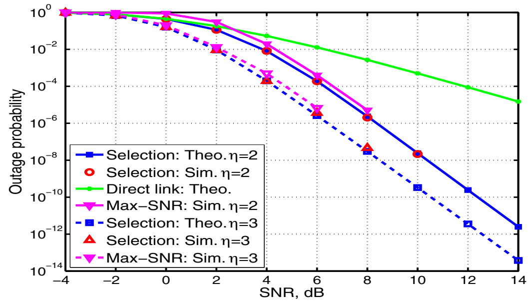

Fig. 3. Outage probabilities versus average SNR for $n_{\rm r} = 3$.

of the selection scheme. Although considered simulation examples show that the proposed method outperforms max-SNR method, we do not claim that this is always the case due to the difficulty of obtaining analytical relation between the outage probabilities of two methods.

V. Conclusions

We proposed a local CSI based selection scheme for a relay system that consists of multi-antenna source, relay and destination nodes. By switching from the direct link to the two-hop link, only when the former is in outage, the proposed scheme lends itself to providing better average spectral efficiency than the max-SNR scheme. For the same spectral efficiency, simulation results show that, for the cases considered, the proposed scheme provides improved outage probability compared to the max-SNR scheme.

Appendix

Using (2), $\tilde{P}_1$ can be expressed as

$$\tilde{P}_1 = \Pr\left\{\frac{ef\alpha_1\alpha_2}{e\alpha_1 + f\alpha_2 + 1} \le R_2\right\} \tag{14}$$
$$= \int_0^\infty \Pr\left\{\alpha_1(efx - R_2 e) \le R_2(fx+1)\right\} f_{\alpha_2}(x)\,dx$$

Since $\Pr\{\alpha_1(efx - R_2e) \le R_2fx + R_2\} = 1$ for $0 \le x < \frac{R_2}{f}$, we can express the integral of (14) as

$$\tilde{P}_1 = \int_0^{\frac{R_2}{f}} f_{\alpha_2}(x)\,dx + \int_{\frac{R_2}{f}}^\infty \Pr\left\{\alpha_1 \le \frac{R_2fx + R_2}{efx - R_2e}\right\} f_{\alpha_2}(x)\,dx. \tag{15}$$

With the help of (4) and eq. (3.381.1) of [10], we get

$$\tilde{P}_1 = \sum_{i=1}^{s_2}\sum_{m=a_2}^{b_2}\frac{d_{i,m}}{m!}\gamma_{\rm L}\left(m+1, \frac{iR_2}{f}\right) + \sum_{j=1}^{s_1}\sum_{n=a_1}^{b_1}\frac{d_{j,n}}{n!} \times \int_{\frac{R_2}{f}}^\infty \gamma_{\rm L}\left(n+1, \frac{jR_2(fx+1)}{efx - R_2e}\right) f_{\alpha_2}(x)\,dx. \tag{16}$$

For better exposition, let us use the following relation

$$\frac{R_2(fx+1)}{efx - R_2e} = d_1 + d_2\left(x - \frac{R_2}{f}\right)^{-1} \tag{17}$$

where $d_1 \triangleq \frac{R_2}{e}$, and $d_2 \triangleq \frac{R_2(R_2+1)}{ef}$. Using (17), (4) and eq. (8.352.1) of [10], the second term of (16) can be written as

$$I_1 = \sum_{i=1}^{s_2}\sum_{m=a_2}^{b_2}\sum_{j=1}^{s_1}\sum_{n=a_1}^{b_1}\frac{d_{i,m}d_{j,n}i^{m+1}}{m!n!} \times \int_{\frac{R_2}{f}}^\infty n!\left[1 - {\rm e}^{-j\left[d_1 + d_2\left(x - \frac{R_2}{f}\right)^{-1}\right]}\sum_{p=0}^{n}\frac{j^p}{p!} \times \left[d_1 + d_2\left(x - \frac{R_2}{f}\right)^{-1}\right]^p\right] x^m {\rm e}^{-ix}\,dx. \tag{18}$$

Using eq. (3.381.3) of [10], and making the substitution $x' = x - \frac{R_2}{f}$ into (18), the integral in (18) can be written as

$$I_2 = n!i^{-(m+1)}\Gamma\left(m+1, \frac{iR_2}{f}\right) - n!{\rm e}^{-\frac{iR_2}{f} - jd_1}\sum_{p=0}^{n}\frac{d_1^p j^p}{p!} \underbrace{\int_0^\infty \left(x' + \frac{d_2}{d_1}\right)^p \left(x' + \frac{R_2}{f}\right)^m (x')^{-p}{\rm e}^{-ix' - \frac{jd_2}{x'}}\,dx'}_{I_3} \tag{19}$$

where $\Gamma(\cdot,\cdot)$ is the upper-incomplete Gamma function. With the help of eq. (1.111) and eq. (3.471.9) of [10], we get

$$I_3 = \sum_{r=0}^{p}\sum_{s=0}^{m}\binom{p}{r}\binom{m}{s}\left(\frac{d_2}{d_1}\right)^{p-r}\left(\frac{R_2}{f}\right)^{m-s} \times 2\left(\frac{jd_2}{i}\right)^{\frac{r+s-p+1}{2}} K_{r+s-p+1}\left(2\sqrt{jd_2 i}\right). \tag{20}$$

Substituting I_3 from (20) into (19), I_2 can be obtained. Then, using I_2 in (18), I_1 can be determined. Finally, resubstituting d_1 and d_2 in terms of R_2, using I_1 from (18) in (16), noting that $\sum_{i=1}^{s_2}\sum_{m=a_2}^{b_2} d_{i,m} = \sum_{j=1}^{s_1}\sum_{n=a_1}^{b_1} d_{j,n} = 1$ [9], and using eq. (8.356.3) of [10], we can get the expression (8).

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