Outlines

- Sampling Process
  - Sampling Theorem
  - Nyquist Rate
- Pulse-Amplitude Modulation (PAM)
- Quantization
  - Uniform and non-uniform quantization
  - Quantization noise
- Pulse-Code Modulation (PCM)
- Delta Modulation
  - Delta-Sigma Modulation
- Line Codes

Pulse Modulations

Continuous-Wave Modulation

Sampling

Analog Pulse Modulation

Quantization

Digital Pulse Modulation

... PAM

... PCM
Sampling Process: Time-Domain Representations

- Ideally sampled signal of a continuous-wave signal \( g(t) \)
  - Through periodic sampling
  - Product of the continuous-wave signal and a sequence of impulses
    \[
    p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)
    \]
    \[
    g(t) \times p(t) = g(t) \sum_{n=-\infty}^{\infty} \delta(t - nT_s)
    \]
    \[
    g_s(t) = g(t) p(t) = g(t) \sum_{n=-\infty}^{\infty} \delta(t - nT_s)
    \]
- \( T_s \): sampling period; \( f_s = 1/T_s \): sampling rate

Sampling Process: Frequency-Domain Representations

- Multiplication in the time domain yields convolution in the frequency domain.
  \[
  g_s(t) = g(t) \left[ \sum_{n=-\infty}^{\infty} \delta(t - nT_s) \right]
  \]
  \[
  \Leftrightarrow G_s(f) = G(f) * P(f) = f_s \sum_{k=-\infty}^{\infty} G(f - kf_s)
  \]
- If \( f_s \geq 2W \), the replicas of \( G(f) \) do not overlap.
Sampling Theorem

- **[Sampling Theorem]** If \( G(f) = 0 \) for \( |f| \geq W \) (band limited signal) and \( f_s \geq 2W \) (with sufficient sampling rate). Then, \( G(f) \) can be obtained by properly filtering \( G_\delta(f) \).
- Otherwise, aliasing phenomenon occurs.
- **\( 2W \): Nyquist rate; \( 1/(2W) \): Nyquist interval**

\[ f_s \geq 2W : \text{Complete recovery of the original signal is possible.} \]

\[ f_s < 2W (\text{Undersampled}): \text{original signal cannot be completely recovered.} \]

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**Sampling Process: Signal Recovery**

- Ideal low-pass filter with cut-off frequency \( W \)
  \[
  H_{\text{LPF}}(f) = \begin{cases} 
  \frac{1}{2W}, & |f| \leq W \\
  0, & |f| > W
  \end{cases}
  \]
  The impulse response is
  \[
  h_{\text{LPF}}(t) = \frac{\sin(2\pi W t)}{2\pi W} = \text{sinc}(2Wt)
  \]

- Recovery of the sampled signal
  (assume \( f_s = 2W; T_s = 1/(2W) \))
  \[
  G_r(f) = G_\delta(f)H_{\text{LPF}}(f)
  \]
  \[
  g_r(t) = g_\delta(t) * h_{\text{LPF}}(t)
  \]
  \[
  = \int_{-\infty}^{\infty} g_\delta(t - nT_s) \delta(t - nT_s) \text{sinc}(2W(t - n)) dt
  \]
  \[
  = \sum_{n=-\infty}^{\infty} g_\delta\left(\frac{T_s}{2W}\right) \text{sinc}(2W(t - n))
  \]
Sampling Process: Examples

g(t) = \cos(4000 \pi t) \Rightarrow W = 2000 \text{ Hz}, \text{ Nyquist rate } 2W = 4000 \text{ Hz}

At sampling freq. \( f_s = 6000 \text{ Hz} \):

No aliasing: Nyquist sampling theorem is satisfied.

At sampling freq. \( f_s = 1500 \text{ Hz} \):

Aliasing occurred: Nyquist sampling theorem not satisfied.

Pulse-Amplitude Modulation (PAM)

- A train of impulse sequences is good, but not practical.
- PAM uses flat-top samples to substitute impulses.
  - Pros: limit the bandwidth; can use higher power
  - Cons: aperture effect – discussed below

PAM waveform \( s(t) \) can be considered as convolution of impulse train and rectangular window

PAM: Aperture Effect

- Distortion in the magnitude and time delay due to the use of flap-top pulse.
  \[ H(f) = T \text{sinc}(fT) \exp(-j \pi fT) \]
  (linear phase delay causes constant time delay of \( T/2 \) without no waveform distortion)

- If the duty cycle \( T/T_s \) is small \( (T/T_s \leq 0.1) \), then the aperture effect is insignificant.

- Otherwise, equalizer has to be applied, with magnitude response

\[
\left| \frac{1}{H(f)} \right| = \frac{1}{T \text{sinc}(fT)} = \frac{\pi f}{\sin(\pi fT)}
\]
Other Forms of Pulse Modulation

- Pulse-Duration Modulation (PDM) or Pulse-Width Modulation (PWM)
- Pulse-Position Modulation (PPM)

Figures: PDM and PPM for the case of a sinusoidal modulating wave.
(a) Modulating wave.
(b) Pulse carrier.
(c) PDM wave.
(d) PPM wave.

PDM & PPM: Observations

- PDM is energy-inefficient
- PPM
  - For perfect rectangular pulse (which requires high bandwidth), the effect of noise is negligible.
  - Exhibit a threshold effect as the SNR is low
  - Used in Ultra-Wideband (UWB) communications

Effective Data Representation

- How many bits you need to represent 8 levels of information (0 ~ 7)?

<table>
<thead>
<tr>
<th>Amplitude based</th>
<th>Code based</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level 6</td>
<td>Binary code 110</td>
</tr>
</tbody>
</table>

Quantization: Example

- 24 bits (8-bit in each color)  
  - A 75% reduction of file size in raw data format can be expected; It does not necessarily reflect that in a compressed file format.
  - Quantization is a lossy operation.

- 6 bits (2-bit in each color)
Quantization

- Quantization: Transform the continuous-amplitude \( m = m(nT_s) \) to discrete approximate amplitude \( v = v(nT_s) \).
- Such a discrete approximate is adequately good in the sense that any human ear or eye can detect only finite intensity differences.

![Quantizer Diagram]

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Quantization Noise

- Define the quantization noise to be
  \[
  Q = M - V = M - g(M)
  \]
- Consider uniform quantization. Assume message \( M \) is uniformly distributed in \((-m_{\text{max}}, m_{\text{max}})\), so \( M \) has zero mean.
- Assume \( g(\cdot) \) is symmetric. Then \( V = g(M) \) also has zero mean. Therefore, \( Q = M - V \) also has zero mean.
- If we choose \([-m_{\text{max}}, m_{\text{max}}]\) as the quantization range (full-load quantizer), then the step size of the quantizer is given by
  \[
  \Delta = \frac{2m_{\text{max}}}{L}
  \]
  where \( L \) is the total number of representation levels.

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Quantization Noise

- Quantization error is bounded by $-\Delta / 2 \leq q \leq \Delta / 2$

- If the step size $D$ is sufficiently small, it is reasonable to assume that $Q$ is a uniformly distributed random variable. That is,

$$f_q(q) = \begin{cases} \frac{1}{\Delta}, & -\Delta / 2 \leq q \leq \Delta / 2 \\ 0, & \text{otherwise} \end{cases}$$

- Its mean-square value is given by

$$\sigma_q^2 = E[Q^2] = \int_{-\Delta / 2}^{\Delta / 2} q^2 f_q(q) dq = \frac{1}{\Delta} \int_{-\Delta / 2}^{\Delta / 2} q^2 dq = \frac{\Delta^2}{12}$$

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Example: Sinusoidal Modulating Signal

- Let $m(t) = A_m \cos(2\pi f_t t)$. Then, $P = \frac{A_m^2}{2}$, $m_{\text{max}} = A_m$,

$$(SNR)_{O} = \frac{3P}{m_{\text{max}}^2} 2^{2R} = \frac{3m_{\text{max}}^2 / 2}{m_{\text{max}}^2} 2^{2R} = \frac{3}{2} 2^{2R} \Rightarrow (1.8 + 6R) \text{ dB}$$

<table>
<thead>
<tr>
<th>$L$</th>
<th>$R$</th>
<th>$(SNR)_O$</th>
</tr>
</thead>
<tbody>
<tr>
<td>32</td>
<td>5</td>
<td>31.8</td>
</tr>
<tr>
<td>64</td>
<td>6</td>
<td>37.8</td>
</tr>
<tr>
<td>128</td>
<td>7</td>
<td>43.8</td>
</tr>
<tr>
<td>256</td>
<td>8</td>
<td>49.8</td>
</tr>
</tbody>
</table>

- In this example, we assumed full-load quantizer, in which no quantization loss is encountered due to saturation.
- In general, $(SNR)_{O}$ is expressed $(a + 6R)$ dB, with $a$ depending on the waveform and the quantization load.

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Effect of Quantization Loading Level

- If we construct a binary code, and denote $R$ as the bits per samples, then

$$L = 2^R \quad R = \log_2(L)$$

- Rewrite the mean-square value of the quantization noise as

$$\sigma_q^2 = \Delta^2 / 12 = \frac{1}{12} \left( \frac{2m_{\text{max}}}{L} \right)^2 = \frac{m_{\text{max}}^2}{3L^2} = \frac{1}{3} \cdot \frac{m_{\text{max}}^2}{2^{2R}}$$

Then, the output SNR (signal to quantization noise ratio) of a uniform quantizer is given by

$$(SNR)_{O} = \frac{P}{\sigma_q^2} = \left( \frac{3P}{m_{\text{max}}^2} \right) 2^{2R}$$

- The SNR increases exponentially with the $R$:

$$(SNR)_{O} (\text{dB}) = a + 6R \text{ (the value of } a \text{ varies)}$$

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Non-Uniform Quantization

**Why:** To give more protection to weak signals

**How:** Apply compressor before quantization.

**Compression laws**

\[
|v| = \frac{\log(1 + \mu |m|)}{\log(1 + \mu)}
\]

When \(\mu|m| \ll 1, |v| \approx |m|\)

\[\log(1 + \mu |m|) \approx \mu |m|\]

When \(\mu|m| \gg 1, |v| \approx \log(\mu|m|)\)

**ITU-T G.711: PCM of Voice frequencies (1972)**

- A-law: Mainly used in Europe
  - 8-bit code represents 13-bit uniformly quantized info.
- \(\mu\)-law: Mainly used in US and Japan
  - 8-bit code represents 14-bit uniformly quantized info.

**G.711 using the \(\mu\)-law**

14-bit uniform quantization \((2^{13} = 8192)\)

**A-law**

\[
|v| = \begin{cases} 
\frac{A |m|}{1 + \log A}, & 0 \leq |m| \leq \frac{1}{A} \\
\frac{1 + \log(1 + |m|)}{1 + \log(1 + A)} & \frac{1}{A} \leq |m| \leq 1 
\end{cases}
\]

Compression: at transmitter according to the \(\mu\)-law or A-law

Expansion: at receiver to restore the signal samples to their correct relative level

[Compressor + Expander = Compander]
Encoding

**Encoding process:** translate the discrete set of sample values to a more appropriate form of signal.

**Symbol:** one discrete event in a code.

<table>
<thead>
<tr>
<th>Ordinal Number of Representation Level</th>
<th>Level Number Expresed as Sum of Powers of 2</th>
<th>Binary Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$2^0$</td>
<td>0000</td>
</tr>
<tr>
<td>1</td>
<td>$2^1$</td>
<td>0001</td>
</tr>
<tr>
<td>2</td>
<td>$2^2$</td>
<td>0010</td>
</tr>
<tr>
<td>3</td>
<td>$2^1 + 2^0$</td>
<td>0011</td>
</tr>
<tr>
<td>4</td>
<td>$2^2 + 2^0$</td>
<td>0100</td>
</tr>
<tr>
<td>5</td>
<td>$2^2 + 2^1$</td>
<td>0101</td>
</tr>
<tr>
<td>6</td>
<td>$2^2 + 2^1$</td>
<td>0110</td>
</tr>
<tr>
<td>7</td>
<td>$2^2 + 2^1 + 2^0$</td>
<td>0111</td>
</tr>
<tr>
<td>8</td>
<td>$2^3$</td>
<td>1000</td>
</tr>
<tr>
<td>9</td>
<td>$2^3 + 2^0$</td>
<td>1001</td>
</tr>
<tr>
<td>10</td>
<td>$2^3 + 2^1$</td>
<td>1010</td>
</tr>
<tr>
<td>11</td>
<td>$2^3 + 2^1 + 2^0$</td>
<td>1011</td>
</tr>
<tr>
<td>12</td>
<td>$2^3 + 2^1 + 2^0$</td>
<td>1100</td>
</tr>
<tr>
<td>13</td>
<td>$2^3 + 2^1 + 2^0$</td>
<td>1101</td>
</tr>
<tr>
<td>14</td>
<td>$2^3 + 2^1 + 2^0$</td>
<td>1110</td>
</tr>
<tr>
<td>15</td>
<td>$2^3 + 2^1 + 2^0$</td>
<td>1111</td>
</tr>
</tbody>
</table>

Gray Code

**Gray Code:** A Gray code (also known as the reflected binary code) is a way of encoding binary numbers so that only one digit changes from one number to the next. That means that a single 0 can change to a 1, or a single 1 can change to a zero.

It results in lower errors in the signal level when bit errors occur.

Frank Gray (1887-1969) was a physicist and researcher at Bell Labs who made numerous innovations in television and is remembered for the Gray code.

Differential Encoding

**Differential Encoding:** Bit streams going through many channels can be unintentionally inverted. Most circuits cannot tell if the entire stream is inverted. Differential encoding is commonly used to protect against this possibility.

Need reference bit in the beginning.

Modula 2 addition

$$0 + 0 = 1 + 1 = 0, \quad 0 + 1 = 1$$

$$e_{out} = d_{in} + e_{n-1} + 1$$

Regenerative Repeater

**Regenerative Repeater:** Ideally, except for a delay, the regenerated signal is exactly the same as the signal originally transmitted.

In practice, the regenerated signal departs from the original signal because:

1. Channel noise and interference
2. Jitter

![Regenerative Repeater Diagram](image)
**Delta Modulation**

- **Delta Modulation:**
  - Oversampling at a rate much higher than the Nyquist rate.
  - Use staircase approximation; The output only takes two values: $\Delta$ and $-\Delta$.

**Delta Modulation System**

Two types of quantization error:

- **Slope-overload distortion:** Occurs when $\Delta$ is too small. To avoid this problem, $\Delta$ must satisfy
  \[
  \frac{\Delta}{T_s} \geq \max \left| \frac{dm(t)}{dt} \right|
  \]

- **Granular noise:** Occurs when $\Delta$ is large relative to the local slope. Adaptive step size may be used.
Delta-Sigma (Δ–Σ) Modulation

- Delta modulation may be viewed as an approximation to the derivative of the message signal.
- A period of disturbances such as noise may result in accumulative error.
- Integrating the message signal prior to the delta modulation can overcome this problem.
- Delta-Sigma (Δ–Σ) modulation can be considered as a smoothed version of delta modulation:
  - Low-frequency component is pre-emphasized;
  - Correlation between adjacent samples is increased;
  - Design of the receiver is simplified.

Δ–Σ modulation is used in A/D converter.

Combining two integrators into one results in a simpler structure.

Line Codes

How to design the waveform to represent binary data?

(a) Unipolar nonreturn-to-zero (NRZ) signaling
(b) Polar NRZ signaling
(c) Unipolar return-to-zero (RZ) signaling
(d) Bipolar RZ signaling
(e) Manchester code (split-phase code)

Unipolar NRZ:
[Also referred to as on-off signaling]

Disadvantages:
- Waste of power due to the transmission of DC level (that does not carry information).
- Power spectrum near zero frequency is high.

DC (direct current): zero-frequency component.
**Line Code: Polar NRZ**

**Polar NRZ:**

**Disadvantage:**
- Power spectrum near zero frequency is high.

**Line Code: Unipolar RZ**

**Unipolar RZ:**

**Disadvantages:**
- Waste of power due to the transmission of DC level;
- Wider signal spectrum.

**Line Code: Bipolar RZ**

**Bipolar RZ:**
[Also referred to as Alternative Mark Inversion (AMI)]

**Advantages:**
- No DC component;
- Insignificant low-frequency components.

**Disadvantages:**
Long strings of zeros may cause the receivers to lose lock.

**Line Code: Manchester Code**

**Manchester Code:**

**Advantages:**
- No DC component;
- Insignificant low-frequency components.

**Disadvantage:**
- Wide spectrum.
Outlines

- Sampling Process
  - Sampling Theorem
  - Nyquist rate
- Pulse-Amplitude Modulation (PAM)
- Quantization
  - Uniform and non-uniform quantization
  - Quantization noise
- Pulse-Code Modulation (PCM)
- Delta Modulation
  - Delta-Sigma Modulation
- Line Codes

Homework

- Review Reading
  - Chapter 6
- Preview Reading
  - Chapter 7
- Homework (due 2/26 at 6:15 pm)
  - From textbook 6.2, 6.4, 6.9, 6.35, 6.36
    - Note: sinc function is defined as (Table 2.2, page 24)
    - \[ \text{sinc}(t) = \frac{\sin(\pi t)}{\pi t} \]
    - Notice “flat-top” pulses in Problem 6.4.